Physics 442 Formula Sheet
(27 Jun 2024 version)

Chapter 7

$\mathbf{J}_p = \frac{\partial \mathbf{P}}{\partial t}$

$\mathbf{J_{tot}} = \mathbf{J}_f + \mathbf{J}_b + \mathbf{J}_p$

$\nabla \times \mathbf{H} = \mathbf{J_f} + \frac{\partial \mathbf{D}}{\partial t}$

$\mathbf{J} = \sigma \mathbf{E}$

$\sigma = \frac{ne^2\tau}{m}$

$M_{12} = \frac{\Phi_{loop\ 2}}{I_1}$

$L = \frac{\Phi_{loop\ 1}}{I_1}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U = \int \left(\frac{\epsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2\right) d\tau$

$u = \frac{\epsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2$

$u = \frac{1}{2}\mathbf{E}\cdot\mathbf{D} + \frac{1}{2}\mathbf{B}\cdot\mathbf{H}$

Chapter 8

$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}$

$\mathbf{S} = \mathbf{E}\times\mathbf{H}$

$\nabla\cdot\mathbf{S} + \mathbf{E}\cdot\mathbf{J} = -\frac{\partial u}{\partial t}$

$\oint \mathbf{S}\cdot d\mathbf{a} + \frac{\partial W}{\partial t} = -\frac{\partial U}{\partial t}$

$\frac{\partial u}{\partial t} = -\nabla\cdot\mathbf{S}$ (if no $\mathbf{J}$)

$\mathbf{g} = \frac{1}{c^2}\mathbf{S} = \mu_0\epsilon_0\mathbf{S} = \epsilon_0\mathbf{E}\times\mathbf{B}$

$T_{ij} = \epsilon_0\left(E_iE_j - \frac{1}{2}\delta_{ij}E^2\right) + \frac{1}{\mu_0}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right)$

$\mathbf{F} = \oint_S \overleftrightarrow{\mathbf{T}}\cdot d\mathbf{a} - \epsilon_0\mu_0\frac{\partial}{\partial t}\int_V \mathbf{S}\, d\tau$

$\begin{pmatrix}F_x\\F_y\\F_z\end{pmatrix} = \oint_S \begin{pmatrix}T_{xx} & T_{xy} & T_{xz}\\ T_{yx} & T_{yy} & T_{yz}\\ T_{zx} & T_{zy} & T_{zz}\end{pmatrix}\begin{pmatrix}n_x\\n_y\\n_z\end{pmatrix} da - \epsilon_0\mu_0\frac{\partial}{\partial t}\int_V \begin{pmatrix}S_x\\S_y\\S_z\end{pmatrix} d\tau$

$\mathbf{f} = \nabla\cdot\overleftrightarrow{\mathbf{T}} - \mu_0\epsilon_0\frac{\partial \mathbf{S}}{\partial t}$

$\frac{\partial \mathbf{g}}{\partial t} = -\nabla\cdot\left(-\overleftrightarrow{\mathbf{T}}\right)$ (if no $\mathbf{f}$)

Chapter 9

$\frac{\partial^2 f}{dz^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$

$\nabla^2 f = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$

$c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$

$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = 0$ (free space)

$\nabla^2\mathbf{B} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} = 0$ (free space)

$v = \lambda f$

$\lambda = \frac{2\pi}{k}$

$f = \frac{1}{T} = \frac{kv}{2\pi}$

$\omega = 2\pi f = kv$

$\nabla^2\mathbf{E} = \mu\epsilon\frac{\partial^2\mathbf{E}}{\partial t^2} + \mu\sigma\frac{\partial\mathbf{E}}{\partial t}$

$\nabla^2\mathbf{B} = \mu\epsilon\frac{\partial^2\mathbf{B}}{\partial t^2} + \mu\sigma\frac{\partial\mathbf{B}}{\partial t}$

$\tilde{\mathbf{E}}(\mathbf{r},t) = \tilde{\mathbf{E}}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

$\tilde{\mathbf{E}}(\mathbf{r},t) = \tilde{\mathrm{E}}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\hat{\mathbf{n}}$

$\tilde{\mathbf{B}}(\mathbf{r},t) = \frac{1}{c}\hat{\mathbf{k}}\times\mathbf{E}(\mathbf{r},t)$

$\tilde{\mathbf{B}}(\mathbf{r},t) = \frac{1}{c}\tilde{\mathrm{E}}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\hat{\mathbf{k}}\times\hat{\mathbf{n}}$

