

Physics 442 Formula Sheet
(11 Aug 2024 version)

Chapter 7

$$c = 1/\sqrt{\mu_0 \epsilon_0}$$

$$\mathbf{J}_p = \frac{\partial \mathbf{P}}{\partial t}; \mathbf{J}_{\text{tot}} = \mathbf{J}_f + \mathbf{J}_b + \mathbf{J}_p$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$$

$$\mathbf{J} = \sigma \mathbf{E}$$

$$\sigma_0 = \frac{ne^2 \tau}{m}; \sigma = \frac{\sigma_0}{1 - i\omega \tau}$$

$$M = \frac{\Phi_{\text{loop}2}}{I_1}; L = \frac{\Phi_{\text{loop}1}}{I_1}$$

$$\mathcal{E} = -L \frac{dI}{dt}$$

$$U = \int \left(\frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2 \right) d\tau$$

$$u = \frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2$$

$$u = \frac{1}{2} \mathbf{E} \cdot \mathbf{D} + \frac{1}{2} \mathbf{B} \cdot \mathbf{H}$$

Chapter 8

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$$

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

$$\nabla \cdot \mathbf{S} + \mathbf{E} \cdot \mathbf{J} = -\frac{\partial u}{\partial t}$$

$$\oint \mathbf{S} \cdot d\mathbf{a} + \frac{\partial W}{\partial t} = -\frac{\partial U}{\partial t}$$

$$\frac{\partial u}{\partial t} = -\nabla \cdot \mathbf{S} \text{ (if no } \mathbf{J})$$

$$\text{mom. dens.} = \frac{1}{c^2} \mathbf{S} = \mu_0 \epsilon_0 \mathbf{S} = \epsilon_0 \mathbf{E} \times \mathbf{B}$$

$$T_{ij} = \epsilon_0 \left(E_i E_j - \frac{1}{2} \delta_{ij} E^2 \right) + \frac{1}{\mu_0} \left(B_i B_j - \frac{1}{2} \delta_{ij} B^2 \right)$$

$$\mathbf{F} = \oint_S \vec{\mathbf{T}} \cdot d\mathbf{a} - \epsilon_0 \mu_0 \frac{\partial}{\partial t} \int_V \mathbf{S} d\tau$$

$$\begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = \oint_S \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix} \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} da$$

$$- \epsilon_0 \mu_0 \frac{\partial}{\partial t} \int_V \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} d\tau$$

$$\mathbf{f} = \nabla \cdot \vec{\mathbf{T}} - \mu_0 \epsilon_0 \frac{\partial \mathbf{S}}{\partial t}$$

$$\frac{\partial(\text{mom dens.})}{\partial t} = -\nabla \cdot (-\vec{\mathbf{T}}) \text{ (if no } \mathbf{f})$$

Chapter 9

$$\frac{\partial^2 f}{dz^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

$$\nabla^2 f = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \text{ (free space)}$$

$$\nabla^2 \mathbf{B} - \frac{1}{c^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0 \text{ (free space)}$$

$$v = \lambda f$$

$$\lambda = \frac{2\pi}{k}$$

$$f = \frac{1}{T} = \frac{kv}{2\pi}$$

$$\omega = 2\pi f = kv$$

$$\nabla^2 \mathbf{E} = \mu \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu \sigma \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla^2 \mathbf{B} = \mu \epsilon \frac{\partial^2 \mathbf{B}}{\partial t^2} + \mu \sigma \frac{\partial \mathbf{B}}{\partial t}$$

$$\vec{\mathbf{E}}(\mathbf{r}, t) = \vec{\mathbf{E}}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

$$\vec{\mathbf{E}}(\mathbf{r}, t) = \vec{\mathbf{E}}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} \hat{\mathbf{n}}$$

$$\vec{\mathbf{B}}(\mathbf{r}, t) = \frac{1}{c} \hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r}, t)$$

$$\vec{\mathbf{B}}(\mathbf{r}, t) = \frac{1}{c} \vec{\mathbf{E}}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} \hat{\mathbf{k}} \times \hat{\mathbf{n}}$$

