Winter 2024
Physics 471, Exam 3
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No time limit. Student calculators are allowed. One page of notes allowed (with printed formulas on one side). Books not allowed.

Name: Solutions

Instructions: Please label & circle/box your answers. **Show your work**, where appropriate.

(16 pts) **Problem 1**: Multiple choice/short answer questions, 2 pts each. Circle/write down the answer.

1.1. In an interferometer, the term "fringe visibility" refers to:
- a. closeness/separation of fringe peaks
- b. height/depth of fringe peaks
- c. the wavelengths which contribute to fringes
- d. the frequencies which contribute to fringes

1.2. Given that the SI units of $E(t)$ are V/m, what are the SI units of $E(\omega)$?
- a. V/m
- b. V/(s·m)
- c. V/(s²·m)
- d. V·s/m
- e. V·s²/m

$E(\omega) = \frac{1}{\sqrt{2\pi}} \int E(t)\, e^{i\omega t}\, dt$

$E(t)$: V/m; $e^{i\omega t}$: No units; dt: sec

1.3. Given that the SI units of $I(t)$ are J/(s·m²), what are the SI units of $I(\omega)$?
- a. J/m²
- b. J/(s·m²)
- c. J/(s²·m²)
- d. J/(s³·m²)
- e. J·s/m²

$\int I(t)\, dt = \int I(\omega)\, d\omega$

$I(t)$: $\frac{J}{s m^2}$; dt: s; $d\omega$: $\frac{1}{s}$

1.4. For rays traveling from point 1 to point 2, imaging occurs when:
- a. y_1 is independent of θ_1
- b. y_1 is independent of θ_2
- c. y_2 is independent of θ_1
- d. y_2 is independent of θ_2

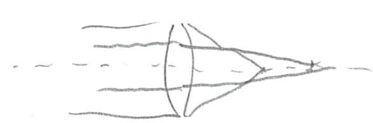
all angles converge to the same point at the same height

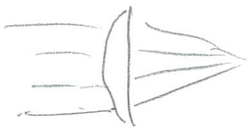
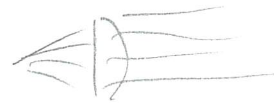
1.5. Chromatic aberration can be mostly corrected by using a specially designed:
- a. curved detector
- b. cylindrical lens
- c. doublet lens
- d. plano-convex lens

main focusing

undoes dispersion

1.6. Spherical aberration causes rays on the _____ of the lens to focus closer than rays on the _____ of the lens :
- a. bottom, top
- b. top, bottom
- c. inside, outside
- d. outside, inside



1.7. To partially correct for spherical aberration using a "plano-convex" lens you should follow which rule:

a. If the rays are parallel, place the flat side first; if the rays are diverging, place the curved side first.

b. If the rays are parallel, place the curved side first; if the rays are diverging, place the flat side first.

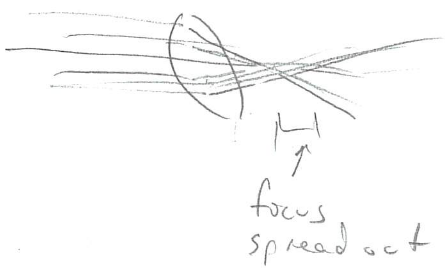
1.8. To correct for coma, you should:

a. change the angle of your lens relative to the beam path

b. image to a curved surface

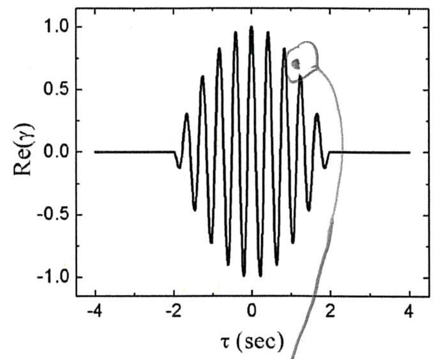
c. introduce an asymmetry between vertical and horizontal planes of light

d. use monochromatic light



(18 pts) **Problem 2**. A Michelson interferometer is used to measure the coherence properties of a certain beam of light. The degree of coherence function, $\gamma(\tau)$, is found to be the following:

$$\gamma(\tau) = \begin{cases} 0, & \tau < -\dfrac{1}{\Delta\omega} \\ e^{i\omega_0\tau}(1-\tau^2(\Delta\omega)^2), & -\dfrac{1}{\Delta\omega} < \tau < \dfrac{1}{\Delta\omega} \\ 0, & \tau > \dfrac{1}{\Delta\omega} \end{cases}$$



In this equation, ω_0 represents the main frequency component and $\Delta\omega$ is a measure of the frequency bandwidth. A plot of the real part of the function is given for $\omega_0 = 15$ rad/s and $\Delta\omega = 0.5$ rad/s.