$\langle S\rangle = \frac{1}{2}c\epsilon_0E_0^2$ (free space)

$\langle S\rangle = \frac{1}{2}nc\epsilon_0E_0^2$

$P = \frac{1}{A}\frac{\Delta p}{\Delta t} = \frac{1}{2}\epsilon_0E^2 = \frac{I}{c}$

$n = \sqrt{\epsilon_r\mu_r} \approx \sqrt{\epsilon_r}$

$v = \frac{c}{n} = \frac{\omega}{k}$

$\theta_I = \theta_R$

$n_1\sin\theta_1 = n_2\sin\theta_2$

$\alpha = \frac{\cos\theta_T}{\cos\theta_I}$

$\beta = \frac{\mu_1 n_2}{\mu_2 n_1} \approx \frac{n_2}{n_1}$

$r = \frac{\tilde{E}_{0R}}{\tilde{E}_{0I}}$

$t = \frac{\tilde{E}_{0T}}{\tilde{E}_{0I}}$

$R = \frac{P_R}{P_I} = r^2$

$T = \frac{P_T}{P_I} = \alpha\beta t^2 = 1 - R$

$r = \frac{n_1-n_2}{n_1+n_2}$ (normal incidence)

$t = \frac{2n_1}{n_1+n_2}$ (normal)

$R = \left(\frac{n_1-n_2}{n_1+n_2}\right)^2$ (normal)

$T = \frac{4n_1n_2}{(n_1+n_2)^2}$ (normal)

$r = \frac{\tilde{E}_{0R}}{\tilde{E}_{0I}} = \frac{\alpha-\beta}{\alpha+\beta}$ (p-polar)

$t = \frac{\tilde{E}_{0T}}{\tilde{E}_{0I}} = \frac{2}{\alpha+\beta}$ (p-polar)

$R = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)^2$ (p-polar)

$T = \frac{4\alpha\beta}{(\alpha+\beta)^2}$ (p-polar)

$\tan\theta_B = \frac{n_2}{n_1}$

$r = \frac{\tilde{E}_{0R}}{\tilde{E}_{0I}} = \frac{1-\alpha\beta}{1+\alpha\beta}$ (s-polar)

$t = \frac{\tilde{E}_{0T}}{\tilde{E}_{0I}} = \frac{2}{1+\alpha\beta}$ (s-polar)

$R = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)^2$ (s-polar)

$T = \frac{4\alpha\beta}{(1+\alpha\beta)^2}$ (s-polar)

$\tilde{\epsilon}_r = \tilde{n}^2$

$\frac{\tilde{k}}{\omega} = \frac{\tilde{n}}{c}$

$\tilde{k}^2 = \mu\epsilon\omega^2 + i\mu\sigma\omega$

$\delta = \frac{1}{k_{imag}}$

$\tilde{\epsilon}_r = \epsilon_{r,real} + i\frac{\sigma}{\epsilon_0\omega}$

$\omega_p = \sqrt{\frac{Nq^2}{m\epsilon_0}}$

$\tilde{\chi} = \frac{qE_0}{m}\frac{1}{\omega_0^2-\omega^2-i\omega\gamma}$

$\tilde{\epsilon}_r = 1 + \frac{\omega_p^2}{\omega_0^2-\omega^2-i\omega\gamma}$

$\tilde{\epsilon}_r = 1 - \frac{\omega_p^2}{\omega^2+i\omega\gamma}$ (conductor)

$k = \sqrt{\frac{\omega^2}{c^2} - \frac{m^2\pi^2}{a^2} - \frac{n^2\pi^2}{b^2}}$ (rect)

$k = \sqrt{\frac{\omega^2}{c^2} - \frac{u_{\alpha m}^2}{R^2}}$ (cylindrical)

($u_{\alpha m}$ is Bessel function zero for TM, Bessel function derivative zero for TE)

Chapter 10

$\mathbf{E} = -\nabla V - \frac{\partial\mathbf{A}}{\partial t}$

$\mathbf{B} = \nabla\times\mathbf{A}$

$\mathbf{A}' = \mathbf{A} + \nabla\lambda$

$V' = V - \frac{\partial\lambda}{\partial t}$

$\nabla\cdot\mathbf{A} = 0$ (Coulomb gauge)

$\nabla^2 V = -\rho/\epsilon_0$ (Coulomb)

$\nabla^2\mathbf{A} - \mu_0\epsilon_0\frac{\partial^2\mathbf{A}}{\partial t^2} = -\mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial}{\partial t}(\nabla V)$ (Coulomb)

$\nabla\cdot\mathbf{A} = -\frac{1}{c^2}\frac{\partial V}{\partial t}$ (Lorentz gauge)

$\nabla^2 V - \mu_0\epsilon_0\frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon_0}$ (Lorentz)

$\nabla^2\mathbf{A} - \mu_0\epsilon_0\frac{\partial^2\mathbf{A}}{\partial t^2} = -\mu_0\mathbf{J}$ (Loren.)

