Winter 2024
Physics 471, Exam 2
Dr. Colton, cell: 801-358-1970

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40132760

No time limit. Student calculators are allowed. One page of notes allowed (with printed formulas on one side). Books not allowed.

Name: Solutions

Instructions: Please label & circle/box your answers. **Show your work**, where appropriate. Here are some Stokes vectors & Mueller matrices.

LIGHT	Jones	Stokes
unpolar	n/a	$\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$
Horiz linear	$\begin{pmatrix}1\\0\end{pmatrix}$	$\begin{pmatrix}1\\1\\0\\0\end{pmatrix}$
Vert linear	$\begin{pmatrix}0\\1\end{pmatrix}$	$\begin{pmatrix}1\\-1\\0\\0\end{pmatrix}$
45° linear	$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$	$\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$
–45° linear	$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$	$\begin{pmatrix}1\\0\\-1\\0\end{pmatrix}$
Linear at θ	$\begin{pmatrix}\cos\theta\\\sin\theta\end{pmatrix}$	$\begin{pmatrix}1\\\cos 2\theta\\\sin 2\theta\\0\end{pmatrix}$
RCP	$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$	$\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$
LCP	$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$	$\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}$
Elliptical	$\begin{pmatrix}A\\Be^{i\delta}\end{pmatrix}$ with $A^2+B^2=1$	$\frac{1}{E_0^2}\begin{pmatrix}E_0^2\\E_{0x}^2-E_{0y}^2\\2E_{0x}E_{0y}\cos\delta\\2E_{0x}E_{0y}\sin\delta\end{pmatrix}$ δ is the phase shift of vert relative to horiz

Angle of elliptical: $\alpha = \frac{1}{2}\tan^{-1}\left(\frac{2AB\cos\delta}{A^2-B^2}\right)$

$E_\alpha = |E_{eff}| \times \sqrt{A^2\cos^2\alpha + B^2\sin^2\alpha + AB\cos\delta\sin 2\alpha}$

$E_{\alpha\pm 90°} = |E_{eff}| \times \sqrt{A^2\sin^2\alpha + B^2\cos^2\alpha - AB\cos\delta\sin 2\alpha}$