(a) For the values of ω_0 and $\Delta\omega$ given in the plot what is the visibility at $\tau = 1$ s?

$$\text{visibility} = |\gamma| = |1 - \tau^2(\Delta\omega)^2|$$
$$= |1 - 1(.5)^2|$$
$$= .75$$

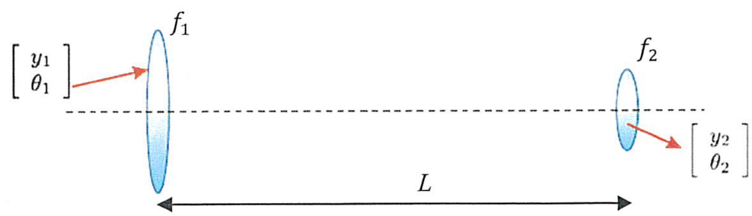
can also be seen on the graph as the envelope value at 1 sec

(b) In terms of arbitrary values of ω_0 and $\Delta\omega$, what is the coherence time of the light?

$$\tau = \int_{-\infty}^{\infty} |\gamma|^2 \, d\tau$$
$$= \int_{-\frac{1}{\Delta\omega}}^{\frac{1}{\Delta\omega}} (1 - \tau^2 \Delta\omega^2)^2 \, d\tau$$
$$= \int_{-\frac{1}{\Delta\omega}}^{\frac{1}{\Delta\omega}} (1 - 2\tau^2 \Delta\omega^2 + \tau^4 \Delta\omega^4) \, d\tau$$
$$= 2\left[\tau - \frac{2}{3}\tau^3 \Delta\omega^2 + \frac{1}{5}\tau^5 \Delta\omega^5\right]\Bigg|_{\tau=0}^{1/\Delta\omega}$$
$$= 2\,\frac{1}{\Delta\omega}\left[1 - \frac{2}{3} + \frac{1}{5}\right]$$
$$= \boxed{\frac{1.0662}{\Delta\omega}} \quad \text{or} \quad = \frac{16}{15}\,\frac{1}{\Delta\omega}$$

if you prefer fractions

(20 pts) **Problem 3.** (a) A thin lens of focal length f_1 is followed by a distance of length L and then by a second thin lens with focal length f_2. Give the combined ABCD matrix for this system in terms of L, f_1, and f_2.



$$M = \begin{pmatrix} \text{thin lens,} \\ f_2 \end{pmatrix} \times \begin{pmatrix} \text{translation,} \\ L \end{pmatrix} \times \begin{pmatrix} \text{thin lens,} \\ f_1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -1/f_2 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1/f_1 & 1 \end{pmatrix}}_{\begin{pmatrix} 1-L/f_1 & L \\ -1/f_1 & 1 \end{pmatrix}}$$

$$\boxed{M = \begin{pmatrix} 1-L/f_1 & L \\ -\frac{1}{f_2}\left(1-\frac{L}{f_1}\right)-\frac{1}{f_1} & -\frac{L}{f_2}+1 \end{pmatrix}}$$

(b) If the angle of the exiting ray should not depend on the height of the entering ray but rather only on the angle of the entering ray, derive a condition on L that will guarantee this. (That choice of L is called the telescope configuration.)

$$\begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix} \longrightarrow \theta_2 = C y_1 + D\theta_1$$

