Ohm's Law

by Dr. Colton, Physics 442 (last updated: Winter 2021)

When electric fields act on mobile electrons in materials, the electrons do not accelerate without limit. This can be explained in a model where collisions between electrons and atoms cause a damping force on the electrons. This model is not in Griffiths, although it's related to the work leading up to Eq. 7.6 (both editions).

In this model the damping force is proportional to velocity and is characterized by a damping time which is called τ, i.e. $F_{damping} = -mv/\tau$. Then, together with the electrical force $\mathbf{F} = q\mathbf{E}$ added in, Newton's 2nd Law becomes:

$$\mathbf{F_{net}} = m\frac{d\mathbf{v}}{dt}$$

$$-e\mathbf{E} - \frac{m\mathbf{v}}{\tau} = m\frac{d\mathbf{v}}{dt}$$

Solving for the steady state velocity (i.e. when $\frac{d\mathbf{v}}{dt} = 0$), this rapidly leads to

$$\mathbf{v} = -\frac{e\tau}{m}\mathbf{E}$$

Since $\mathbf{J} = \rho\mathbf{v}$ for a moving charge density ρ, and since the charge density of a group of electrons is $\rho = -ne$ (where n is number of electrons per volume), those equations can all be put together to yield Ohm's Law in the form $\mathbf{J} = \sigma\mathbf{E}$, where the proportionality constant between current density and electric field, σ, is known as the conductivity of the material. Specifically, we have:

$$\mathbf{J} = \frac{ne^2\tau}{m}\mathbf{E}$$

In other words, the conductivity $\sigma_0 = \frac{ne^2\tau}{m}$. I have called that σ_0 instead of just σ here since there are other types of conductivities, two of which are explored in a homework problem. The quantity $\frac{ne^2\tau}{m}$ is sometimes called the "DC conductivity" of the material.