Converting T^{tot} into More Useful Form

By Dr. Colton, Physics 471 (last updated 30 Jan 2024)

Start with this:

$$T^{tot} = \frac{n_2}{n_0}\frac{\cos\theta_2}{\cos\theta_0}\frac{|t^{01}|^2|t^{12}|^2}{|e^{-ik_1 d\cos\theta_1} - r^{10}r^{12}e^{ik_1 d\cos\theta_1}|^2} \qquad \text{P\&W Eq. (4.14)}$$

$$\begin{aligned}
\text{denom} &= (e^{-ik_1 d\cos\theta_1} - r^{10}r^{12}e^{ik_1 d\cos\theta_1}) \times \text{complex conjugate} \\
&= (e^{-ik_1 d\cos\theta_1} - r^{10}r^{12}e^{ik_1 d\cos\theta_1})(e^{+ik_1 d\cos\theta_1} - r^{10*}r^{12*}e^{-ik_1 d\cos\theta_1}) \\
\text{(FOIL)} \quad &= 1 \underbrace{-r^{10}r^{12}e^{2ik_1 d\cos\theta_1} - r^{10*}r^{12*}e^{-2ik_1 d\cos\theta_1}} + |r^{10}|^2|r^{12}|^2
\end{aligned}$$

$$\begin{aligned}
&= -(r^{10}r^{12}e^{2ik_1 d\cos\theta_1} + \text{complex conjugate}) \\
&= -2 \times \text{Real}[r^{10}r^{12}e^{2ik_1 d\cos\theta_1}]
\end{aligned}$$

Write $r^{10} = |r^{10}|e^{i\phi^{10}}$,

$r^{12} = |r^{12}|e^{i\phi^{12}}$,

$\delta = 2k_1 d\cos\theta_1$

$$\text{denom} = 1 + |r^{10}|^2|r^{12}|^2 - 2\text{Real}\left[\underbrace{|r^{10}|e^{i\phi^{10}}|r^{12}|e^{i\phi^{12}}e^{i\delta}}_{= |r^{10}||r^{12}|e^{i(\phi^{10}+\phi^{12}+\delta)}}\right]$$

$$= 1 + |r^{10}|^2|r^{12}|^2 - 2|r^{10}||r^{12}|\underbrace{\cos(\phi^{10}+\phi^{12}+\delta)}$$

Trig trick: $\cos\Phi = 1 - 2\sin^2\frac{\Phi}{2}$

$$= 1 + |r^{10}|^2|r^{12}|^2 - 2|r^{10}||r^{12}| + 4|r^{10}||r^{12}|\sin^2\frac{\Phi}{2}$$

where $\Phi = \phi^{10} + \phi^{12} + 2k_1 d\cos\theta_1$

$$\text{denom} = (1 - |r^{10}||r^{12}|)^2 + 4|r^{10}||r^{12}|\sin^2\frac{\Phi}{2}$$

Put denominator back into T^{tot} equation:

$$T^{tot} = \frac{n_2}{n_0}\frac{\cos\theta_2}{\cos\theta_0}\frac{|t^{10}|^2|t^{12}|^2}{(1-|r^{10}||r^{12}|)^2 + 4|r^{10}||r^{12}|\sin^2\frac{\Phi}{2}} \times \frac{\frac{1}{(1-|r^{10}||r^{12}|)^2}}{\frac{1}{(1-|r^{10}||r^{12}|)^2}}$$

$$T^{tot} = \frac{n_2}{n_0}\frac{\cos\theta_2}{\cos\theta_0}\frac{\frac{|t^{10}|^2|t^{12}|^2}{(1-|r^{10}||r^{12}|)^2}}{1 + \frac{4|r^{10}||r^{12}|}{(1-|r^{10}||r^{12}|)^2}\sin^2\frac{\Phi}{2}}$$

Define some symbols to make it look simpler:

$$\boxed{T^{tot} = \frac{T^{max}}{1+F\sin^2\frac{\Phi}{2}}}$$ P&W Eq. (4.15)

$$\boxed{T^{max} = \frac{n_2\cos\,\theta_2}{n_0\cos\theta_0}\frac{\left|t^{01}\right|^2\left|t^{12}\right|^2}{(1-|r^{10}||r^{12}|)^2}}$$ P&W Eq (4.16.) equiv.

$$\boxed{F = \frac{4\left|r^{10}\right|\left|r^{12}\right|}{(1-|r^{10}||r^{12}|)^2}}$$ P&W Eq. (4.18)

called “coefficient of finesse”

and Φ as defined on the previous page,

$$\boxed{\Phi = \phi^{10} + \phi^{12} + 2k_1 d\cos\theta_1}$$ P&W Eq. (4.17)

phase of r^{10} phase of r^{12}

Further note: my T^{max} equation does not look exactly like Eq (4.16)

To complete things, write numerator like this:

$$T^{max}\text{ numer} = \frac{n_2}{n_1}\frac{\cos\theta_2}{\cos\theta_1}|t^{12}|^2\ \frac{n_1}{n_0}\ \frac{\cos\theta_1}{\cos\theta_0}|t^{01}|^2$$

(because n_1’s and cos θ_1’s will cancel out)

$$= T^{12}T^{01}$$

Write $|r^{10}|$ as $\sqrt{R^{10}}$ and $|r^{12}|$ as $\sqrt{R^{12}}$, then

(alternate version) $\boxed{T^{max} = \frac{T^{01}T^{12}}{(1-\sqrt{R^{10}R^{12}})^2}}$ P&W Eq. (4.16)