Amount polarized: $\sqrt{S_1^2+S_2^2+S_3^2}$

Amount unpolarized: $S_0 - \sqrt{S_1^2+S_2^2+S_3^2}$

Deg of polarization $= \frac{1}{S_0}\sqrt{S_1^2+S_2^2+S_3^2}$

OPTIC	Jones	Mueller (Stokes)
Horiz linear polar	$\begin{pmatrix}1&0\\0&0\end{pmatrix}$	$\frac{1}{2}\begin{pmatrix}1&1&0&0\\1&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$
Vert linear polar	$\begin{pmatrix}0&0\\0&1\end{pmatrix}$	$\frac{1}{2}\begin{pmatrix}1&-1&0&0\\-1&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$
45° linear polar	$\frac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix}$	$\frac{1}{2}\begin{pmatrix}1&0&1&0\\0&0&0&0\\1&0&1&0\\0&0&0&0\end{pmatrix}$
–45° linear polar	$\frac{1}{2}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}$	$\frac{1}{2}\begin{pmatrix}1&0&-1&0\\0&0&0&0\\-1&0&1&0\\0&0&0&0\end{pmatrix}$
angle θ linear polar	$\begin{pmatrix}\cos^2\theta&\sin\theta\cos\theta\\\sin\theta\cos\theta&\sin^2\theta\end{pmatrix}$	$\frac{1}{2}\begin{pmatrix}1&\cos 2\theta&\sin 2\theta&0\\\cos 2\theta&\cos^2 2\theta&\cos 2\theta\sin 2\theta&0\\\sin 2\theta&\cos 2\theta\sin 2\theta&\sin^2 2\theta&0\\0&0&0&0\end{pmatrix}$
$\lambda/4$ fast axis horiz	$\begin{pmatrix}1&0\\0&i\end{pmatrix}$	$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&-1&0\end{pmatrix}$
$\lambda/4$ fast axis vert	$\begin{pmatrix}1&0\\0&-i\end{pmatrix}$	$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix}$
$\lambda/4$ fast axis 45°	$\frac{1}{2}\begin{pmatrix}1+i&1-i\\1-i&1+i\end{pmatrix}$	$\begin{pmatrix}1&0&0&0\\0&0&0&-1\\0&0&1&0\\0&1&0&0\end{pmatrix}$
$\lambda/4$ fast axis angle θ	$\begin{pmatrix}\cos^2\theta+i\sin^2\theta&(1-i)\sin\theta\cos\theta\\(1-i)\sin\theta\cos\theta&\sin^2\theta+i\cos^2\theta\end{pmatrix}$	$\begin{pmatrix}1&0&0&0\\0&\cos^2 2\theta&\frac{1}{2}\sin 4\theta&-\sin 2\theta\\0&\frac{1}{2}\sin 4\theta&\sin^2 2\theta&\cos 2\theta\\0&\sin 2\theta&-\cos 2\theta&0\end{pmatrix}$
$\lambda/2$ fast axis horiz	$\begin{pmatrix}1&0\\0&-1\end{pmatrix}$	$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}$
$\lambda/2$ fast axis vert	$\begin{pmatrix}-1&0\\0&1\end{pmatrix}$	$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}$
$\lambda/2$ fast axis at θ	$\begin{pmatrix}\cos 2\theta&\sin 2\theta\\\sin 2\theta&-\cos 2\theta\end{pmatrix}$	$\begin{pmatrix}1&0&0&0\\0&\cos 4\theta&\sin 4\theta&0\\0&\sin 4\theta&-\cos 4\theta&0\\0&0&0&-1\end{pmatrix}$

(12 pts) **Problem 1**: Multiple choice questions, 2 pts each. Circle/write down the answer.

1.1. In a uniaxial crystal, which vector will always obey Snell's law?

a. k-vector
b. Poynting vector
c. both
d. neither

1.2. If a right circular polarized beam reflects from a mirror at normal incidence, the resulting beam will likely be:

a. right circular polarized
b. left circular polarized
c. linearly polarized
d. unpolarized

1.3. A half wave plate with fast axis at θ with respect to the x axis, and $\theta < 90°$, will do what to horizontal linearly polarized light?

a. Reduce the intensity of the light by a factor of $\cos\theta$
b. Reduce the intensity of the light by a factor of $\cos^2\theta$
c. Rotate the linear polarization by θ
d. Rotate the linear polarization by 2θ
e. Change the light to right circular polarization
f. Change the light to left circular polarization

1.4. The SI units of electric field $E(t)$ are V/m. What are the units of its Fourier transform, $E(\omega)$?

a. V
b. V·m
c. V/m
d. V/m^2
e. V/(m·s)
f. V·s/m

$$E(\omega) = \frac{1}{\sqrt{2\pi}} \int \underset{\frac{V}{m}}{\underset{\uparrow}{E(t)}}\, e^{i\omega t}\, \underset{s}{\underset{\uparrow}{dt}}$$

1.5. What is meant by "optic axis" in the context of uniaxial crystals?

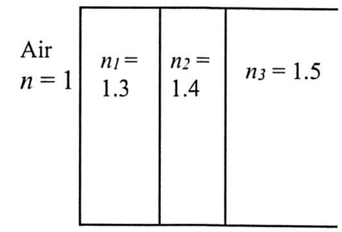
a. The direction of the k-vector
b. The direction of the Poynting vector
c. The direction in which both polarizations have the same speed
d. The direction of polarization which gives rise to $n = n_o$
e. The direction of polarization which gives rise to $n = n_e$
f. The direction of polarization which gives rise to $n =$ (the smaller of n_o and n_e)
g. The direction of polarization which gives rise to $n =$ (the larger of n_o and n_e)

