

Converting T^{tot} into More Useful Form

By Dr. Colton, Physics 471 (last updated 30 Jan 2024)

Start with this:

$$T^{tot} = \frac{n_2 \cos \theta_2}{n_0 \cos \theta_0} \frac{|t^{01}|^2 |t^{12}|^2}{|e^{-ik_1 d \cos \theta_1} - r^{10} r^{12} e^{ik_1 d \cos \theta_1}|^2} \quad \text{P\&W Eq. (4.14)}$$

$$\begin{aligned} \text{denom} &= (e^{-ik_1 d \cos \theta_1} - r^{10} r^{12} e^{ik_1 d \cos \theta_1}) \times \text{complex conjugate} \\ &= (e^{-ik_1 d \cos \theta_1} - r^{10} r^{12} e^{ik_1 d \cos \theta_1})(e^{+ik_1 d \cos \theta_1} - r^{10*} r^{12*} e^{-ik_1 d \cos \theta_1}) \\ \text{(FOIL)} \quad &= 1 - \underbrace{r^{10} r^{12} e^{2ik_1 d \cos \theta_1} - r^{10*} r^{12*} e^{-2ik_1 d \cos \theta_1}}_{= -(r^{10} r^{12} e^{2ik_1 d \cos \theta_1} + \text{complex conjugate})} + |r^{10}|^2 |r^{12}|^2 \\ &= -2 \times \text{Real}[r^{10} r^{12} e^{2ik_1 d \cos \theta_1}] \end{aligned}$$

$$\begin{aligned} \text{Write } r^{10} &= |r^{10}| e^{i\phi^{10}}, \\ r^{12} &= |r^{12}| e^{i\phi^{12}}, \\ \delta &= 2k_1 d \cos \theta_1 \end{aligned}$$

$$\begin{aligned} \text{denom} &= 1 + |r^{10}|^2 |r^{12}|^2 - 2 \text{Real} \left[\underbrace{|r^{10}| e^{i\phi^{10}} |r^{12}| e^{i\phi^{12}} e^{i\delta}}_{= |r^{10}| |r^{12}| e^{i(\phi^{10} + \phi^{12} + \delta)}} \right] \\ &= 1 + |r^{10}|^2 |r^{12}|^2 - 2 |r^{10}| |r^{12}| \cos(\phi^{10} + \phi^{12} + \delta) \end{aligned}$$

$$\text{Trig trick: } \cos \Phi = 1 - 2 \sin^2 \frac{\Phi}{2}$$

$$= 1 + |r^{10}|^2 |r^{12}|^2 - 2 |r^{10}| |r^{12}| + 4 |r^{10}| |r^{12}| \sin^2 \frac{\Phi}{2}$$

$$\text{where } \Phi = \phi^{10} + \phi^{12} + 2k_1 d \cos \theta_1$$

$$\text{denom} = (1 - |r^{10}| |r^{12}|)^2 + 4 |r^{10}| |r^{12}| \sin^2 \frac{\Phi}{2}$$

Put denominator back into T^{tot} equation:

$$T^{tot} = \frac{n_2 \cos \theta_2}{n_0 \cos \theta_0} \frac{|t^{10}|^2 |t^{12}|^2}{(1 - |r^{10}| |r^{12}|)^2 + 4 |r^{10}| |r^{12}| \sin^2 \frac{\Phi}{2}} \times \frac{\frac{1}{(1 - |r^{10}| |r^{12}|)^2}}{\frac{1}{(1 - |r^{10}| |r^{12}|)^2}}$$

$$T^{tot} = \frac{n_2 \cos \theta_2}{n_0 \cos \theta_0} \frac{\frac{|t^{10}|^2 |t^{12}|^2}{(1 - |r^{10}| |r^{12}|)^2}}{1 + \frac{4 |r^{10}| |r^{12}|}{(1 - |r^{10}| |r^{12}|)^2} \sin^2 \frac{\Phi}{2}}$$

Define some symbols to make it look simpler:

$$T^{tot} = \frac{T^{max}}{1 + F \sin^2 \frac{\Phi}{2}}$$

P&W Eq. (4.15)

$$T^{max} = \frac{n_2 \cos \theta_2}{n_0 \cos \theta_0} \frac{|t^{01}|^2 |t^{12}|^2}{(1 - |r^{10}| |r^{12}|)^2}$$

P&W Eq (4.16.) equiv.

$$F = \frac{4|r^{10}||r^{12}|}{(1 - |r^{10}||r^{12}|)^2}$$

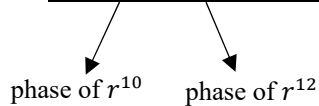
P&W Eq. (4.18)

called “coefficient of finesse”

and Φ as defined on the previous page,

$$\Phi = \phi^{10} + \phi^{12} + 2k_1 d \cos \theta_1$$

P&W Eq. (4.17)



Further note: my T^{max} equation does not look exactly like Eq (4.16)

To complete things, write numerator like this:

$$T^{max} \text{ numer} = \frac{n_2 \cos \theta_2}{n_1 \cos \theta_1} |t^{12}|^2 \frac{n_1 \cos \theta_1}{n_0 \cos \theta_0} |t^{01}|^2$$

(because n_1 's and $\cos \theta_1$'s will cancel out)

$$= T^{12} T^{01}$$

Write $|r^{10}|$ as $\sqrt{R^{10}}$ and $|r^{12}|$ as $\sqrt{R^{12}}$, then

(alternate version)
$$T^{max} = \frac{T^{01} T^{12}}{(1 - \sqrt{R^{10}} \sqrt{R^{12}})^2}$$

P&W Eq. (4.16)