$$\langle S \rangle = \frac{1}{2} c \epsilon_0 E_0^2 \text{ (free space)}$$

$$\langle S \rangle = \frac{1}{2} n c \epsilon_0 E_0^2 \text{ (general)}$$

$$P = \frac{1}{A} \frac{\Delta p}{\Delta t} = \frac{1}{2} \epsilon_0 E^2 = \frac{I}{c}$$

$$n = \sqrt{\epsilon_r \mu_r} \approx \sqrt{\epsilon_r}$$

$$v = \frac{c}{n} = \frac{\omega}{k}$$

$$\theta_l = \theta_R$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\alpha = \frac{\cos \theta_l}{\cos \theta_T}$$

$$\beta = \frac{\mu_1 n_1}{\mu_2 n_2} \approx \frac{n_2}{n_1}$$

$$r = \frac{\vec{E}_{0R}}{\vec{E}_{0I}}$$

$$t = \frac{\vec{E}_{0T}}{\vec{E}_{0I}}$$

$$R = \frac{P_R}{P_I} = r^2 \text{ (= } |r|^2 \text{ if complex)}$$

$$T = \frac{P_T}{P_I} = \alpha \beta t^2 = 1 - R$$

$$r = \frac{n_1 - n_2}{n_1 + n_2} \text{ (normal incidence)}$$

$$t = \frac{2n_1}{n_1 + n_2} \text{ (normal)}$$

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2 \text{ (normal)}$$

$$T = \frac{4n_1 n_2}{(n_1 + n_2)^2} \text{ (normal)}$$

$$r = \frac{\vec{E}_{0R}}{\vec{E}_{0I}} = \frac{\alpha - \beta}{\alpha + \beta} \text{ (p-polar)}$$

$$t = \frac{\vec{E}_{0T}}{\vec{E}_{0I}} = \frac{2}{\alpha + \beta} \text{ (p-polar)}$$

$$R = \left(\frac{\alpha - \beta}{\alpha + \beta} \right)^2 \text{ (p-polar)}$$

$$T = \frac{4\alpha\beta}{(\alpha + \beta)^2} \text{ (p-polar)}$$

$$\tan \theta_B = \frac{n_2}{n_1} \text{ (p-polar)}$$

$$r = \frac{\vec{E}_{0R}}{\vec{E}_{0I}} = \frac{1 - \alpha\beta}{1 + \alpha\beta} \text{ (s-polar)}$$

$$t = \frac{\vec{E}_{0T}}{\vec{E}_{0I}} = \frac{2}{1 + \alpha\beta} \text{ (s-polar)}$$

$$R = \left(\frac{1 - \alpha\beta}{1 + \alpha\beta} \right)^2 \text{ (s-polar)}$$

$$T = \frac{4\alpha\beta}{(1 + \alpha\beta)^2} \text{ (s-polar)}$$

$$\tilde{\epsilon}_r = \tilde{n}^2; \tilde{k} = \frac{\omega}{c} \tilde{n}$$

$$\lambda = 2\pi/k_{\text{real}}; \delta = 1/k_{\text{imag}}$$

$$\tilde{k}^2 = \mu \epsilon \omega^2 + i \mu \sigma \omega$$

$$\tilde{\epsilon}_r = \epsilon_{r,\text{real}} + i \frac{\sigma}{\epsilon_0 \omega}$$

$$\text{Case 1: } \tilde{k} \approx \sqrt{\mu \epsilon \omega^2}$$

$$\text{Case 2: } \tilde{k} \approx i \sqrt{\mu |\epsilon| \omega^2}$$

$$\text{Case 3: } \tilde{k} \approx \sqrt{\frac{\mu \sigma \omega}{2}} + \sqrt{\frac{\mu \sigma \omega}{2}} i$$

$$\omega_p = \sqrt{\frac{Nq^2}{m\epsilon_0}}$$

$$\tilde{\epsilon}_r = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\omega\gamma} \text{ (insul.)}$$

$$\tilde{\epsilon}_r = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\gamma} \text{ (conduct.)}$$

$$k = \sqrt{\frac{\omega^2}{c^2} - \frac{m^2 \pi^2}{a^2} - \frac{n^2 \pi^2}{b^2}} \text{ (rect.)}$$

$$k = \sqrt{\frac{\omega^2}{c^2} - \frac{u_{am}^2}{R^2}} \text{ (cyl.)}$$

(u_{am} is Bessel function zero for TM, Bessel function derivative zero for TE)

$$u'_z = \frac{u_z}{\gamma(1 - v u_x/c^2)}$$

(prime frame is moving to the right)

