Winter 2024
Physics 471
Exam 1
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barcode here

No time limit. Student calculators are allowed. One page of notes allowed (with printed formulas on one side). Books not allowed.

Name: Solutions

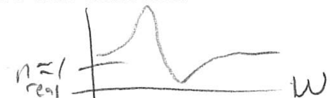
Instructions: Please label & circle/box your answers. **Show your work**, where appropriate.

(16 pts) **Problem 1**: Multiple choice conceptual questions, 2 pts each. Circle the best answer.

1.1. For insulators, the *real* part of the index of refraction cannot be less than one.

a. true
b. false

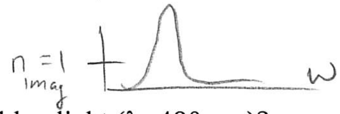
Example: Lorentz model



1.2. For insulators, the *imaginary* part of the index of refraction cannot be less than one.

a. true
b. false

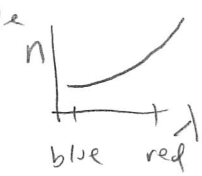
Lorentz model



1.3. In transparent glass, which travels faster: red (λ=680 nm) or blue light (λ=480 nm)?

a. red light
b. blue light
c. both travel at the same speed, which is *c*
d. both travel at the same speed, which is some velocity less than *c*

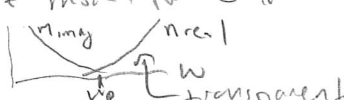
Dispersion from here looks like this in visible



1.4. In a conductor, the most transparent region is likely to be at _______ wavelengths.

a. short (shorter than UV)
b. intermediate (visible light)
c. long (longer than infrared)

Lorentz model for Conductors



At low freqs the electrons can keep up with the oscillation and generate a reflected wave. At high freqs, they can't.

1.5. When light is incident upon a material interface at Brewster's angle, only one polarization will be reflected.

a. true
b. false

$r_p = 0$ at Brewster's angle

1.6. For which polarization does Brewster's angle occur?

a. p-polarization
b. s-polarization
c. both

1.7. For which polarization does total internal reflection occur?

a. p-polarization
b. s-polarization
c. both

Whenever $\theta > \theta_{critical}$

1.8. Suppose you want to make the transmission peaks in a Fabry-Perot interferometer as tall and narrow as possible. You should choose partial reflectors (i.e. reflective coatings on the inside of the glass pieces) with:

a. very high reflectivity
b. very low reflectivity

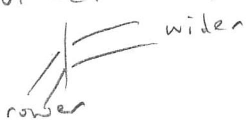
$F = \frac{4|r_{10}||r_{12}|}{(1-|r_{10}||r_{12}|)^2}$ → highest when high reflectivities

(16 pts) **Problem 2**. Short answers.

(a) The equation for the transmitted power across an interface is $T = \frac{n_2 \cos\theta_2}{n_1 \cos\theta_1}|t|^2$. Explain why it is not just $T = |t|^2$; specifically explain where the ratio of the n's and the ratio of the cosines each come from (one sentence each).

(1) Ratio of n's comes from the Poynting vector which describes the intensity of plane waves, $\langle S \rangle = \frac{1}{2} n \varepsilon_0 c E^2$

(2) Ratio of cosines comes from the difference between power and intensity, and that the angle of refraction causes the beam diameter to change



(b) The general form of a plane wave traveling in the z-direction is $\tilde{\mathbf{E}} = \tilde{\mathbf{E}}_0 e^{i(kz-\omega t)}$. Briefly explain why **E** is complex, and what the complex magnitude, complex phase, and direction of $\tilde{\mathbf{E}}_0$ all represent (one sentence each).

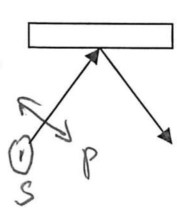
E is complex to allow the mathematics of phase shifts of sinusoidal waves to be easier.

magnitude of $\tilde{E}_0$: the amplitude of the actual electric field

phase of $\tilde{E}_0$: the phase shift ϕ of the field, eg $\cos(kz - \omega t + \phi)$

direction of $\tilde{E}_0$: the actual direction (polarization) that the field is oscillating in

(c) A laser beam traveling parallel to the surface of an optical table reflects off of a mirror as shown (top view). Draw in the picture what is meant by "s-polarization" and what is meant by "p-polarization" for this situation.



where these directions refer to the polarization of the electric field

(18 pts) **Problem 3**. For the following, assume that $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$ are complex plane waves that exist in a material with a complex $\tilde{\mathbf{k}}$. The coordinate axes are aligned such that the wave is propagating in the z-direction and the electric field is polarized in the x-direction.

(a) At a frequency of $\omega = 3 \times 10^{14}$ rad/s the material has $\tilde{n} = 2 + 2i$. What are k_{real} and k_{imag}?