1.6. What is meant by "fast axis" in the context of polarization optics?

a. The direction of the k-vector
b. The direction of the Poynting vector
c. The direction in which both polarizations have the same speed
d. The direction of polarization which gives rise to $n = n_o$
e. The direction of polarization which gives rise to $n = n_e$
f. The direction of polarization which gives rise to $n =$ (the smaller of n_o and n_e)
g. The direction of polarization which gives rise to $n =$ (the larger of n_o and n_e)

All $\cos\theta$ terms $= 1$
Both polarizations are equivalent

(19 pts) **Problem 2.** For the structure as shown, light comes in at normal incidence. The n_1 and n_2 layers have the right thickness to make them $\lambda/4$ for the wavelength of interest, where λ refers to the wavelength inside the material.



(a) The equation to find the overall matrix **A** involves an initial number, multiplied by a product of four matrices. Explicitly write out the initial number and each of the four matrices. Simplify each as much as possible but don't do any matrix multiplication.

setting $\cos\theta$'s to 1, ...

$$A = \frac{1}{2n_0}\begin{pmatrix} n_0 & 1 \\ n_0 & -1 \end{pmatrix}\begin{pmatrix} \cos\beta_1 & -i\sin\beta_1/n_1 \\ -in_1\sin\beta_1 & \cos\beta_1 \end{pmatrix}\begin{pmatrix} \cos\beta_2 & -i\sin\beta_2/n_2 \\ -in_2\sin\beta_2 & \cos\beta_2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ n_3 & 0 \end{pmatrix}$$

$$\beta_j = k_j \ell_j \cos\theta_j = k_j \ell_j = \frac{2\pi}{\lambda_j}\ell_j$$

"All thicknesses are $\lambda/4$" $\rightarrow$ $= \frac{2\pi}{\lambda}\left(\frac{1}{4}\lambda\right) = \frac{\pi}{2}$

so all $\cos\beta$ terms $= 0$

all $\sin\beta$ terms $= 1$

plugging in the n's and these, we have

$$\boxed{A = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 & \frac{-i}{1.3} \\ -1.3i & 0 \end{pmatrix}\begin{pmatrix} 0 & \frac{-i}{1.4} \\ -1.4i & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1.5 & 0 \end{pmatrix}}$$

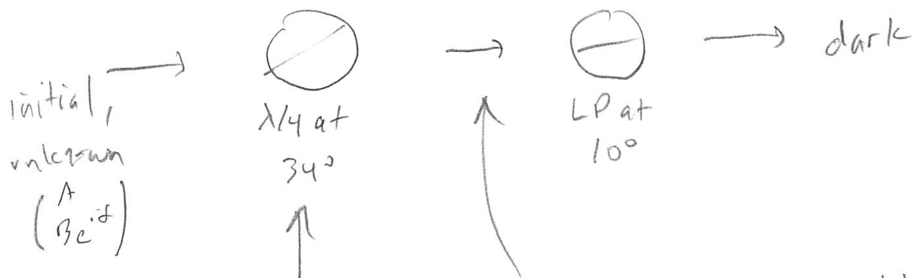
(b) Suppose after doing the matrix multiplication in the previous part, you determine $\mathbf{A} = \begin{pmatrix} 4.1 & 3.1 \\ 2.4 & 1.5 \end{pmatrix}$.
What would be the overall reflectivity r of the structure for that wavelength?

$r = \frac{a_{21}}{a_{11}}$ which would be $\frac{2.4}{4.1}$ for this matrix

$= \boxed{.5854}$

(13 pts) **Problem 3.** A light beam of unknown polarization goes through a 1/4 wave plate followed by a linear polarizer. By trial and error, you manage to extinguish the light when the quarter wave plate fast axis is at +34° from the horizontal and the linear polarization transmission axis is at +10° from the horizontal. Set up a Jones matrix equation you could use with e.g. Mathematica to determine the polarization state of the initial light. Don't do any of the matrix math, just set up the equation.

