Parallel Equations for the Electric and Magnetic Fields

by Dr. Colton, Spring 2016 (based on a document by former student Gus Borstad)

Electric

1. $\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$
2. $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
3. $\nabla \cdot \mathbf{D} = \rho_f$
4. $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$
5. $\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}$
6. $\mathbf{D} = \varepsilon \mathbf{E}$
7. $\mathbf{F} = Q\mathbf{E}$
8. $U = \frac{\varepsilon_0}{2} \int E^2 d\tau$
9. $\mathbf{F} = \frac{1}{4\pi\varepsilon_0} \frac{qQ}{|\mathbf{r}-\mathbf{r}'|^3} (\mathbf{r} - \mathbf{r}')$
10. $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{|\mathbf{r}-\mathbf{r}'|^3} (\mathbf{r} - \mathbf{r}')$

    $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda(\mathbf{r}') dl'}{|\mathbf{r}-\mathbf{r}'|^3} (\mathbf{r} - \mathbf{r}')$

    $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int_S \frac{\sigma(\mathbf{r}') da'}{|\mathbf{r}-\mathbf{r}'|^3} (\mathbf{r} - \mathbf{r}')$

    $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho(\mathbf{r}') d\tau'}{|\mathbf{r}-\mathbf{r}'|^3} (\mathbf{r} - \mathbf{r}')$
11. $\Phi_E = \int_S \mathbf{E} \cdot d\mathbf{a}$
12. $\oint_S \mathbf{E} \cdot d\mathbf{a} = \frac{Q_{enc}}{\varepsilon_0}$
13. $V(\mathbf{r}) = -\int_{\mathbf{ref}}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l}$
14. $\mathbf{E} = -\nabla V$
15. $\nabla^2 V = -\frac{\rho}{\varepsilon_0}$
16. $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} dl'$

    $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} da'$

    $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\tau'$
17. $E_1^{\perp} - E_2^{\perp} = \frac{\sigma}{\varepsilon_0}$
18. $\mathbf{E}_1^{\parallel} = \mathbf{E}_2^{\parallel}$
19. $V_1 = V_2$
20. $C = \frac{Q}{V}$
21. $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int (r')^n P_n(\cos\theta') \rho(\mathbf{r}') d\tau'$

Magnetic

1. $\nabla \cdot \mathbf{B} = 0$
2. $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$
3. $\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$
4. $\mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}$
5. $\mathbf{M} = \chi_m \mathbf{H}$
6. $\mathbf{H} = \mathbf{B}/\mu$
7. $\mathbf{F} = Q(\mathbf{v} \times \mathbf{B})$
8. $U = \frac{1}{2\mu_0} \int B^2 d\tau$
9. $\mathbf{F}_{\text{mag}} = \int (\mathbf{I} \times \mathbf{B}) dl$, combined with $\mathbf{B}(\mathbf{r})$ equation from 10 (the **I**'s are different).
10. $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{I} \times (\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} dl'$

    $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{K}(\mathbf{r}') \times (\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} da'$

    $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}') \times (\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} dl'$
11. $\Phi_B = \int_S \mathbf{B} \cdot d\mathbf{a}$
12. $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}$
13. No parallel for the magnetic field, but the electric field equation is important
14. $\mathbf{B} = \nabla \times \mathbf{A}$, $\nabla \cdot \mathbf{A} = 0$
15. $\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$
16. $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{I}}{|\mathbf{r}-\mathbf{r}'|} dl'$

    $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{K}}{|\mathbf{r}-\mathbf{r}'|} da'$

    $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\tau'$
17. $B_1^{\perp} = B_2^{\perp}$
18. $\mathbf{B}_1^{\parallel} - \mathbf{B}_2^{\parallel} = \mu_0 \mathbf{K}$
19. $\mathbf{A}_1 = \mathbf{A}_2$
20. $L = \frac{\Phi}{I}$
21. $\mathbf{A}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \oint (r')^n P_n(\cos\theta') d\mathbf{l}'$

22. $\mathbf{p} = \int \mathbf{r}'\rho(\mathbf{r}')d\tau'$
23. $V_{\text{dip}} = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{\mathrm{r}^2}$
24. $\mathbf{E}_{\text{dip}}(r,\theta) = \frac{p}{4\pi\varepsilon_0 r^3}(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\theta})$
25. $\mathbf{N} = \mathbf{p} \times \mathbf{E}$
26. $\mathbf{F} = (\mathbf{p}\cdot\nabla)\mathbf{E}$
27. $U = -\mathbf{p}\cdot\mathbf{E}$
28. $\mathbf{P}$ = dipole moment per unit volume

29. $\sigma_b = \mathbf{P}\cdot\hat{\mathbf{n}}$
30. $\rho_b = -\nabla\cdot\mathbf{P}$
31. $\oint \mathbf{D}\cdot d\mathbf{a} = Q_{f_{enc}}$
32. $\varepsilon_r = 1 + \chi_e = \frac{\varepsilon}{\varepsilon_0}$
33. $U = \frac{1}{2}\int \mathbf{D}\cdot\mathbf{E}d\tau$
34. $U = \frac{1}{2}\frac{Q^2}{C}$
35. $q = \int \lambda dl$
$q = \int \sigma da$
$q = \int \rho d\tau$

22. $\mathbf{m} = I\int d\mathbf{a} = I\mathbf{a}$
23. $\mathbf{A}_{\text{dip}} = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\mathbf{r}}}{\boldsymbol{r}^2}$
24. $\mathbf{B}_{\text{dip}}(r,\theta) = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\theta})$
25. $\mathbf{N} = \mathbf{m}\times\mathbf{B}$
26. $\mathbf{F} = \nabla(\mathbf{m}\cdot\mathbf{B})$
27. $U = -\mathbf{m}\cdot\mathbf{B}$
28. $\mathbf{M}$ = magnetic dipole moment per unit volume

29. $\mathbf{J_b} = \nabla\times\mathbf{M}$
30. $\mathbf{K_b} = \boldsymbol{M}\times\hat{\mathbf{n}}$
31. $\oint \mathbf{H}\cdot d\mathbf{l} = I_{f_{enc}}$
32. $\mu_r = 1 + \chi_m = \frac{\mu}{\mu_0}$
33. $U = \frac{1}{2}\int \mathbf{H}\cdot\mathbf{B}d\tau$
34. $U = \frac{1}{2}LI^2$
35. $\sum_{i=1}^{n}(\ )q_i\mathbf{v}_i \sim \int_{\text{line}}(\ )\mathbf{I}dl$
$\sum_{i=1}^{n}(\ )q_i\mathbf{v}_i \sim \int_{\text{surface}}(\ )\mathbf{K}da$
$\sum_{i=1}^{n}(\ )q_i\mathbf{v}_i \sim \int_{\text{volume}}(\ )\mathbf{J}d\tau$