E_z component of TM Modes of a Cylindrical Waveguide, by Dr. Colton
Physics 442, Summer 2016

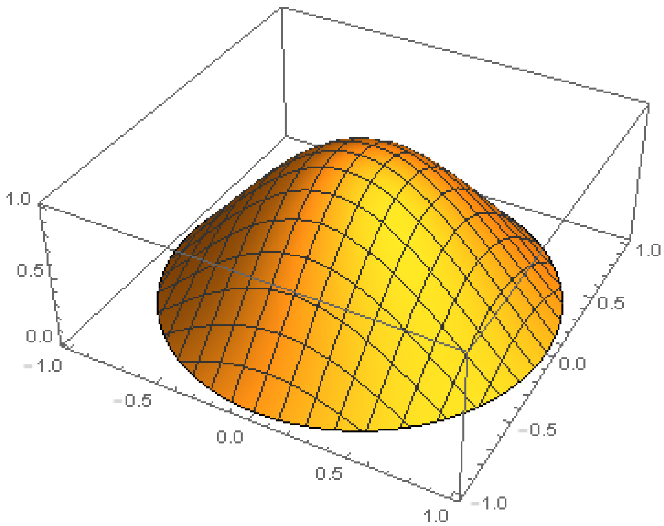
```
In[1]:= uαm[α_, m_] = BesselJZero[α, m];

In[2]:= f01[x_, y_] = BesselJ[0, Sqrt[x^2 + y^2] uαm[0, 1]] // N
        Plot3D[f01[x, y], {x, -1, 1}, {y, -1, 1},
         RegionFunction → Function[{x, y, z}, x^2 + y^2 < 1]]
```

This is $Ez = J_0\left(\frac{u_{01}s}{R}\right)$, with $R = 1$ and $s = \sqrt{x^2+y^2}$.

Out[2]= $\text{BesselJ}\left[0., 2.40483\sqrt{x^2+y^2}\right]$

Out[3]=

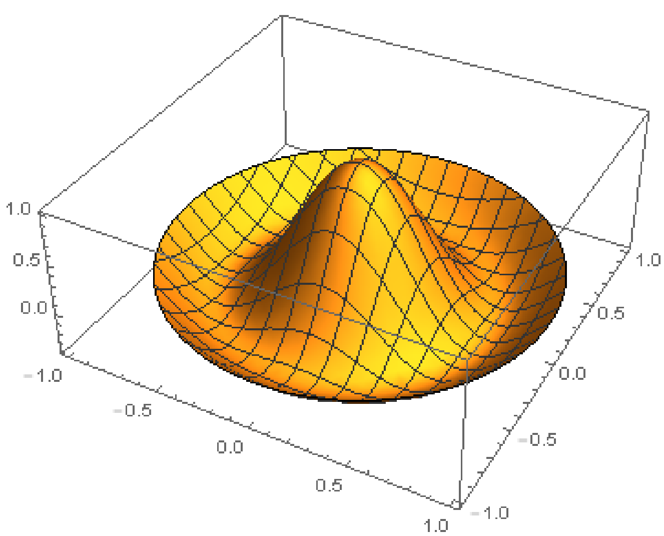


TM$_{01}$

```
In[4]:= f02[x_, y_] = BesselJ[0, Sqrt[x^2 + y^2] uαm[0, 2]] // N
        Plot3D[f02[x, y], {x, -1, 1}, {y, -1, 1},
         RegionFunction → Function[{x, y, z}, x^2 + y^2 < 1]]
```