This is exactly the lab problem, L6.9



Jones matrix for $\lambda/4$ at 34°

the polarization here must be linear, 100°

which is $\begin{pmatrix} \cos 100° \\ \sin 100° \end{pmatrix}$

Matrix equation

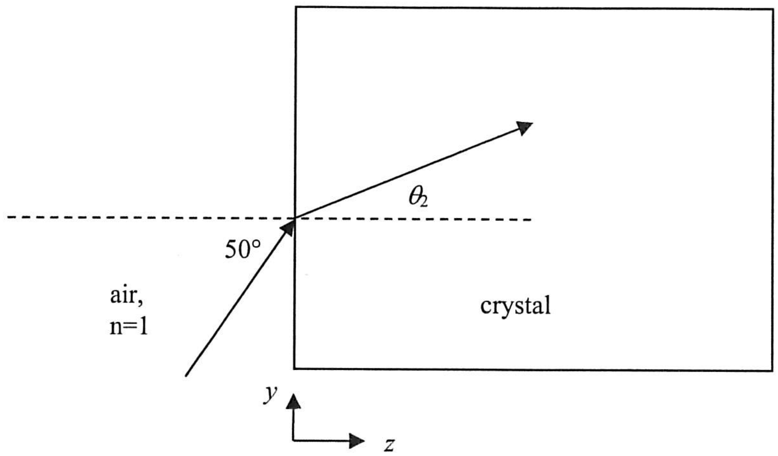
$$\begin{pmatrix} \cos 100° \\ \sin 100° \end{pmatrix} = \begin{pmatrix} M \text{ for} \\ \lambda/4 \text{ at } 34° \end{pmatrix} \begin{pmatrix} A \\ Be^{i\delta} \end{pmatrix}$$

or

$$\boxed{\begin{pmatrix} A \\ Be^{i\delta} \end{pmatrix} = \begin{pmatrix} \cos^2 34° + i\sin^2 34° & (1-i)\sin 34°\cos 34° \\ (1-i)\sin 34°\cos 34° & \sin^2 34° + i\cos^2 34° \end{pmatrix}^{-1} \begin{pmatrix} \cos 100° \\ \sin 100° \end{pmatrix}}$$

$n_x = 1.5$
$n_y = 1.5$
$n_z = 1.7$

(17 pts) **Problem 4.** A beam of light enters a uniaxial crystal with an incident angle of 50° measured from the perpendicular. The crystal has $n_o = 1.5$ and $n_e = 1.7$ for the light's wavelength. The optic axis of the crystal is in the z direction and the surface of the crystal is the plane formed by the x- and y-axes.



(x is into the page)

(a) If the light is s-polarized (i.e. electric field is in the x-direction), what is θ_2?

← $n = 1.5$

Use regular Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\theta_2 = \sin^{-1}\left(\frac{1}{1.5}\sin 50°\right) = \boxed{30.71°}$$

(b) If the light is p-polarized (i.e. electric field is in the y-z plane), what is θ_2?

Have to use the more complicated eqn

$$\tan\theta_2 = \frac{n_e}{n_o}\frac{\sin\theta_1}{\sqrt{n_e^2 - \sin^2\theta_1}}$$

$$\theta_2 = \tan^{-1}\left(\frac{1.7}{1.5}\frac{\sin 50°}{\sqrt{1.7^2 - \sin^2 50°}}\right) = \boxed{29.77°}$$

(21 pts) **Problem 5**. As shown in a homework problem, a 1/4 wave plate at 45 degrees followed by a horizontal polarizer will serve as a left circular polarization filter in the sense that it blocks RCP and passes LCP with no attenuation. The Mueller matrix for that combination turns out to be:

$$\frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \leftarrow M_{LCP\ filter}$$

(a) Similarly, a 1/4 wave plate at 45 degrees followed by a *vertical* polarizer will block LCP and allow RCP to pass. Determine the Mueller matrix for this new combination.

$$M_{RCP\ filter} = \frac{1}{2}\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$= \boxed{\frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}$$

(b) The degree of circular polarization ($DOCP$) can be defined as (RCP-LCP)/(RCP+LCP) where LCP and RCP are the amounts of light that would get through the two filters mentioned in parts (a) and (b). Determine the $DOCP$ for an arbitrary Stokes vector (A, B, C, D) by virtually passing the vector through the two filters just described (i.e. using their Mueller matrices), identifying how much intensity gets through in each case, and using the $DOCP$ definition just given.

