TE Modes of a Rectangular Waveguide – Dr Colton, Winter 2018

```
a = 0.10;
b = 0.07;
wcutoff[m_, n_] := 3*^8 Sqrt[(m Pi / a) ^2 + (n Pi / b) ^2]
cutofftable = Table[wcutoff[m, n], {m, 0, 3}, {n, 0, 3}] ;
cutofftable // MatrixForm
cutofftable  // Flatten // Sort
```

I'm using dimensions of $a = 10$ cm, $b = 7$ cm (chosen arbitrarily).

$$\begin{pmatrix} 0. & 1.3464\times10^{10} & 2.69279\times10^{10} & 4.03919\times10^{10} \\ 9.42478\times10^{9} & 1.64349\times10^{10} & 2.85296\times10^{10} & 4.14769\times10^{10} \\ 1.88496\times10^{10} & 2.31643\times10^{10} & 3.28697\times10^{10} & 4.45737\times10^{10} \\ 2.82743\times10^{10} & 3.13164\times10^{10} & 3.90455\times10^{10} & 4.93046\times10^{10} \end{pmatrix}$$

These are the cutoff frequencies of the first 15 modes (ignore the 0 one); first in table form…

$$\{0.,\ 9.42478\times10^{9},\ 1.3464\times10^{10},\ 1.64349\times10^{10},\ 1.88496\times10^{10},\ 2.31643\times10^{10},\ 2.69279\times10^{10},\ 2.82743\times10^{10},\ 2.85296\times10^{10},\ 3.13164\times10^{10},\ 3.28697\times10^{10},\ 3.90455\times10^{10},\ 4.03919\times10^{10},\ 4.14769\times10^{10},\ 4.45737\times10^{10},\ 4.93046\times10^{10}\}$$