Out[4]= $\text{BesselJ}\left[0., 5.52008\sqrt{x^2+y^2}\right]$

Out[5]=

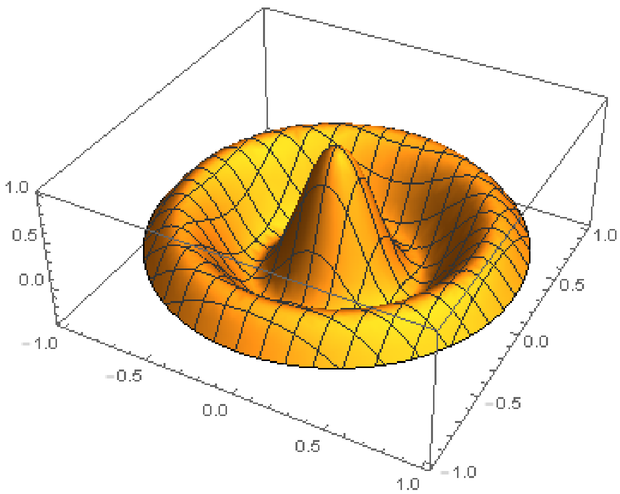


TM$_{02}$

```
In[6]:= f03[x_, y_] = BesselJ[0, Sqrt[x^2 + y^2] uαm[0, 3]] // N
        Plot3D[f03[x, y], {x, -1, 1}, {y, -1, 1},
         RegionFunction → Function[{x, y, z}, x^2 + y^2 < 1]]
```

Out[6]= BesselJ$\left[0., 8.65373 \sqrt{x^2 + y^2}\right]$

Out[7]=



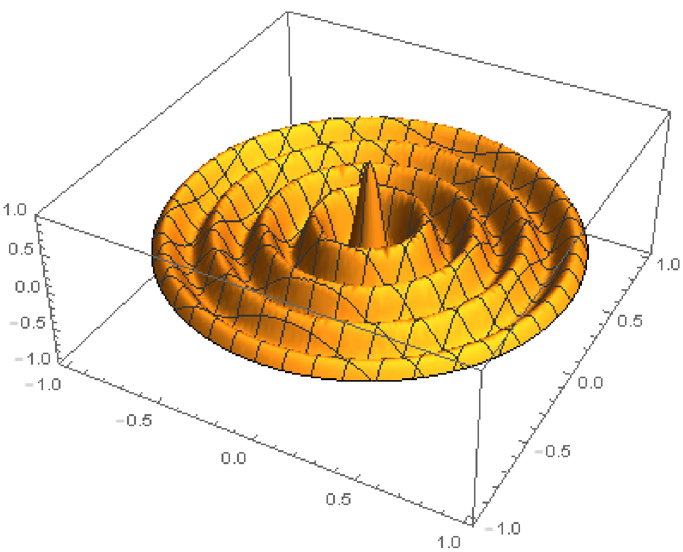
TM_{03}

```
In[8]:= f09[x_, y_] = BesselJ[0, Sqrt[x^2 + y^2] uαm[0, 9]] // N
        Plot3D[f09[x, y], {x, -1, 1}, {y, -1, 1} , PlotRange → {-1, 1},
         PlotPoints → 30,
         RegionFunction → Function[{x, y, z}, x^2 + y^2 < 1]]
```

(Skipping a few, over to mode 09.)