Light through LCP filter $= \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \frac{1}{2}\begin{pmatrix} A-D \\ A-D \\ 0 \\ 0 \end{pmatrix} \rightarrow$ intensity is $\frac{1}{2}(A-D)$ (first vector element)

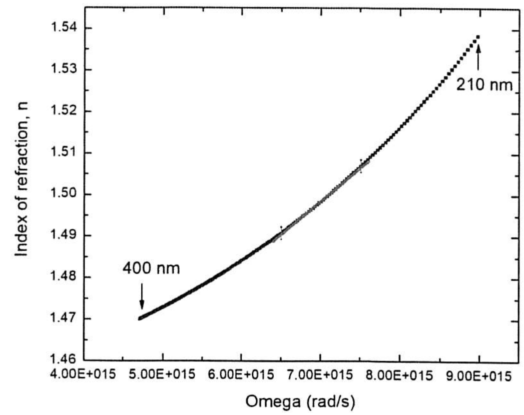
Light through RCP filter $= \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \frac{1}{2}\begin{pmatrix} A+D \\ -A-D \\ 0 \\ 0 \end{pmatrix} \rightarrow$ intensity is $\frac{1}{2}(A+D)$

$$DOCP = \frac{RCP - LCP}{RCP + LCP}$$

$$= \frac{\cancel{\tfrac{1}{2}}(A+D) - \cancel{\tfrac{1}{2}}(A-D)}{\cancel{\tfrac{1}{2}}(A+D) + \cancel{\tfrac{1}{2}}(A-D)}$$

$$= \frac{D+D}{A+A} = \frac{2D}{2A} = \boxed{\frac{D}{A}}$$

(18 pts) **Problem 6**. In my lab we have several lenses made out of fused silica which we use for experiments in the near UV. I looked up the index of refraction for fused silica, and in the wavelength range from 210-400 nm its index of refraction vs ω looks like this figure. Fitting the section of the curve around $7 \cdot 10^{15}$ rad/s (which is 269.3 nm) to a straight line, shown in red, I got this equation: $y = 1.38704 + 1.59168 \cdot 10^{-17}\, x$, where the units are such to make y dimensionless and x in rad/s.



If I tune my laser to that ~~wavelength, set it~~ for a Gaussian pulse with temporal width $T = 24 \cdot 10^{-15}$ s, and shine it into a 1 cm thick piece of fused silica, what will be the temporal width of the pulse when it leaves?

this is $\tilde{T} = T\sqrt{1+\Phi^2}$

$z = .01\,m$

where $\Phi = \dfrac{2\alpha z}{T^2}$

where $\alpha = \dfrac{1}{2c}\left(n''\omega + 2n'\right)\Big|_{\omega=\omega_0}$

$n = 1.38704 + 1.59168 \cdot 10^{-17}\,\omega$

$n' = 1.59168 \cdot 10^{-17}$ (units = seconds)

$n'' = 0$

Piecing together, $\alpha = \dfrac{1}{2 \cdot 3 \cdot 10^{8}\,\frac{m}{s}}\left(0 + 2 \cdot 1.59168 \cdot 10^{-17}\,s\right)$

$\alpha = 5.3056 \cdot 10^{-26}\ s^2/m$

$\Phi = \dfrac{2\left(5.3056 \cdot 10^{-26}\ \frac{s^2}{m}\right)(.01\,m)}{\left(24 \cdot 10^{-15}\,s\right)^2} = 1.842$

$\tilde{T} = 24 \cdot 10^{-15}\,s\,\sqrt{1 + 1.842^2}$

$= 24 \cdot 10^{-15}\,s\,(2.096)$

$\boxed{\tilde{T} = 50.31\ fs}$