$t_r = t - \frac{\mathscr{r}}{c}$

$V(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\mathbf{r},t_r)}{\mathscr{r}}d\tau'$

$\mathbf{A}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r},t_r)}{\mathscr{r}}d\tau'$

$V(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\frac{qc}{(\mathscr{r}c - \boldsymbol{\mathscr{r}}\cdot\mathbf{v})}$ (L-W)

$\mathbf{A}(\mathbf{r},t) = \frac{\mathbf{v}}{c^2}V(\mathbf{r},t)$ (L-W)

$\mathbf{u} = c\hat{\boldsymbol{\mathscr{r}}} - \mathbf{v}$

$\mathbf{E}(\mathbf{r},t) = \frac{q\mathscr{r}}{4\pi\epsilon_0(\boldsymbol{\mathscr{r}}\cdot\mathbf{u})^3}[(c^2 - v^2)\mathbf{u} + \boldsymbol{\mathscr{r}}\times(\mathbf{u}\times\mathbf{a})]$

$\mathbf{B}(\mathbf{r},t) = \frac{1}{c}\hat{\boldsymbol{\mathscr{r}}}\times\mathbf{E}(\mathbf{r},t)$

$\mathbf{E} = \frac{q}{4\pi\epsilon_0}\frac{1-v^2/c^2}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2}}\frac{\hat{\mathbf{R}}}{R^2}$

(constant velocity; $\mathbf{R}$ = from *current* location of charge to field point, θ = angle between $\mathbf{v}$ and $\mathbf{R}$)

$\mathbf{B}(\mathbf{r},t) = \frac{1}{c^2}\mathbf{v}\times\mathbf{E}$

Chapter 11

$P = \int\langle\mathbf{S}\rangle\cdot d\mathbf{a}$

$\langle\mathbf{S}\rangle \sim \frac{\sin^2\theta}{r^2}$; $P = \frac{\mu_0 p_0^2\omega^4}{12\pi c}$ (electric dipole)

$\langle\mathbf{S}\rangle \sim \frac{\sin^2\theta}{r^2}$; $P = \frac{\mu_0 m_0^2\omega^4}{12\pi c^3}$ (magnetic dipole)

$P = \frac{\mu_0\ddot{p}^2}{6\pi c}$ (arb. localized source)

$\langle\mathbf{S}\rangle \sim \frac{\sin^2\theta}{\mathscr{r}^2}$; $P = \frac{\mu_0 q^2a^2}{6\pi c}$ (Larmor)

$\frac{dP}{d\Omega} \sim \frac{\sin^2\theta}{(1-\beta\cos\theta)^5}$; $P = \frac{\mu_0q^2\gamma^6}{6\pi c}\left(a^2 - \left|\frac{\mathbf{v}\times\mathbf{a}}{c}\right|^2\right)$ (Liénard)

$\mathbf{F}_{\mathrm{rr}} = \frac{\mu_0q^2\dot{\mathbf{a}}}{6\pi c}$ (Abraham-Lorentz)

Chapter 12

$\beta = v/c$

$\gamma = \frac{1}{\sqrt{1-\beta^2}}$

$\beta_{13} = \frac{\beta_{12}+\beta_{23}}{1+\beta_{12}\beta_{23}}$

$u_x' = \frac{u_x - v}{1 - vu_x/c^2}$

$u_y' = \frac{u_y}{\gamma(1-vu_x/c^2)}$

$u_z' = \frac{u_z}{\gamma(1-vu_x/c^2)}$

(prime frame is moving to the right)

$\Delta t_2 = \frac{1}{\gamma}\Delta t_1$

$\Delta x_2 = \gamma\Delta x_1$

$\begin{pmatrix}ct\\x\end{pmatrix}_2 = \begin{pmatrix}\gamma & \pm\gamma\beta\\ \pm\gamma\beta & \gamma\end{pmatrix}\begin{pmatrix}ct\\x\end{pmatrix}_1$

$\boldsymbol{\Lambda} = \Lambda^\mu_\nu = \begin{pmatrix}\gamma & \pm\gamma\beta & 0 & 0\\ \pm\gamma\beta & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$

$x^\mu = \Lambda^\mu_\nu x^\nu$ (gen. vector transf.)

