TE Modes of a Rectangular Waveguide

by Dr. Colton, Physics 442 (last updated: Winter 2020)

Calculating TE Modes

Using Mathematica, we can calculate the first 15 TE modes for a rectangular waveguide. I'm using dimensions of $a = 10$ cm and $b = 7$ cm, which were chosen arbitrarily.

Here are the cutoff frequencies of the first 15 modes (ignore the 0 frequency); they are shown first in table form and then in list form in ascending order.

```
a = 0.10;
b = 0.07;
wcutoff[m_, n_] := 3*^8 Sqrt[(m Pi / a) ^2 + (n Pi / b) ^2]
cutofftable = Table[wcutoff[m, n], {m, 0, 3}, {n, 0, 3}] ;
cutofftable // MatrixForm
cutofftable  // Flatten // Sort
```

$$\begin{pmatrix} 0. & 1.3464\times10^{10} & 2.69279\times10^{10} & 4.03919\times10^{10} \\ 9.42478\times10^{9} & 1.64349\times10^{10} & 2.85296\times10^{10} & 4.14769\times10^{10} \\ 1.88496\times10^{10} & 2.31643\times10^{10} & 3.28697\times10^{10} & 4.45737\times10^{10} \\ 2.82743\times10^{10} & 3.13164\times10^{10} & 3.90455\times10^{10} & 4.93046\times10^{10} \end{pmatrix}$$

$$\{0., 9.42478\times10^{9}, 1.3464\times10^{10}, 1.64349\times10^{10}, 1.88496\times10^{10}, 2.31643\times10^{10}, 2.69279\times10^{10}, 2.82743\times10^{10}, 2.85296\times10^{10}, 3.13164\times10^{10}, 3.28697\times10^{10}, 3.90455\times10^{10}, 4.03919\times10^{10}, 4.14769\times10^{10}, 4.45737\times10^{10}, 4.93046\times10^{10}\}$$

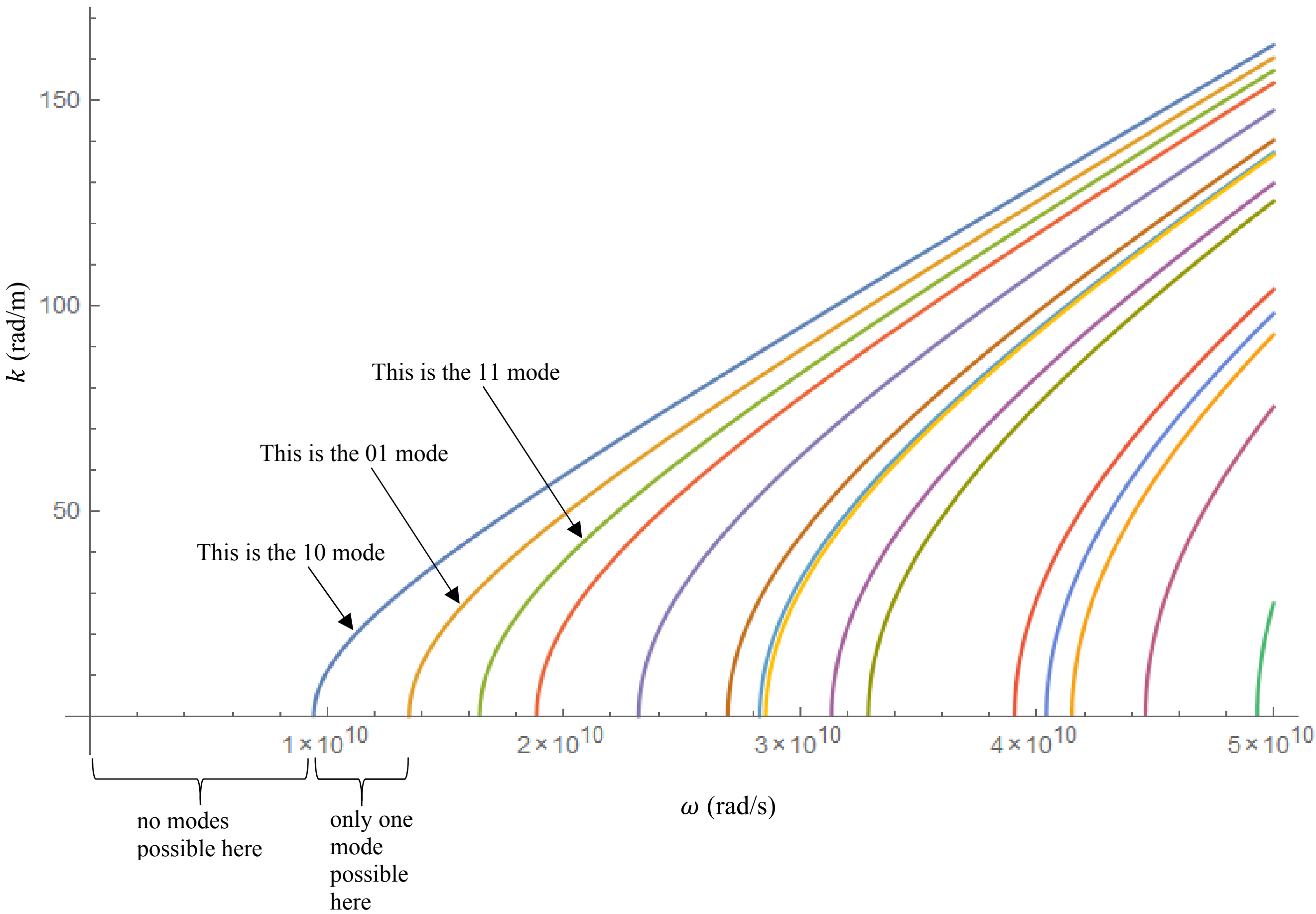
The $k(\omega)$ dispersion relations for the first 15 modes are as follows:

```
c = 3*^8;
k[w_, m_, n_] := Sqrt[w^2 / c^2 - Pi^2 m^2 / a^2 - Pi^2 n^2 / b^2]
Table[k[w, m, n], {m, 0, 3}, {n, 0, 3}] // Flatten // Drop[#, 1] & // Sort // Reverse
Plot[%, {w, 0, 5*^10}]
```