For it to not depend on y_1, $\boxed{C=0}$

$$-\frac{1}{f_2}\left(1-\frac{L}{f_1}\right)-\frac{1}{f_1} = 0$$

$$-\frac{1}{f_2}\left(1-\frac{L}{f_1}\right) = \frac{1}{f_1}$$

$$1-\frac{L}{f_1} = -\frac{f_2}{f_1}$$

$$1+\frac{f_2}{f_1} = \frac{L}{f_1} \longrightarrow \boxed{L = f_1 + f_2}$$

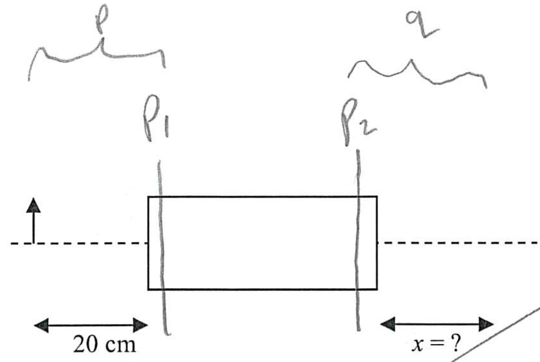
(c) When used as a telescope, rays from a given far-away point all strike the first lens with essentially the same angle θ_1. "Angular magnification" m is defined as θ_2/θ_1 and quantifies the telescope's purpose of enlarging the apparent angle between points in the field of view. Use the ABCD matrix you found above to determine m in terms of f_1 and f_2.

$\theta_2 = C y_1 + D\theta_1$ from above, $= \underline{D\theta_1}$ if $C=0$

$$\frac{\theta_2}{\theta_1} = D = -\frac{L}{f_2}+1$$

$$= \frac{-(f_1+f_2)}{f_2}+1 = \boxed{-\frac{f_1}{f_2}}$$

(18 pts) **Problem 4**



Mistake! The determinant here $= 0$ instead of 1. Consequently the problem is ill-formed. But let's pretend that I had given a matrix with det $= 1$.

A complex system of lenses (the rectangle in the figure) is described by this ABCD matrix: $\begin{pmatrix} 0.7 & -2 \\ -0.28 & 0.8 \end{pmatrix}$. The numbers are given in units such that you can put all distances in cm and all angles in radians.

(a) Note that since the determinant of the matrix = 1, the concept of principal planes applies here. Calculate the principal planes and the ~~effective focal length of the~~ rectangle. Mark the locations of the principal planes on the figure.

$$P_1 = \frac{1-D}{C} = \frac{1-.8}{-.28} = \frac{.2}{-.28} = \underline{-0.714 \text{ cm}}$$

$$P_2 = \frac{1-A}{C} = \frac{1-.7}{-.28} = \frac{.3}{-.28} = \underline{-1.071 \text{ cm}}$$

$$f = -\frac{1}{C} = -\frac{1}{-.28} = \underline{3.571 \text{ cm}}$$

(b) An object is placed 20 cm to the left of the system, as shown by the vertical arrow. Use the principal planes along with the thin lens equation to determine where the image is formed relative to the end of the optical system, i.e. the distance marked x in the figure.

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \rightarrow \frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

p = distance of object from P_1 plane

$$q = \left(\frac{1}{3.571} - \frac{1}{(20+.714)}\right)^{-1}$$

$$q = 4.315 \text{ cm}$$

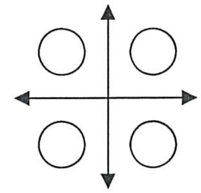
q = distance of image from P_2 plane

From picture, $x = q - |P_2|$

$$x = 4.315 - 1.071$$

$$= \boxed{3.244 \text{ cm}}$$

(18 pts) **Problem 5**. Use the convolution theorem or equivalently the array theorem to calculate the Fraunhofer diffraction pattern intensity for this aperture comprised of four circular holes centered at $(-2a,-2a)$, $(-2a,2a)$, $(2a,-2a)$, and $(2a,2a)$. All circles have radius a. Don't worry about constant factors such as square roots of 2π; just lump them all into one overall constant, I_0. The rest of your answer, however, should be written in terms of the screen coordinates x, y, and z, circle radius a, and wave vector k. Simplify as much as possible using the identities $e^{ix}+e^{-ix}=2\cos x$ and/or $e^{ix}-e^{-ix}=2i\sin x$. In particular, your final simplified answer for the intensity must not have any factors of i in it.



