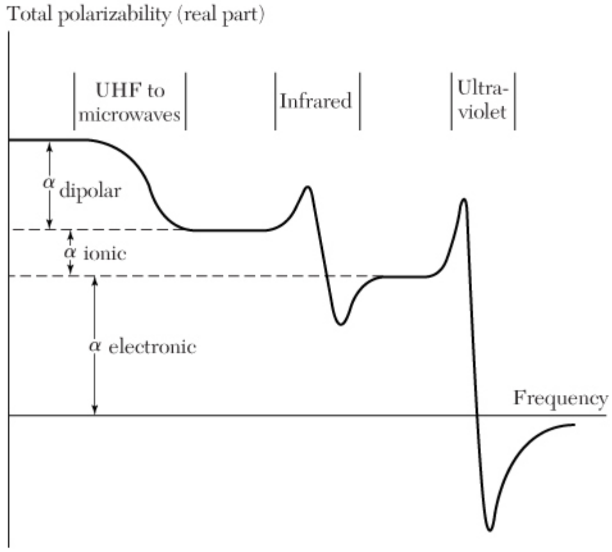
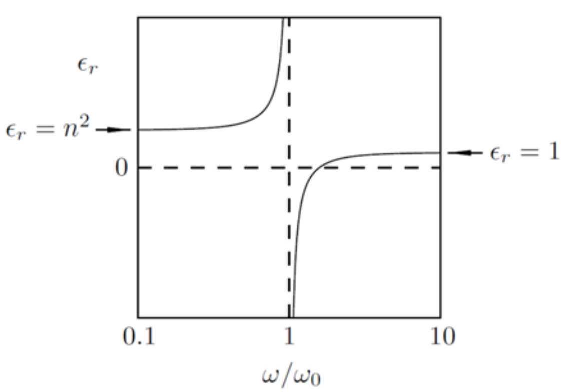
The Power of the Lorentz Model, by Dr. Colton, Winter 2018

$$\epsilon_r = 1 + (stuff)\sum_j \frac{f_j}{\omega_{0j}^2 - \omega^2 - i\gamma_j\omega}$$

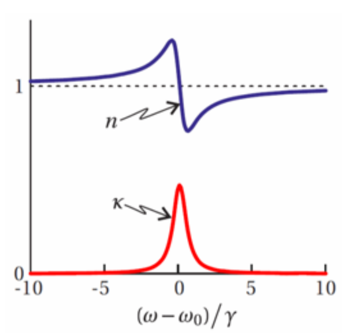


Kittel, *Introduction to Solid State Physics*, Fig. 16-8. Schematic of the frequency dependence of the several contributions to the polarizability.

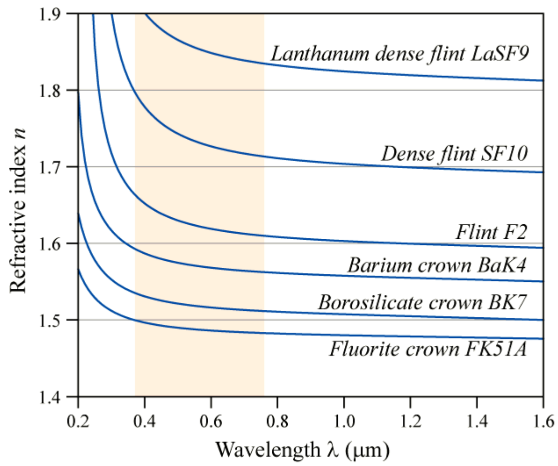
<u>**UV resonances: oscillation of electrons**</u>



Stokes, *Solid State Physics for Advanced Undergraduate Students*, Fig. 16-5. The dielectric constant near the resonant frequency of the orbital electrons in a solid, using the Lorentz model but with no damping. He's using the letter n to refer to the low frequency value of the square root of ϵ_r. The frequency scale is logarithmic.

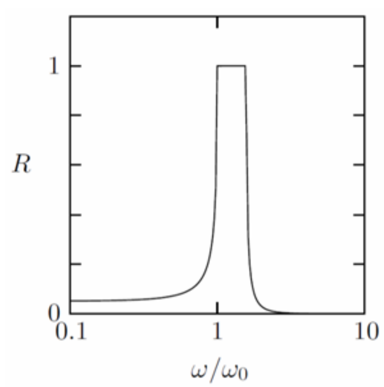


Peatross and Ware, *Physics of Light and Optics*, Fig. 2.5. Real and imaginary parts of the index of refraction for a single Lorentz oscillator dielectric with $\omega_p = 10\gamma$. [Note: this makes it look like n goes to 1 on the far left of the graph. That is not correct; n goes to a constant which is greater than 1.]