$$\left\{\sqrt{-986.96 + \frac{w^2}{90\,000\,000\,000\,000\,000}}, \sqrt{-2014.2 + \frac{w^2}{90\,000\,000\,000\,000\,000}}, \sqrt{-3001.17 + \frac{w^2}{90\,000\,000\,000\,000\,000}},\right.$$
$$\sqrt{-3947.84 + \frac{w^2}{90\,000\,000\,000\,000\,000}}, \sqrt{-5962.05 + \frac{w^2}{90\,000\,000\,000\,000\,000}}, \sqrt{-8056.82 + \frac{w^2}{90\,000\,000\,000\,000\,000}},$$
$$\sqrt{-8882.64 + \frac{w^2}{90\,000\,000\,000\,000\,000}}, \sqrt{-9043.78 + \frac{w^2}{90\,000\,000\,000\,000\,000}}, \sqrt{-10\,896.8 + \frac{w^2}{90\,000\,000\,000\,000\,000}},$$
$$\sqrt{-12\,004.7 + \frac{w^2}{90\,000\,000\,000\,000\,000}}, \sqrt{-16\,939.5 + \frac{w^2}{90\,000\,000\,000\,000\,000}}, \sqrt{-18\,127.8 + \frac{w^2}{90\,000\,000\,000\,000\,000}},$$
$$\left.\sqrt{-19\,114.8 + \frac{w^2}{90\,000\,000\,000\,000\,000}}, \sqrt{-22\,075.7 + \frac{w^2}{90\,000\,000\,000\,000\,000}}, \sqrt{-27\,010.5 + \frac{w^2}{90\,000\,000\,000\,000\,000}}\right\}$$