$$FT(\,) = FT(\,)\cdot FT(\,)$$

$$= \underline{\frac{a^2}{2}\,\mathrm{jinc}(k_\rho a)}\ \cdot$$

sum of four phase factors

$$\to e^{ik_x 2a}e^{ik_y 2a} + e^{ik_x 2a}e^{-ik_y 2a} + e^{-ik_x 2a}e^{ik_y 2a} + e^{-ik_x 2a}e^{-ik_y 2a}$$

simplify

$$= e^{ik_x 2a}\Big(\underbrace{e^{ik_y 2a}+e^{-ik_y 2a}}_{2\cos(k_y 2a)}\Big) + e^{-ik_x 2a}\Big(\underbrace{e^{ik_y 2a}+e^{-ik_y 2a}}_{2\cos k_y 2a}\Big)$$

$$= 2\cos(k_y 2a)\Big[\underbrace{e^{ik_x 2a}+e^{-ik_x 2a}}_{2\cos(k_x 2a)}\Big]$$

$$= \underline{4\cos 2k_x a\cos 2k_y a}$$

there's also a $\frac{1}{2\pi}$ but can be lumped into I_0

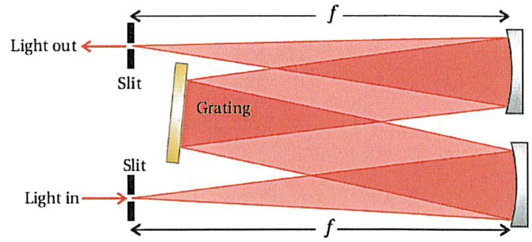
$I \sim |FT|^2$ with $k_x = \frac{kx}{z}$, etc

$$I = I_0\,\mathrm{jinc}^2 k_\rho a\ \cos^2 2k_x a\ \cos^2 2k_y a$$

$$I = I_0\,\mathrm{jinc}^2\!\left(\frac{ka\sqrt{x^2+y^2}}{z}\right)\cos^2\!\left(\frac{2kxa}{z}\right)\cos^2\!\left(\frac{2kya}{z}\right)$$

This is just like lab L11.10

(18 pts) **Problem 6**. In my lab I have a "0.55 meter spectrometer", meaning that the distance from the entrance slit to the first mirror is 0.55 m. The first mirror has a width of 8.6 cm, and let's assume the grating has the same width as the first mirror, as depicted in the book's picture. The grating is used in the first diffraction order, and the exit slit being positioned at the focal length of the second curved mirror guarantees the regular far-field Fraunhofer pattern will occur at the location of the exit slit. The grating has 1200 grooves/mm. → 12000/cm



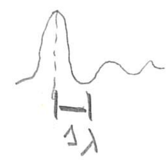
(a) What is the wavelength resolution (in nm) for light that is around $\lambda = 500$ nm? I.e. about how far apart in λ will it be able to tell the difference between two very close wavelengths?

$$\Delta\lambda = \frac{\lambda}{mN} = \frac{500 \cdot 10^{-9}}{(1)(12000\ \frac{\text{grooves}}{\text{cm}} \times 8.6\text{ cm})}$$

$$\Delta\lambda = 4.84 \cdot 10^{-12}$$

$$\boxed{\Delta\lambda = .00484\text{ nm}}$$

this is FWHM, or more specifically from peak to first zero



(b) What will be the width (in millimeters) of the maximum of the diffraction pattern of a 500 nm monochromatic beam, using the exit slit as a "screen". Typically both slits would be set to about this same value, to achieve maximum resolution. Hint: this is just like part of the lab. You can start with the $\lambda = xh/mz$ equation from the formula sheet, where h is the grating period; the spatial width you are looking for here is Δx.

Many ways to do this, one way:

$$\lambda = \frac{xh}{mz} \rightarrow x = \frac{\lambda m z}{h}$$

$$\Delta x = \frac{mz}{h}\Delta\lambda$$

$$= \frac{(1)(.55\text{ m})(.00484 \cdot 10^{-9}\text{ m})}{(\frac{1}{1200} \times 10^{-3}\text{ m})}$$

$$= 3.20 \cdot 10^{-6}\text{ m}$$

$$= \boxed{3.20\ \mu\text{m}}$$

or you can double that if you want to go from first zero on left to first zero on right.