Out[8]= BesselJ$\left[0., 27.4935 \sqrt{x^2 + y^2}\right]$

Out[9]=



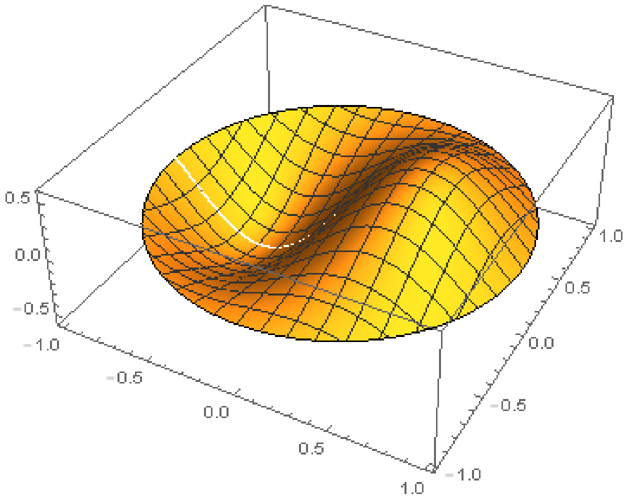
TM_{09}

This is $Ez = J_1\left(\frac{u_{11}s}{R}\right)\cos\phi$. It's the first mode with ϕ dependence. I used the "Arg" function as a trick to get ϕ. Often we say $\phi = \tan^{-1}\left(\frac{y}{x}\right)$, but the tan[-1] function has problems in the 2nd and 3rd quadrants. Arg (the polar angle of a complex number) doesn't suffer from those issues.

```
In[10]:= f11[x_, y_] =
          BesselJ[1, Sqrt[x^2 + y^2] uαm[1, 1]] Cos[Arg[x + I y]] // N
         Plot3D[f11[x, y], {x, -1, 1}, {y, -1, 1},
          RegionFunction → Function[{x, y, z}, x^2 + y^2 < 1]]

Out[10]= BesselJ[1., 3.83171 √(x² + y²)] Cos[Arg[x + (0. + 1. i) y]]
```

Out[11]=

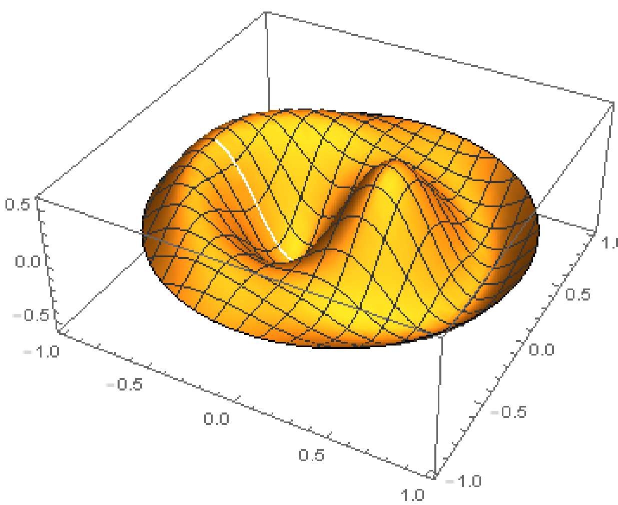


TM$_{11}$

```
In[12]:= f12[x_, y_] =
          BesselJ[1, Sqrt[x^2 + y^2] uαm[1, 2]] Cos[Arg[x + I y]] // N
         Plot3D[f12[x, y], {x, -1, 1}, {y, -1, 1},
          RegionFunction → Function[{x, y, z}, x^2 + y^2 < 1]]

Out[12]= BesselJ[1., 7.01559 √(x² + y²)] Cos[Arg[x + (0. + 1. i) y]]
```

Out[13]=

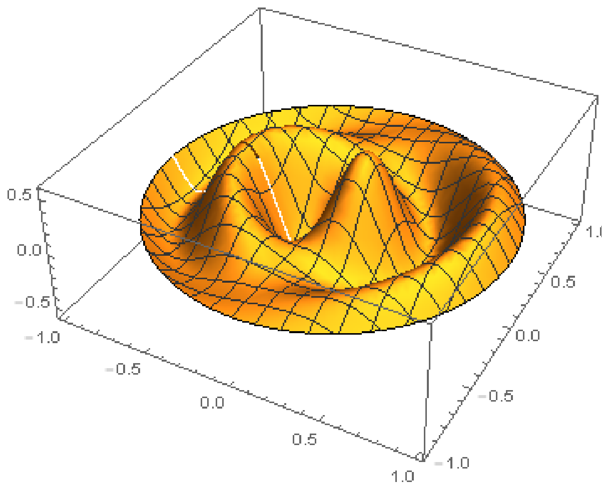


TM$_{12}$

In[14]:=
```
f13[x_, y_] =
 BesselJ[1, Sqrt[x^2 + y^2] uαm[1, 3]] Cos[Arg[x + I y]] // N
Plot3D[f13[x, y], {x, -1, 1}, {y, -1, 1},
 RegionFunction → Function[{x, y, z}, x^2 + y^2 < 1]]
```

