Physics 442 Formula Sheet
(11 Aug 2024 version)

Chapter 7

$c = 1/\sqrt{\mu_0 \varepsilon_0}$

$\mathbf{J}_p = \frac{\partial \mathbf{P}}{\partial t}; \quad \mathbf{J}_{tot} = \mathbf{J}_f + \mathbf{J}_b + \mathbf{J}_p$

$\nabla \times \mathbf{H} = \mathbf{J_f} + \frac{\partial \mathbf{D}}{\partial t}$

$\mathbf{J} = \sigma \mathbf{E}$

$\sigma_0 = \frac{ne^2\tau}{m}; \quad \sigma = \frac{\sigma_0}{1 - i\omega\tau}$

$M = \frac{\Phi_{loop\,2}}{I_1}; \quad L = \frac{\Phi_{loop\,1}}{I_1}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U = \int \left(\frac{\varepsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2 \right) d\tau$

$u = \frac{\varepsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2$

$u = \frac{1}{2}\mathbf{E} \cdot \mathbf{D} + \frac{1}{2}\mathbf{B} \cdot \mathbf{H}$

Chapter 8

$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$

$\mathbf{S} = \mathbf{E} \times \mathbf{H}$

$\nabla \cdot \mathbf{S} + \mathbf{E} \cdot \mathbf{J} = -\frac{\partial u}{\partial t}$

$\oint \mathbf{S} \cdot d\mathbf{a} + \frac{\partial W}{\partial t} = -\frac{\partial U}{\partial t}$

$\frac{\partial u}{\partial t} = -\nabla \cdot \mathbf{S}$ (if no $\mathbf{J}$)

mom. dens. $= \frac{1}{c^2}\mathbf{S} = \mu_0 \varepsilon_0 \mathbf{S} =$
$\quad \varepsilon_0 \mathbf{E} \times \mathbf{B}$

$T_{ij} = \varepsilon_0 \left(E_i E_j - \frac{1}{2}\delta_{ij}E^2 \right) +$
$\quad \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}\delta_{ij}B^2 \right)$

$\mathbf{F} = \oint_S \overleftrightarrow{\mathbf{T}} \cdot d\mathbf{a} - \varepsilon_0\mu_0 \frac{\partial}{\partial t}\int_V \mathbf{S}\, d\tau$

$\begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = \oint_S \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix}\begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} da$

$\quad -\varepsilon_0\mu_0 \frac{\partial}{\partial t}\int_V \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} d\tau$

$\mathbf{f} = \nabla \cdot \overleftrightarrow{\mathbf{T}} - \mu_0 \varepsilon_0 \frac{\partial \mathbf{S}}{\partial t}$

$\frac{\partial(\text{mom dens.})}{\partial t} = -\nabla \cdot \left(-\overleftrightarrow{\mathbf{T}} \right)$ (if no $\mathbf{f}$)

Chapter 9

$\frac{\partial^2 f}{dz^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$

$\nabla^2 f = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$

$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$ (free space)

$\nabla^2 \mathbf{B} - \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$ (free space)

$v = \lambda f$

$\lambda = \frac{2\pi}{k}$

$f = \frac{1}{T} = \frac{kv}{2\pi}$

$\omega = 2\pi f = kv$

$\nabla^2 \mathbf{E} = \mu\varepsilon\frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu\sigma\frac{\partial \mathbf{E}}{\partial t}$

$\nabla^2 \mathbf{B} = \mu\varepsilon\frac{\partial^2 \mathbf{B}}{\partial t^2} + \mu\sigma\frac{\partial \mathbf{B}}{\partial t}$

$\tilde{\mathbf{E}}(\mathbf{r}, t) = \tilde{\mathbf{E}}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$

$\tilde{\mathbf{E}}(\mathbf{r}, t) = \widetilde{\mathbf{E}}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}\hat{\mathbf{n}}$

$\tilde{\mathbf{B}}(\mathbf{r}, t) = \frac{1}{c}\hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r}, t)$

$\tilde{\mathbf{B}}(\mathbf{r}, t) = \frac{1}{c}\widetilde{\mathrm{E}}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}\hat{\mathbf{k}} \times \hat{\mathbf{n}}$

