Physics 471 Formula Sheet (8 Apr 2024)

$c = 2.998 \times 10^8$ m/s $\quad h = 6.626 \times 10^{-34}$ J·s

$k_B = 1.381 \times 10^{-23}$ J/K $\quad N_A = 6.022 \times 10^{23}$

$m_{elec} = 9.109 \times 10^{-31}$ kg $\quad e = 1.602 \times 10^{-19}$ C

$\varepsilon_0 = 8.854 \times 10^{-12}$ C²/N·m²

$\mu_0 = 4\pi \times 10^{-7}$ T·m/A $\quad \sigma = 5.670 \times 10^{-8}$ W/m²·K⁴

General E&M

$\nabla \cdot \mathbf{E} = \rho/\varepsilon_0$

$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$

$\nabla \cdot \mathbf{B} = 0$

$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \partial \mathbf{E}/\partial t$

$\nabla \cdot \mathbf{D} = \rho_{free}$

$\nabla \times \mathbf{H} = \mathbf{J}_{free} + \frac{\partial \mathbf{D}}{\partial t}$

$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$

$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}, \quad = \varepsilon_0 \varepsilon_r \mathbf{E}$ for linear isotropic

$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}, \quad = \frac{\mathbf{B}}{\mu_0 \mu_r}$ for linear isotropic

$\mathbf{P} = \varepsilon_0 \chi \mathbf{E}$ for linear isotropic

$\varepsilon_r = 1 + \chi$ for linear isotropic

$\rho = \rho_{free} + \rho_{bound} \qquad \rho_{bound} = -\nabla \cdot \mathbf{P}$

$\mathbf{J} = \mathbf{J}_{free} + \mathbf{J}_{bound} + \mathbf{J}_{polar}$

$\mathbf{J}_{bound} = \nabla \times \mathbf{M} \quad \mathbf{J}_{polar} = \partial \mathbf{P}/dt$

$$\nabla^2 E - \varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial \mathbf{J}_{free}}{\partial t} + \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} - \frac{1}{\varepsilon_0} \nabla(\nabla \cdot \mathbf{P})$$

Plane wave: $\tilde{\mathbf{E}} = \tilde{\mathbf{E}}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$

$\tilde{\varepsilon}_r = \tilde{n}^2 \quad \frac{\omega}{k} = \frac{c}{n} \quad \tilde{k} = \frac{\omega}{c}\tilde{n} \quad \lambda = \frac{2\pi}{k_{real}} \quad \delta = \frac{1}{k_{imag}}$

Lorentz Model

$\omega_p^2 = \frac{Nq^2}{m\epsilon_0}$

$\chi = \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\omega\gamma}$ (dielectrics)

$\chi = \frac{\omega_p^2}{-\omega^2 - i\omega\gamma}$ (metals)

Poynting

$\nabla \cdot \mathbf{S} + \frac{\partial u_{field}}{\partial t} = -\frac{\partial u_{medium}}{\partial t}$

$\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$

$u_{field} = \frac{\varepsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2$

$\frac{\partial u_{medium}}{\partial t} = \mathbf{E} \cdot \mathbf{J}$

$I(t) = \langle S(t) \rangle = \frac{1}{2} n \varepsilon_0 c E(t)^2$

Fresnel Eqs

$\alpha = \frac{\cos\theta_2}{\cos\theta_1}, \ \beta = \frac{n_2}{n_1}$

$r = \frac{\alpha - \beta}{\alpha + \beta}, \quad t = \frac{2}{\alpha + \beta}$ (p-polar)

$r = \frac{1 - \alpha\beta}{1 + \alpha\beta}, \quad t = \frac{2}{1 + \alpha\beta}$ (s-polar)

$R = |r|^2, \quad T = \alpha\beta|t|^2$

Two Interfaces

$t_{tot} = \frac{t_{01} t_{12}}{\exp(-ik_1 d \cos\theta_1) - r_{10} r_{12} \exp(ik_1 d \cos\theta_1)}$

$T_{tot} = \frac{n_2}{n_0} \frac{\cos\theta_2}{\cos\theta_1} \frac{|t_{01}|^2 |t_{12}|^2}{|1 - r_{10} r_{12} \exp(i2k_1 d \cos\theta_1)|^2}$

$T_{tot} = \alpha_{02} \beta_{02} |t_{02}|^2 = \frac{T_{max}}{1 + F \sin^2 \Phi/2}$

$T_{max} = \frac{T_{01} T_{12}}{(1 - \sqrt{R_{10}}\sqrt{R_{12}})^2}$

$F = \frac{4|r_{10}||r_{12}|}{(1 - |r_{10}||r_{12}|)^2}$

$\Phi = \phi_{10} + \phi_{12} + 2k_1 d \cos\theta_1$

$\Delta\Phi_{\text{FWHM}} = 4/\sqrt{F}$

$\Delta\lambda_{FWHM} = \frac{\lambda^2}{\pi n_1 d \cos\theta_1 \sqrt{F}}$

$\Delta\lambda_{FSR} = \frac{\lambda^2}{2n_1 d \cos\theta_1}$

$f = 2\pi/\Delta\Phi_{\text{FWHM}} = \Delta\lambda_{FSR}/\Delta\lambda_{FWHM} = \pi\sqrt{F}/2$

Multilayers

$t_{tot} = 1/a_{11}$

$r_{tot} = a_{21}/a_{11}$

$\beta_j = k_j \ell_j \cos\theta_j$

p-polar:

$$M_j = \begin{pmatrix} \cos\beta_j & -\frac{i \sin\beta_j \cos\theta_j}{n_j} \\ -\frac{i n_j \sin\beta_j}{\cos\theta_j} & \cos\beta_j \end{pmatrix}$$

$$A = \frac{1}{2n_0 \cos\theta_0} \begin{pmatrix} n_0 & \cos\theta_0 \\ n_0 & -\cos\theta_0 \end{pmatrix} \times \left(\prod_{j=1}^N M_j\right) \begin{pmatrix} \cos\theta_{N+1} & 0 \\ n_{N+1} & 0 \end{pmatrix}$$

s-polar:

$$M_j = \begin{pmatrix} \cos\beta_j & -\frac{i \sin\beta_j}{n_j \cos\theta_j} \\ -i n_j \sin\beta_j \cos\theta_j & \cos\beta_j \end{pmatrix}$$

$$A = \frac{1}{2n_0 \cos\theta_0} \begin{pmatrix} n_0 \cos\theta_0 & 1 \\ n_0 \cos\theta_0 & -1 \end{pmatrix} \times \left(\prod_{j=1}^N M_j\right) \begin{pmatrix} 1 & 0 \\ n_{N+1} \cos\theta_{N+1} & 0 \end{pmatrix}$$