Fresnel's Equation Examples, solving for n in crystals

by Dr. Colton, Physics 471 (last updated: Winter 2024)

Fresnel's equation to find index of refraction for wave travel in crystals is this:

$$\frac{1}{n^2} = \frac{u_x^2}{n^2 - n_x^2} + \frac{u_y^2}{n^2 - n_y^2} + \frac{u_z^2}{n^2 - n_z^2}$$

where (ux,uy,uz) are components of the unit vector in the direction of travel and (nx,ny,nz) are the refractive index parameters with the coordinate system aligned to the principle axes for which the susceptibility tensor is diagonal.

Example 1. nx=1.2, ny = 1.3, nz = 1.4. What are the two n values for wave travel in the (1,1,1) direction?

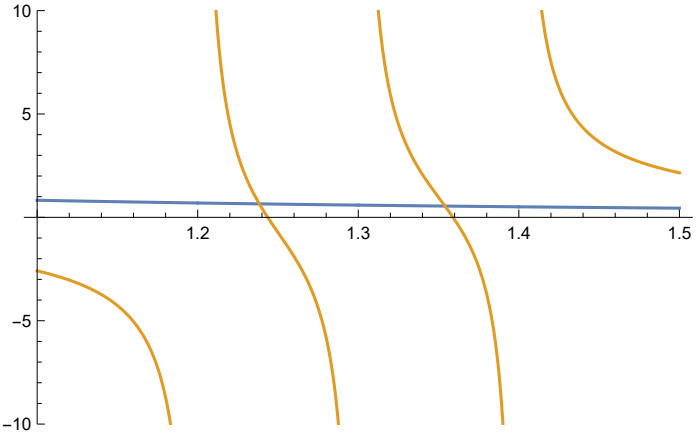
In[1]:=
```
Solve[1 / n^2 == (1 / 3) / (n^2 - 1.2^2) + (1 / 3) / (n^2 - 1.3^2) + (1 / 3) / (n^2 - 1.4^2) , n]
```

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[1]=
```
{{n → -1.35397}, {n → -1.23836}, {n → 1.23836}, {n → 1.35397}}
```

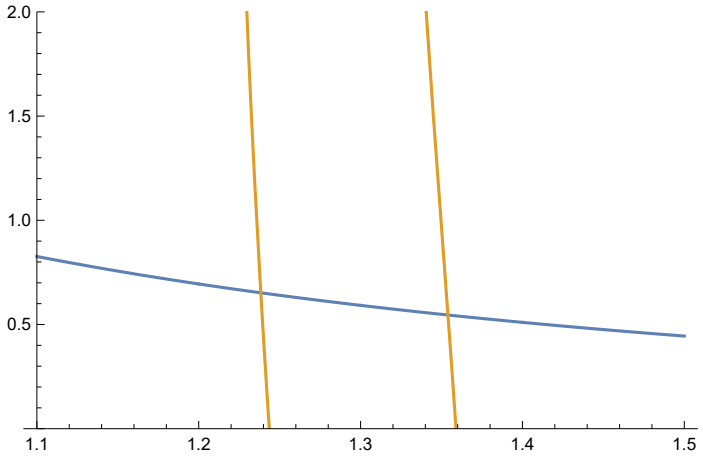
In[2]:=
```
Plot[{1 / n^2 , (1 / 3) / (n^2 - 1.2^2) + (1 / 3) / (n^2 - 1.3^2) + (1 / 3) / (n^2 - 1.4^2)},
 {n, 1.1, 1.5}, PlotRange → {-10, 10}]
```

Out[2]=



In[3]:=
```
Plot[{1 / n^2 , (1 / 3) / (n^2 - 1.2^2) + (1 / 3) / (n^2 - 1.3^2) + (1 / 3) / (n^2 - 1.4^2)},
 {n, 1.1, 1.5}, PlotRange → {0, 2}, Exclusions → {n == 1.2, n == 1.3, n == 1.4}]
```

Out[3]=



In[4]:=
```
FindRoot[1 / n^2 ==
  (1 / 3) / (n^2 - 1.2^2) + (1 / 3) / (n^2 - 1.3^2) + (1 / 3) / (n^2 - 1.4^2) , {n, 1.24}]
FindRoot[1 / n^2 ==
  (1 / 3) / (n^2 - 1.2^2) + (1 / 3) / (n^2 - 1.3^2) + (1 / 3) / (n^2 - 1.4^2) , {n, 1.35}]
```

Out[4]= {n → 1.23836}

Out[5]= {n → 1.35397}

Example 2. nx=1.2, ny = 1.3, nz = 1.4. What are the two n values for wave travel in the (1,0,0) direction?

In[6]:=
```
(* As discussed in class, the two indices must be n = 1.3 and n = 1.4. *)
```

Example 3. nx=1.2, ny = 1.3, nz = 1.4. What are the two n values for wave travel in the (1,1,0) direction?

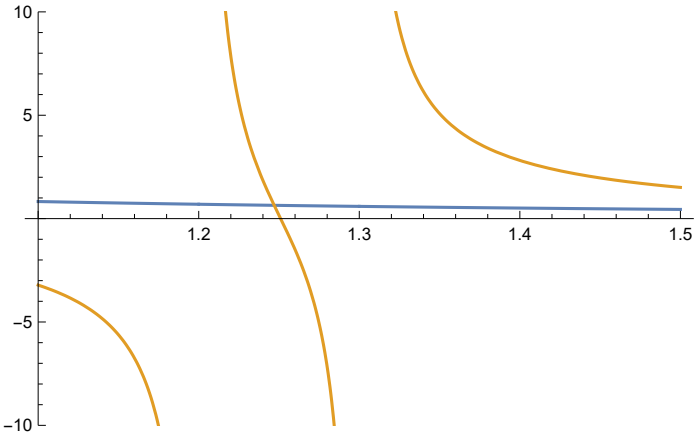
In[7]:=
```
(* As discussed in class, one of the indices must be n = 1.4. With that identification,
Fresnel's Eq becomes this:  *)
Solve[1 / n^2 == (1 / 2) / (n^2 - 1.2^2) + (1 / 2) / (n^2 - 1.3^2) , n]
```

••• Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[7]= {{n → -1.247}, {n → 1.247}}

In[8]:=
```
Plot[{1 / n^2 , (1 / 2) / (n^2 - 1.2^2) + (1 / 2) / (n^2 - 1.3^2)},
 {n, 1.1, 1.5}, PlotRange → {-10, 10}]
```

Out[8]=



In[9]:=
```
FindRoot[1 / n^2 == (1 / 2) / (n^2 - 1.2^2) + (1 / 2) / (n^2 - 1.3^2) , {n, 1.25}]
```

Out[9]=
```
{n → 1.247}
```

In[10]:=
```
(* Alternately, we can use the original Fresnel's equation and
 the "limit method" to get both n's at once. Assume the direction is
 very very close to (1,1,0) but not quite exactly in that direction.
  For example, let u = 1/sqrt(20001) * (100,100,1). Then:  *)

Solve[1 / n^2 == (10000 / 20001) / (n^2 - 1.2^2) +
   (10000 / 20001) / (n^2 - 1.3^2) + (1 / 20001) / (n^2 - 1.4^2) , n]
```

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[10]=
```
{{n → -1.39999}, {n → -1.247}, {n → 1.247}, {n → 1.39999}}
```