$\langle S \rangle = \frac{1}{2}c\varepsilon_0 E_0^2$ (free space)

$\langle S \rangle = \frac{1}{2}nc\varepsilon_0 E_0^2$ (general)

$P = \frac{1}{A}\frac{\Delta p}{\Delta t} = \frac{1}{2}\varepsilon_0 E^2 = \frac{I}{c}$

$n = \sqrt{\varepsilon_r \mu_r} \approx \sqrt{\varepsilon_r}$

$v = \frac{c}{n} = \frac{\omega}{k}$

$\theta_I = \theta_R$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\alpha = \frac{\cos\theta_T}{\cos\theta_I}$

$\beta = \frac{\mu_1 n_2}{\mu_2 n_1} \approx \frac{n_2}{n_1}$

$r = \frac{\tilde{E}_{0R}}{\tilde{E}_{0I}}$

$t = \frac{\tilde{E}_{0T}}{\tilde{E}_{0I}}$

$R = \frac{P_R}{P_I} = r^2$ ($= |r|^2$ if complex)

$T = \frac{P_T}{P_I} = \alpha\beta t^2 = 1 - R$

$r = \frac{n_1 - n_2}{n_1 + n_2}$ (normal incidence)

$t = \frac{2n_1}{n_1 + n_2}$ (normal)

$R = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2$ (normal)

$T = \frac{4n_1 n_2}{(n_1 + n_2)^2}$ (normal)

$r = \frac{\tilde{E}_{0R}}{\tilde{E}_{0I}} = \frac{\alpha - \beta}{\alpha + \beta}$ (p-polar)

$t = \frac{\tilde{E}_{0T}}{\tilde{E}_{0I}} = \frac{2}{\alpha + \beta}$ (p-polar)

$R = \left(\frac{\alpha - \beta}{\alpha + \beta} \right)^2$ (p-polar)

$T = \frac{4\alpha\beta}{(\alpha + \beta)^2}$ (p-polar)

$\tan\theta_B = \frac{n_2}{n_1}$ (p-polar)

$r = \frac{\tilde{E}_{0R}}{\tilde{E}_{0I}} = \frac{1 - \alpha\beta}{1 + \alpha\beta}$ (s-polar)

$t = \frac{\tilde{E}_{0T}}{\tilde{E}_{0I}} = \frac{2}{1 + \alpha\beta}$ (s-polar)

$R = \left(\frac{1 - \alpha\beta}{1 + \alpha\beta} \right)^2$ (s-polar)

$T = \frac{4\alpha\beta}{(1 + \alpha\beta)^2}$ (s-polar)

$\tilde{\varepsilon}_r = \tilde{n}^2; \quad \tilde{k} = \frac{\omega}{c}\tilde{n}$

$\lambda = 2\pi/k_{real}; \quad \delta = 1/k_{imag}$

$\tilde{k}^2 = \mu\varepsilon\omega^2 + i\mu\sigma\omega$

$\tilde{\varepsilon}_r = \varepsilon_{r,real} + i\frac{\sigma}{\varepsilon_0 \omega}$

Case 1: $\tilde{k} \approx \sqrt{\mu\varepsilon\omega^2}$

Case 2: $\tilde{k} \approx i\sqrt{\mu|\varepsilon|\omega^2}$

Case 3: $\tilde{k} \approx \sqrt{\frac{\mu\sigma\omega}{2}} + \sqrt{\frac{\mu\sigma\omega}{2}}\,i$

$\omega_p = \sqrt{\frac{Nq^2}{m\varepsilon_0}}$

$\tilde{\varepsilon}_r = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\omega\gamma}$ (insul.)

$\tilde{\varepsilon}_r = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\gamma}$ (conduct.)

$k = \sqrt{\frac{\omega^2}{c^2} - \frac{m^2\pi^2}{a^2} - \frac{n^2\pi^2}{b^2}}$ (rect.)

$k = \sqrt{\frac{\omega^2}{c^2} - \frac{u_{\alpha m}^2}{R^2}}$ (cyl.)