Out[14]= $\text{BesselJ}\left[1.,\ 10.1735\sqrt{x^2+y^2}\right]\ \text{Cos}[\text{Arg}[x+(0.+1.\,i)\ y]]$

Out[15]=



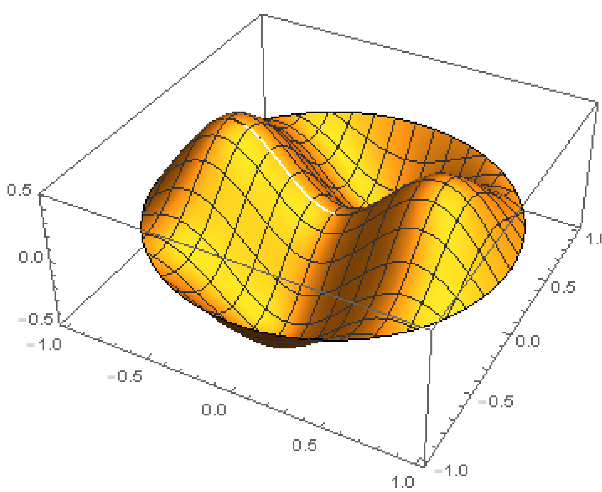
TM_{13}

In[16]:=
```
f21[x_, y_] =
 BesselJ[2, Sqrt[x^2 + y^2] uαm[2, 1]] Cos[2 Arg[x + I y]]  // N
Plot3D[f21[x, y], {x, -1, 1}, {y, -1, 1},
 RegionFunction → Function[{x, y, z}, x^2 + y^2 < 1]]
```

This is $E_z = J_2\left(\frac{u_{21}s}{R}\right)\cos 2\phi$.

Out[16]= $\text{BesselJ}\left[2.,\ 5.13562\sqrt{x^2+y^2}\right]\ \text{Cos}[2.\ \text{Arg}[x+(0.+1.\,i)\ y]]$

Out[17]=

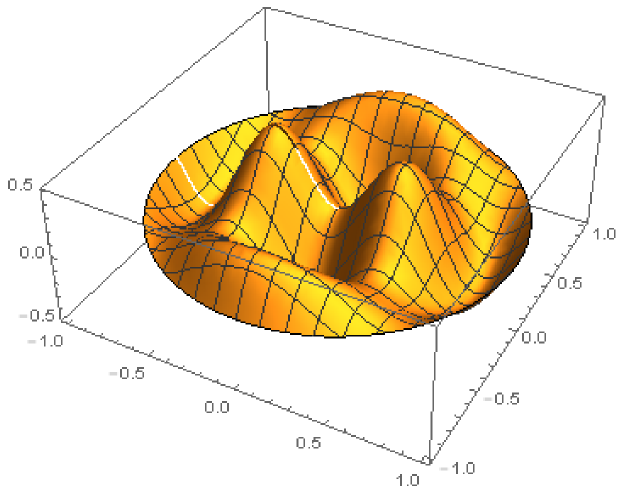


TM_{21}

In[18]:=
```
f22[x_, y_] =
 BesselJ[2, Sqrt[x^2 + y^2] uαm[2, 2]] Cos[2 Arg[x + I y]] // N
Plot3D[f22[x, y], {x, -1, 1}, {y, -1, 1},
 RegionFunction → Function[{x, y, z}, x^2 + y^2 < 1]]
```

Out[18]= BesselJ$\left[2., 8.41724 \sqrt{x^2 + y^2}\right]$ Cos[2. Arg[x + (0. + 1. i) y]]

Out[19]=



TM_{22}