$\frac{w}{k} = \frac{c}{\tilde{n}} \rightarrow \tilde{k} = \frac{w}{c}\tilde{n}$

$k_{real} = \frac{3\cdot10^{14}}{3\cdot10^{8}}(2) = \boxed{2\cdot10^{6}\ \text{rad}/\text{m}}$

k_{imag} = same thing since $n_{real} = n_{imag}$ $= \boxed{2\cdot10^{6}\ \text{rad}/\text{m}}$

(b) How do the phases of $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$ relate? Be specific and quantitative. Hint: for a complex plane wave, $\nabla \cdot \tilde{\mathbf{E}} = i\tilde{\mathbf{k}} \cdot \tilde{\mathbf{E}}$ and $\nabla \times \tilde{\mathbf{E}} = i\tilde{\mathbf{k}} \times \tilde{\mathbf{E}}$. (Sorry for the typo on the exam, missing i's)

From Faraday's law

$\nabla \times \vec{\tilde{E}} = -\frac{\partial \vec{B}}{\partial t}$

$\cancel{i}\tilde{k} \times \tilde{E} = -(-\cancel{i}w)\vec{B}$

because E, B, and k are perpendicular we can lose the cross product and the vectors

$\tilde{B} = \frac{1}{w}\tilde{k}\tilde{E}$

$\tilde{k} = 2\sqrt{2}\angle 45°$, $\tilde{E} \propto e^{i(kz - wt + \phi)}$

$\underbrace{\qquad}_{e^{i(kz - wt + \phi + 45°)}}$

$\boxed{\text{phase of } B = 45° \text{ more than phase of } E}$

(c) Quick derive/justify the equation: $\tilde{\mathbf{E}} = \tilde{E}_0 e^{-\cancel{i}k_{imag}z} e^{i(k_{real}z - \omega t)}\hat{\mathbf{x}}$ and use it to make a sketch of the oscillating electric field inside the material. Calculate the wavelength and the skin depth and mark them on your sketch. Another typo! Apologies

$\tilde{E} = \tilde{E}_0 e^{i(\tilde{k}z - wt)}\hat{x}$

$= \tilde{E}_0 e^{i[(k_{real} + i k_{imag})z - wt]}\hat{x}$

$\boxed{\tilde{E} = \tilde{E}_0 e^{-k_{imag}z} e^{i(k_{real}z - wt)}\hat{x}}$

skin depth $= \frac{1}{k_{imag}}$

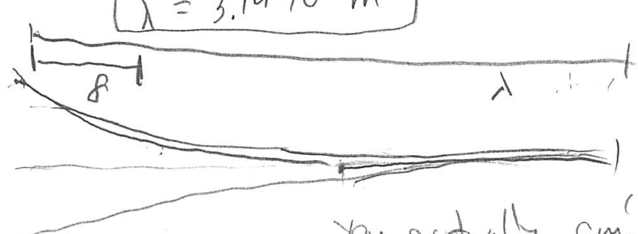
$\delta = \frac{1}{2\cdot10^{6}\ \text{rad}/\text{m}}$

$\boxed{\delta = 5\cdot10^{-7}\ \text{m}}$

Wavelength $= \frac{2\pi}{k_{real}}$

$\lambda = \frac{2\pi}{2\cdot10^{6}\ \text{rad}/\text{m}}$

$\boxed{\lambda = 3.14\cdot10^{-6}\ \text{m}}$



You actually can't even see a single oscillation for this situation since δ is so small, relatively

(18 pts) **Problem 4**. A certain insulator has a plasma frequency of $\omega_p = 1.2 \times 10^{16}$ rad/s and a resonant frequency of $\omega_0 = 1.5 \times 10^{16}$ rad/s. The damping is small enough as to be negligible for frequencies which are not extremely close to ω_0.

$\rightarrow \gamma = 0$

(a) What wavelength does the resonant frequency correspond to?

$\hookrightarrow$ assuming vacuum wavelength

$$\frac{\omega}{k} = c \rightarrow \frac{\omega\lambda}{2\pi} = c \rightarrow \lambda = \frac{2\pi c}{\omega} = \frac{2\pi\, 3\cdot 10^8}{1.5\cdot 10^{16}} = \boxed{125.7 \text{ nm}} \text{ for } \omega_0$$

(b) P-polarized light with a frequency of $\omega_0 = 0.75 \times 10^{16}$ rad/s is incident from air onto this material at $\theta_i = 30°$. Predict n_{real} and n_{imag} from the Lorentz oscillator model.

$$\varepsilon_r = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2} \quad \text{with no damping}$$

$$= 1 + \frac{(1.2\cdot 10^{16})^2}{(1.5\cdot 10^{16})^2 - (0.75\cdot 10^{16})^2}$$

$$= 1.853$$

$$n = \sqrt{\varepsilon_r} = \sqrt{1.853}$$

$$\boxed{n = 1.361}$$

(c) What will be the angle of transmission into the material?