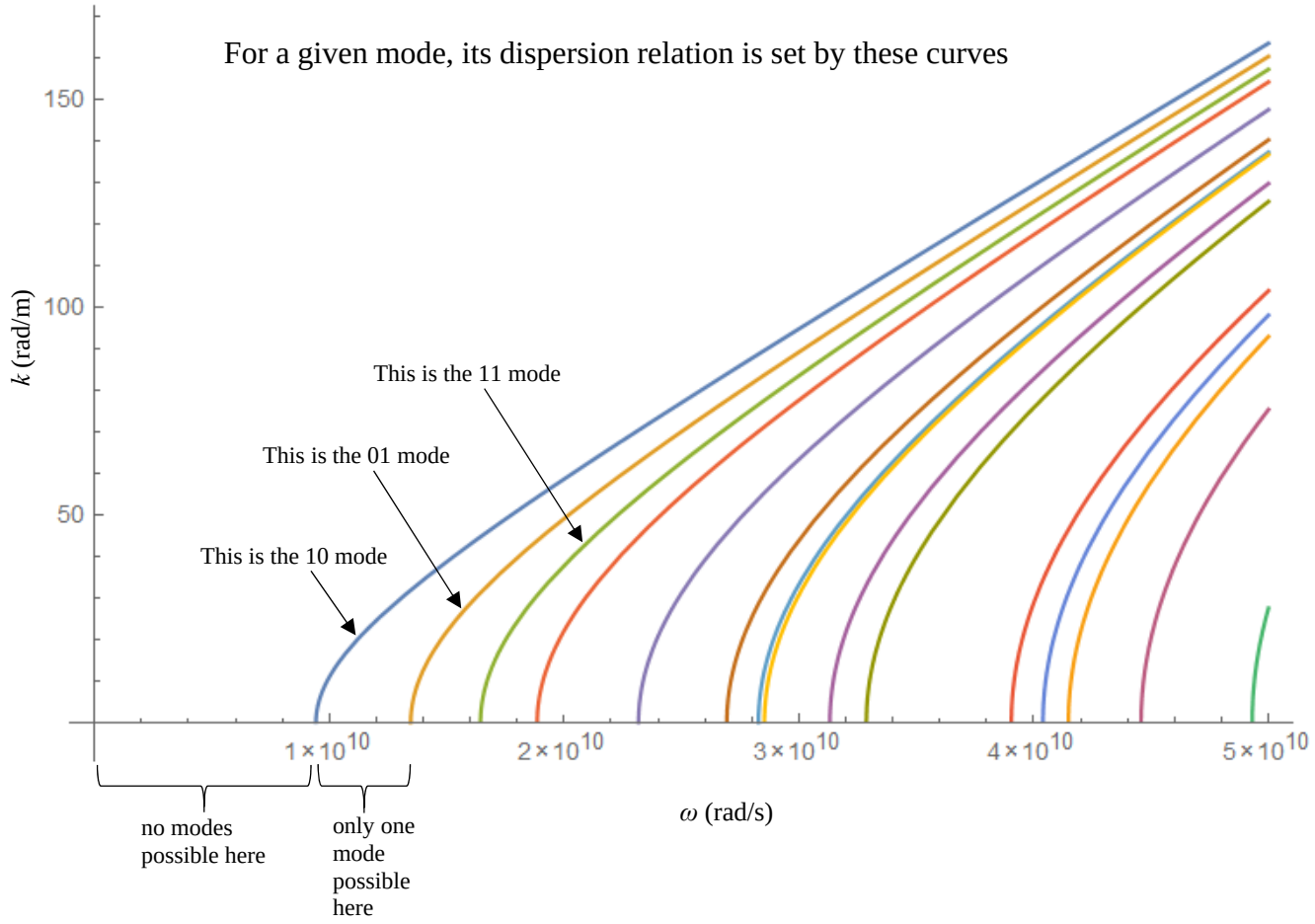
…and then in list form

These are the $k(\omega)$ dispersion relations for the first 15 modes.

```
c = 3*^8;
k[w_, m_, n_] := Sqrt[w^2 / c^2 - Pi^2 m^2 / a^2 - Pi^2 n^2 / b^2]
Table[k[w, m, n], {m, 0, 3}, {n, 0, 3}] // Flatten // Drop[#, 1] & // Sort // Reverse
Plot[%, {w, 0, 5*^10}]
```

$$\left\{\sqrt{-986.96+\frac{w^2}{90\,000\,000\,000\,000\,000}},\ \sqrt{-2014.2+\frac{w^2}{90\,000\,000\,000\,000\,000}},\ \sqrt{-3001.17+\frac{w^2}{90\,000\,000\,000\,000\,000}},\ \sqrt{-3947.84+\frac{w^2}{90\,000\,000\,000\,000\,000}},\ \sqrt{-5962.05+\frac{w^2}{90\,000\,000\,000\,000\,000}},\ \sqrt{-8056.82+\frac{w^2}{90\,000\,000\,000\,000\,000}},\ \sqrt{-8882.64+\frac{w^2}{90\,000\,000\,000\,000\,000}},\ \sqrt{-9043.78+\frac{w^2}{90\,000\,000\,000\,000\,000}},\ \sqrt{-10\,896.8+\frac{w^2}{90\,000\,000\,000\,000\,000}},\ \sqrt{-12\,004.7+\frac{w^2}{90\,000\,000\,000\,000\,000}},\ \sqrt{-16\,939.5+\frac{w^2}{90\,000\,000\,000\,000\,000}},\ \sqrt{-18\,127.8+\frac{w^2}{90\,000\,000\,000\,000\,000}},\ \sqrt{-19\,114.8+\frac{w^2}{90\,000\,000\,000\,000\,000}},\ \sqrt{-22\,075.7+\frac{w^2}{90\,000\,000\,000\,000\,000}},\ \sqrt{-27\,010.5+\frac{w^2}{90\,000\,000\,000\,000\,000}}\right\}$$

(Plot is on next page.)


For a given mode, its dispersion relation is set by these curves
150
100
50
k (rad/m)
This is the 11 mode
This is the 01 mode
This is the 10 mode
1×10^10
2×10^10
3×10^10
4×10^10
5×10^10
ω (rad/s)
no modes possible here
only one mode possible here

Pictures of B_z for the first 15 modes (ignore the upper left one).
Tan = positive antinode, blue = negative antinode

```
Table[DensityPlot[Cos[m Pi x / a] Cos[n Pi y / b] , {x, 0, 0.10}, {y, 0, 0.07}], {m, 0, 3}, {n, 0, 3}] //
 TableForm
```