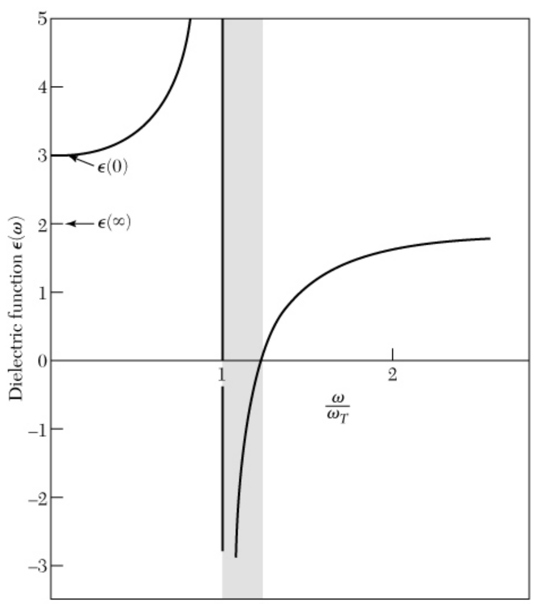
Wikipedia, *Dispersion (optics)*

From the section of n vs ω plot on previous page that is just to the left of ω_0 (plotted vs λ instead of ω)



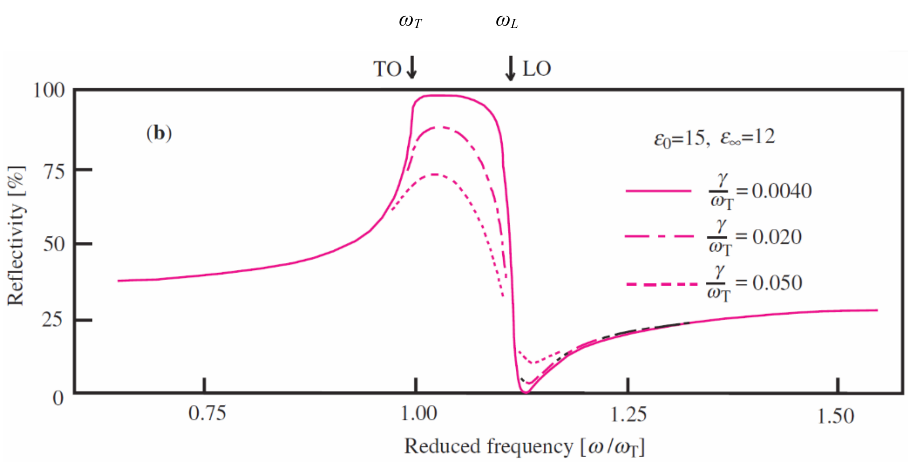
Stokes, Fig. 16-6. The reflectivity near the resonant frequency of the orbital electrons in a solid. (No damping.) From these equations: $\tilde{n} = \sqrt{\tilde{\epsilon}_r}$ and $R = \left|\frac{1-\tilde{n}}{1+\tilde{n}}\right|^2$ (i.e., normal incidence, going from air to material).

<u>**Infrared resonances: oscillation of ions (in ionic or partially ionic materials)**</u>

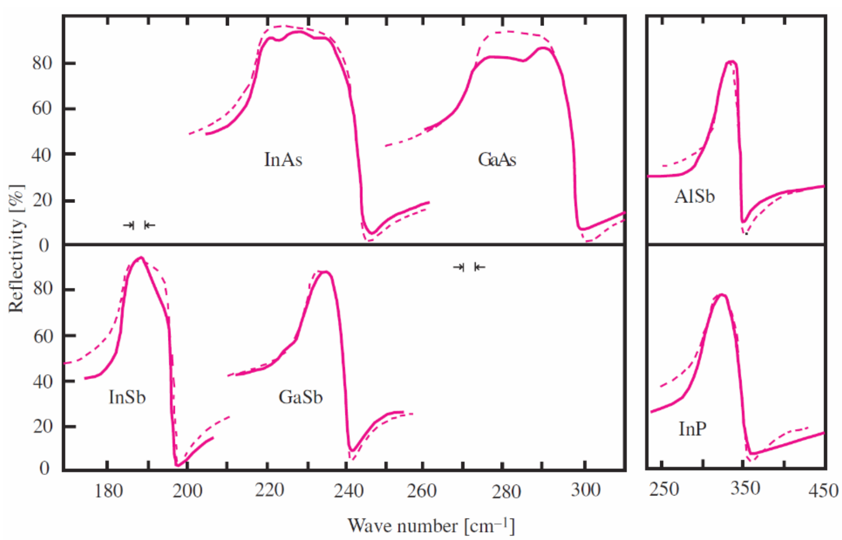


(a)

Kittel, Fig. 14.13a. The dielectric constant near the resonant frequency of the oscillating ions in a solid, using the Lorentz model with no damping. Note that the right hand side doesn't go to 1 because this feature is to the left (lower energy) of the "oscillating electrons" UV feature. Plotted here with $\varepsilon(\infty) = 2$ and $\varepsilon(0) = 3$. ω_T is defined to be the frequency where ω crosses from positive infinity to negative infinity. ω_L is defined to be the frequency where ϵ_r goes positive again. Electromagnetic waves with frequencies in the shaded region, between ω_T and ω_L, will not propagate in the medium but instead will be reflected at the boundary just like the Stokes figure for electron resonances directly above.