$\rightarrow$ from air $(n = 1)$ into material $(n = 1.361)$

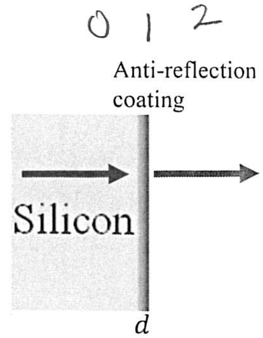
Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\frac{(1)(\sin 30°)}{1.361} = \cancel{1.361} \sin\theta_2$$

$$\theta_2 = \sin^{-1}\left(\frac{.5}{1.361}\right)$$

$$\boxed{\theta_2 = 21.55°}$$

I'm using p-polar eqns but it doesn't matter at normal incidence



(32 pts) **Problem 5**. Infrared light goes through a piece of silicon with optical index $n = 3.4$; the silicon has thin coatings on each side, as shown. Focus on the right hand side, which is marked "anti-reflection coating".

(a) First, determine the percent of the light (power) which would go from the silicon into the air if there were no coating.

Normal incidence: $\alpha = 1$

$$\beta = \frac{n_2}{n_1} = \frac{1}{3.4} = .294$$

Fresnel Eqn

$$t = \frac{2}{\alpha + \beta} = \frac{2}{1.294} = 1.545$$

$$T = \alpha\beta|t|^2 = (1)(.294)(1.545)^2 = \boxed{70.2\%}$$

Alternately

$$r = \frac{\alpha - \beta}{\alpha + \beta} = \frac{1 - .294}{1 + .294} = .545$$

then $R = r^2$

$T = 1 - R = 70.2\%$

(b) Now, suppose you want to add a coating. You have these three common materials to use as coatings: ZnS ($n = 2.32$), CeF_3 ($n = 1.63$) and MgF_2 ($n = 1.38$). Based on your knowledge of the Fresnel coefficients, make an argument about which is most likely to be the best at reducing reflections both at the silicon-coating interface and also the coating-air interface. (You can still get nearly all the credit for this problem if you choose incorrectly.)

$$r = \frac{1 - \beta}{1 + \beta} = \frac{1 - \frac{n_2}{n_1}}{1 + \frac{n_2}{n_1}}$$

to reduce reflections at a given interface you want $\frac{n_2}{n_1}$ to be close to 1, ie $n_2 \approx n_1$

the material which does this best for both interfaces is when $n_{coating} = \sqrt{n_{Si} \times n_{air}}$

$$n_{coating} = \sqrt{3.4 \times 1} = 1.84 \rightarrow$$ Looks like $\boxed{CeF_3 \text{ is likely best}}$

(c) Using the "two interfaces" theory, and for the coating you selected, determine the percent of the light (power) which goes all the way through the prism for the optimal thickness which gives you the maximum transmission.

Plan: use $T_{tot} = \frac{T_{max}}{1 + F\sin^2\frac{\Phi}{2}}$

The max T_{tot} is T_{max}.

$$T_{max} = \frac{T_{01}T_{12}}{\left(1 + \sqrt{R_{10}}\sqrt{R_{12}}\right)^2}$$

So we need to find the T and R values.

Normal incidence, so $\alpha = 1$, β = ratio of n's

$$r_{01} = \frac{1 - \frac{1.63}{3.4}}{1 + \frac{1.63}{3.4}} = .352$$

(Could also use r_{10}, which is $-.352$)

$$R_{01} = |r_{01}|^2 = .124$$

$$T_{01} = 1 - R_{01} = .876$$

$$r_{12} = \frac{1 - \frac{1}{1.63}}{1 + \frac{1}{1.63}} = .240$$

$$R_{12} = |r_{12}|^2 = .057$$

$$T_{12} = 1 - R_{12} = .943$$

(more space on next page)

So $T_{max} = \dfrac{(.876)(.943)}{(1+\sqrt{.124}\sqrt{.057})^2} = \boxed{98.49\%}$

Remarkably, by adding this coating we're increased the transmission from 70.2% to over 98%!

vacuum wavelength

(d) Determine the optimal thickness in terms of the wavelength λ, which will give you the transmission predicted in the previous part. There are actually an infinite number of optimal thicknesses; just determine the smallest one.

The optimal thickness $(T = T_{max})$ occurs when $\sin^2\frac{\phi}{2} = 0 \rightarrow \frac{\phi}{2} = m\pi$.

Minimum optimal thickness when $\frac{\phi}{2} = \pi \rightarrow \phi = 2\pi$

$$\underbrace{\phi_{10}}_{\pi} + \underbrace{\phi_{12}}_{0} + 2\underbrace{k_1}_{\frac{2\pi n_{coating}}{\lambda_{vac}}} d \cos \underbrace{\theta_1}_{0^\circ} = 2\pi$$

$$2\left(\frac{2\cancel{\pi} n_{coating}}{\lambda_{vac}}\right)(d)(1) = \cancel{\pi}$$

$$d = \frac{\lambda_{vac}}{4 n_{coating}} = \frac{1}{4(1.63)} \lambda_{vac} = \boxed{.153\,\lambda_{vac}}$$

(or $\frac{1}{4}\lambda_{coating}$)