Chapter 10

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{A}' = \mathbf{A} + \nabla \lambda$$

$$V' = V - \frac{\partial \lambda}{\partial t}$$

$$\nabla \cdot \mathbf{A} = 0 \text{ (Coulomb gauge)}$$

$$\nabla^2 V = -\rho/\epsilon_0 \text{ (Coulomb)}$$

$$\nabla^2 \mathbf{A} - \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} (\nabla V) \text{ (Coulomb)}$$

$$\nabla \cdot \mathbf{A} = -\frac{1}{c^2} \frac{\partial V}{\partial t} \text{ (Lorentz gauge)}$$

$$\nabla^2 V - \mu_0 \epsilon_0 \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon_0} \text{ (Lorentz)}$$

$$\nabla^2 \mathbf{A} - \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J} \text{ (Loren.)}$$

$$t_r = t - \frac{z}{c}$$

$$V(\mathbf{r}, t) = \frac{1}{4\pi \epsilon_0} \int \frac{\rho(\mathbf{r}', t_r)}{z} d\tau'$$

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}', t_r)}{z} d\tau'$$

$$V(\mathbf{r}, t) = \frac{1}{4\pi \epsilon_0} \frac{qc}{(2c - \mathbf{z} \cdot \mathbf{v})} \text{ (L-W)}$$

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mathbf{v}}{c^2} V(\mathbf{r}, t) \text{ (L-W)}$$

$$\mathbf{u} = c \hat{\mathbf{z}} - \mathbf{v}$$

$$\mathbf{E}(\mathbf{r}, t) = \frac{qz}{4\pi \epsilon_0 (\mathbf{z} \cdot \mathbf{u})^3} [(c^2 - v^2) \mathbf{u} + \mathbf{z} \times (\mathbf{u} \times \mathbf{a})]$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{1}{c} \hat{\mathbf{z}} \times \mathbf{E}(\mathbf{r}, t)$$

$$\mathbf{E} = \frac{q}{4\pi \epsilon_0} \frac{1 - v^2/c^2}{(1 - \frac{v^2}{c^2} \sin^2 \theta)^{3/2}} \frac{\hat{\mathbf{R}}}{R^2}$$

(constant velocity; \mathbf{R} = from *current* location of charge to field point, θ = angle between \mathbf{v} and \mathbf{R})

$$\mathbf{B}(\mathbf{r}, t) = \frac{1}{c^2} \mathbf{v} \times \mathbf{E}$$

Chapter 11

$$P = \int \langle \mathbf{S} \rangle \cdot d\mathbf{a}$$

$$\langle \mathbf{S} \rangle \sim \frac{\sin^2 \theta}{r^2}; P = \frac{\mu_0 p_0^2 \omega^4}{12\pi c} \text{ (electric dipole)}$$

$$\langle \mathbf{S} \rangle \sim \frac{\sin^2 \theta}{r^2}; P = \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3} \text{ (magnetic dipole)}$$

$$P = \frac{\mu_0 \dot{p}^2}{6\pi c} \text{ (arb. localized source)}$$

$$\langle \mathbf{S} \rangle \sim \frac{\sin^2 \theta}{r^2}; P = \frac{\mu_0 q^2 a^2}{6\pi c} \text{ (Larmor)}$$

$$\frac{dP}{d\Omega} \sim \frac{\sin^2 \theta}{(1 - \beta \cos \theta)^5}; P = \frac{\mu_0 q^2 \gamma^6}{6\pi c} \left(a^2 - \left| \frac{\mathbf{v} \times \mathbf{a}}{c} \right|^2 \right) \text{ (Liénard)}$$

$$\mathbf{F}_{r.r.} = \frac{\mu_0 q^2 \dot{a}}{6\pi c} \text{ (Abraham-Lorentz)}$$

Chapter 12

$$\beta = v/c$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

$$\beta_{13} = \frac{\beta_{12} + \beta_{23}}{1 + \beta_{12}\beta_{23}}$$

$$u'_x = \frac{u_x - v}{1 - v u_x/c^2}$$

$$u'_y = \frac{u_y}{\gamma(1 - v u_x/c^2)}$$