$x^\mu = \begin{pmatrix}ct\\x\\y\\z\end{pmatrix}$

$\eta^\mu = \begin{pmatrix}\gamma_u c\\ \gamma_u u_x\\ \gamma_u u_y\\ \gamma_u u_z\end{pmatrix}$

$E = \gamma mc^2$

$\mathbf{p} = \gamma m\mathbf{u}$

$p^\mu = \begin{pmatrix}E/c\\p_x\\p_y\\p_z\end{pmatrix}$

$a^\mu = \begin{pmatrix}\gamma_u{}^4\frac{\mathbf{u}\cdot\mathbf{a}}{a}\\ \gamma_u{}^2\left(a_x + \frac{u_x^2a_x}{c^2-u^2}\right)\\ \gamma_u{}^2\left(a_y + \frac{u_y^2a_y}{c^2-u^2}\right)\\ \gamma_u{}^2\left(a_z + \frac{u_z^2a_z}{c^2-u^2}\right)\end{pmatrix}$

$K^\mu = ma^\mu$

$A^\mu = \begin{pmatrix}V/c\\A_x\\A_y\\A_z\end{pmatrix}$

$\mathbf{J} = \rho\mathbf{u}$

$J^\mu = \begin{pmatrix}\rho c\\J_x\\J_y\\J_z\end{pmatrix}$

$a_\mu = (-a^0, a^1, a^2, a^3)$ (covar.)

$E^2 = (pc)^2 + (mc^2)^2$

$E_x' = E_x$

$E_y' = \gamma(E_y - vB_z)$

$E_z' = \gamma(E_z + vB_y)$

$B_x' = B_x$

$B_y' = \gamma\left(B_y + \frac{v}{c^2}E_z\right)$

$B_z' = \gamma\left(B_z - \frac{v}{c^2}E_y\right)$

(prime frame is moving to the right)

$F^{\mu\nu} = \Lambda^\mu_\lambda\Lambda^\nu_\sigma F^{\lambda\sigma}$ (tensor transf.)

$F^{\mu\nu} = \begin{pmatrix}0 & E_x/c & E_y/c & E_z/c\\ -E_x/c & 0 & B_z & -B_y\\ -E_y/c & -B_z & 0 & B_x\\ -E_z/c & B_y & -B_x & 0\end{pmatrix}$

$G^{\mu\nu} = \begin{pmatrix}0 & B_x & B_y & B_z\\ -B_x & 0 & -E_z/c & E_y/c\\ -B_y & E_z/c & 0 & -E_x/c\\ -B_z & -E_y & E_x/c & 0\end{pmatrix}$

$$T^{\mu\nu} = \begin{pmatrix} u & S_x/c & S_y/c & S_z/c \\ S_x/c & -T_{1,1} & -T_{1,2} & -T_{1,3} \\ S_y/c & -T_{2,1} & -T_{2,2} & -T_{2,3} \\ S_z/c & -T_{3,1} & -T_{3,2} & -T_{3,3} \end{pmatrix}$$

$\frac{\partial F^{\mu\nu}}{\partial x^{\nu}} = \mu_0 J^{\mu}$ (two of Maxwell)

$\frac{\partial G^{\mu\nu}}{\partial x^{\nu}} = 0$ (other two of Maxwell)

$K^{\mu} = q\eta_{\nu} F^{\mu\nu}$ (Lorentz force)

$F^{\mu\nu} = \frac{\partial A^{\nu}}{\partial x^{\mu}} - \frac{\partial A^{\mu}}{\partial x^{\nu}}$ (**E**,**B** from V,**A**)

$\frac{\partial A^{\mu}}{\partial x^{\mu}} = 0$ (Lorentz gauge)

$\Box^2 A^{\mu} = -\mu_0 J^{\mu}$ (V,**A** from ρ,**J**)

$\frac{\partial J^{\mu}}{\partial x^{\mu}} = 0$ (Eq of cont. for charge)

$\frac{\partial T^{\mu\nu}}{\partial x^{\mu}} = 0$ (Eq. of cont. for E & **p**)