$k(\omega)$ dispersion relation plots

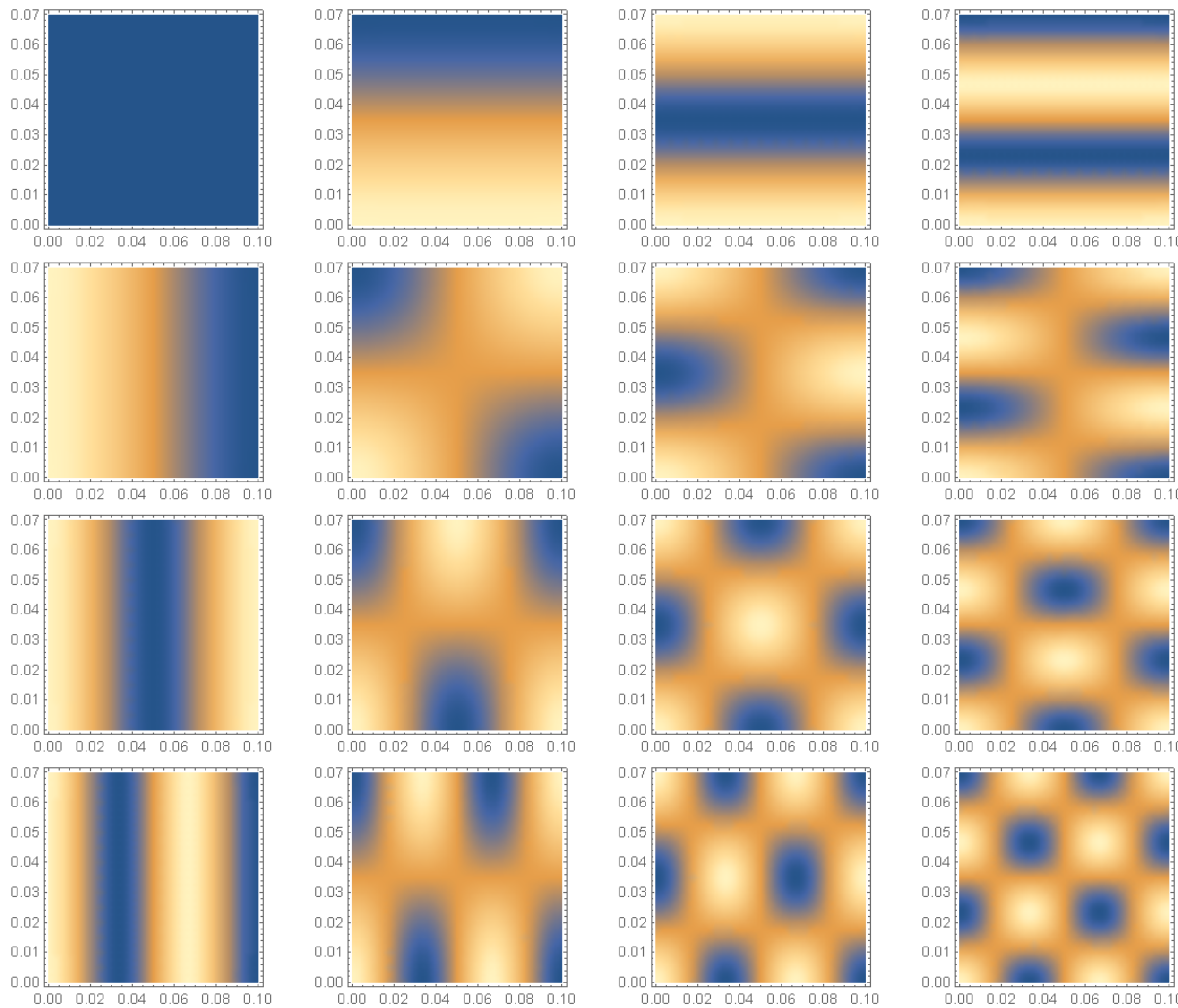
For a given mode its dispersion relation is set by one of the following curves.



***B_z* plots**

Recall that the governing field for the TE modes is the z component of the magnetic field (because the electric field has no z-component). Here are plots of B_z for the first 15 modes (ignore the upper left one). Tannish white is the positive antinode and blue is the negative antinode.

```
Table[DensityPlot[Cos[m Pi x / a] Cos[n Pi y / b] , {x, 0, 0.10}, {y, 0, 0.07}], {m, 0, 3}, {n, 0, 3}] //
 TableForm
```



$E_z = 0$ by definition, and all of the other nonzero components of the fields, namely $E_x, E_y, B_x,$ and B_y, can be calculated from B_z.