Yu & Cardona, *Fundamentals of Semiconductors*, Fig. 6.31(b). The infrared reflectivity from ionic oscillations, calculated from the Lorentz model with dispersion for different values of γ/ω_T; compare to the Stokes Fig. 16-6 above.



Yu & Cardona, Fig. 6.32. Actual experimental reflectivity data (solid curves), and theoretical fits from the Lorentz model (dashed curves) using ω_T, ω_L, and γ as fitting parameters.

Note on units: As you can see, the *x*-axis is labeled “Wave number [cm^{-1}]". CAUTION: that is not what we’ve been calling the wavenumber! The wavenumber k, as we’ve been using it, would be labeled as rad/cm. By contrast, when you see experimental data that is labeled “cm^{-1}”, particularly with optical data like this, they nearly always mean 1/λ instead of 2π/λ. (In the “olden days” k was defined as 1/λ, and this has persisted in some settings even today.) So, if you see a feature on a graph like this at, say, 185 cm^{-1}, you can convert it to regular wavelength like this:

185 cm^{-1} = 18500 m^{-1} $\rightarrow$ take inverse, λ = 5.405e-5 meters ≈ 54 µm

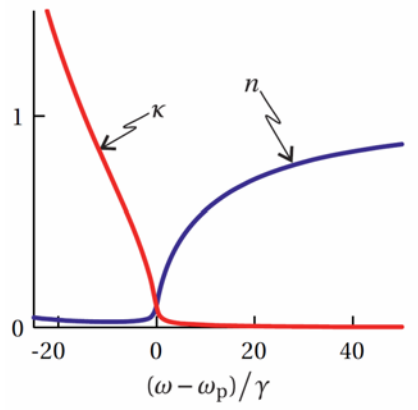
From there you can convert to frequency if you want, using $\lambda f = c$, or angular frequency using $\omega = 2\pi f$.

<u>The Lorentz Model Applied to Metals</u>

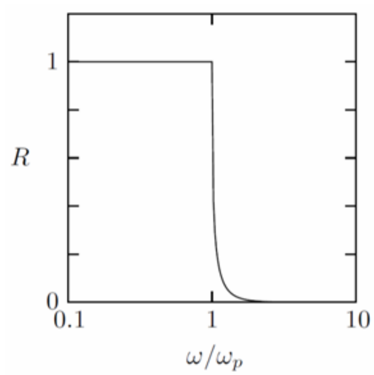
Throw out our previous complicated dispersion relationship for conductors, and just use the Lorentz model! No restoring force because electrons are free to move means

$$\epsilon_r = 1 + \frac{stuff}{-\omega^2 - i\gamma\omega}$$

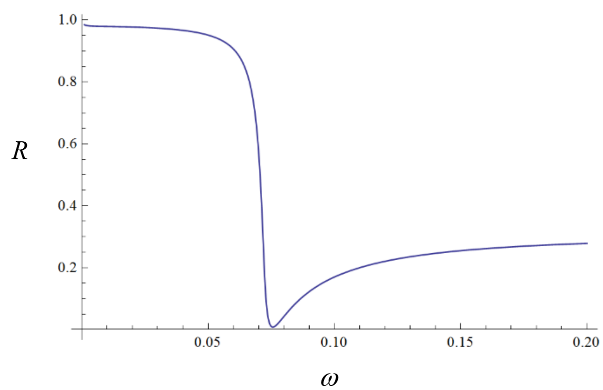
The “stuff” in the numerator has units of ω^2, and its square root is called the plasma frequency, symbol ω_p.



Peatross and Ware, Fig. 2.6. Real and imaginary parts of the index for a conductor with $\omega_p = 50\gamma$.



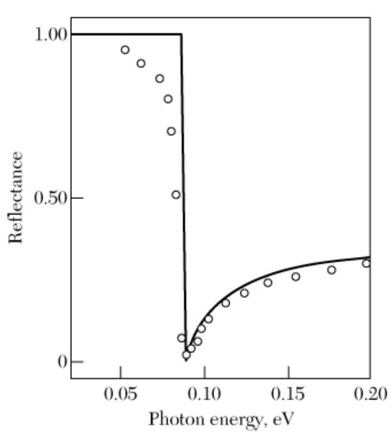
Stokes, Fig. 16-10. The reflectivity of metal near its plasma frequency. The frequency scale is logarithmic. [No damping.]



Colton Plot. Reflectivity of a metal, using the above equation for ϵ_r, with damping and with one additional modification. To account for large ω values of ϵ_r not going to one, we change the 1 in our equation to a different constant:

$$\epsilon_r = C + \frac{\omega_p^2}{-\omega^2 - i\gamma\omega}$$

Here I've used $C = 12$, $\omega_p = 0.25$, and $\gamma = \omega_p/100$.



Kittel, Fig. 14.3. Experimental reflectance of indium antimonide (empty points), fitted with the Lorentz model with no damping (solid line). My plot directly above with damping makes it look like they should have used damping for their fit to smooth out the sharp features in the solid line.