Review for Optics Exam 1

Info

- Take in the Testing Center, 2/7 (Thurs) through 2/12 (Tues)
- 3 hour time limit, 1% penalty per minute for going over (my guess: will take most of you ~2 hrs)
- Late fee if you start it 2/12 after 6 pm
- Closed book, closed notes
- I will give you the most difficult equations; the simpler equations (definitions, fundamental laws & relationships, etc) you will be expected to have memorized—see following pages
- It's worth 16% of your final course grade
- calculators - No constants/formulas/etc stored. (You will sign a statement)

What to study - complex numbers

- HW problems
- Sample exam problems from pages 111-117 in *P&W* (except the ones with "crystals"—those are from chap 5)
- Class notes
- Reading quizzes
- Textbook
- Old exams from students who took this class from other profs in the past
- Problems from the book that were not assigned
- Problems from other Optics books, such as *Hecht*

Important equations

Will be given if needed
(unless e.g. part of a multiple-choice quiz-like problem)

$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

Fundamental vector calc thms:

- gradient $\int_a^b (\nabla f)\cdot d\vec{l} = f_b - f_a$
- divergence $\int (\nabla\cdot\vec{A})\,dv = \oint \vec{A}\cdot d\vec{a}$
- Stokes' $\int (\vec{\nabla}\times\vec{A})\cdot d\vec{a} = \oint \vec{A}\cdot d\vec{l}$

Need to have memorized
(not necessarily comprehensive)

Values of fundamental constants in standard units to three ~~decimal places~~: sig figs

- c $3\cdot 10^{8}$ m/s
- ε_0 $8.85\cdot 10^{-12}$
- μ_0 $4\pi\cdot 10^{-7}$
- m_e $9.11\cdot 10^{-31}$ kg
- q_e $1.602\cdot 10^{-19}$ C

In rectangular coordinates, how to take:

- gradient $\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$
- divergence $\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$
- curl
- Laplacian

$$\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 \vec{A} = \text{same thing} \quad \frac{\partial^2 (A_x\hat{x} + A_y\hat{y} + A_z\hat{z})}{\partial x^2} + \dots$$

Random vector theorems, like $\nabla\times(\nabla\times\vec{A})=\nabla(\nabla\cdot\vec{A})-\nabla^2\vec{A}$, $\nabla\cdot\left(\nabla\times\vec{A}\right)=0$, etc.

Coulomb's law in vector form

B-S law in vector form

Equation of continuity

$$\nabla\cdot\vec{J}=-\frac{\partial\rho}{\partial t}$$

$$\rho_b=-\vec{\nabla}\cdot\vec{P}$$

Eqn for bound charge (often)

Defn of D $=\epsilon_0 E+P=\epsilon_0\epsilon_r\vec{E}$

~~Defn of~~ $H=\frac{B}{\mu_0}-M=\frac{B}{\mu_0\mu_r}$

Defn of χ

$$\vec{P}=\epsilon_0\chi\vec{E}$$

Defn of polarization current

$$\vec{J}_p=\frac{\partial\vec{P}}{\partial t}$$

$$\vec{E}=\frac{1}{4\pi\epsilon_0}\int\frac{\rho\,dv\,\hat{r}}{r^2}$$

$$\vec{B}=\frac{\mu_0}{4\pi}\int\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

$\nabla\cdot E=\rho/\epsilon_0$

$\nabla\times E=-\partial B/\partial t$

$\nabla\cdot B=0$

$\nabla\times B=\mu_0\vec{J}+\epsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$

4 fundamental Maxwell's equations (aka vacuum eqns)

$c=\frac{1}{\sqrt{\mu_0\epsilon_0}}$

Relationship between c, ε_0, μ_0

Defn of dipole moment $\vec{p}=q\vec{d}$

Defn of polarization

$\vec{P}=\frac{d\vec{p}}{dv}$ $\quad\vec{p}=\int\vec{P}\,dv$

$m=\int p\,dv$

Relationship between ε_r and χ

$\epsilon_r=1+\chi$

$n=\sqrt{1+\chi}$

Two modified Maxwell equations in matter

$\nabla\cdot\vec{D}=\rho_{free}$

$\nabla\times\vec{H}=J_f+\frac{\partial D}{\partial t}$

Basic wave stuff: $\lambda f=v$, $f=1/T$, $k=2\pi/\lambda$, $\omega=2\pi/T$

$$\nabla^2 f = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

Basic wave equation

$\omega/k = v = c/n$

$$\frac{\omega}{k} = \frac{c}{n} \quad \frac{k}{\omega} = \frac{\tilde{n}}{c}$$

Complex number stuff from handout and section 0.2

$e^{ix} = \cos x + i\sin x$

rectangular $\Leftrightarrow$ polar

amp $\angle$ angle

E and B as complex plane waves

$$\tilde{E} = \tilde{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\text{real } E = E_0 \cos(\vec{k}\cdot\vec{r}-\omega t+\phi)$$

Magnitudes of E and B

$\omega / \underline{\tilde{k}} = c / \underline{\tilde{n}}$

$$\tilde{k} = k_{real} + i k_{imag}$$

Defn of κ $\quad \tilde{n} = n + i\kappa$ $\quad \tilde{n} = \sqrt{1+\tilde{\chi}}$

Relationship between n and χ

k_{real} and k_{imag}: decaying sine wave, relationship to n & κ

absorption length, $1/\alpha$

$$I \propto E^2$$

$$I \sim I_0 e^{-2k_{imag} z}$$

$$\alpha = 2k_{imag}$$

$$\text{abs. length} = \frac{1}{2k_{imag}}$$

$$\nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 3 \text{ terms}$$

P&W's complicated wave equation with 3 sources on RHS

Solution to driven/damped oscillator $\tilde{X}_0 = \frac{qE_0}{m}\frac{1}{\omega_0^2-\omega^2-i\omega\gamma}$

Lorentz oscillator model's value for complex χ & n

$$\tilde{\chi} = \frac{\omega_p^2}{\omega_0^2-\omega^2-i\omega\gamma}$$

$$\tilde{n}^2 = 1 + \frac{\omega_p^2}{\text{denom.}}$$

Defn of plasma freq

$$\omega_p = \sqrt{\frac{Nq^2}{\epsilon_0 m}}$$

How to convert χ formula for use with multiple resonant freqs; oscillator strength

$$\tilde{\chi} = \sum_j \frac{f_j\,\omega_{pj}^2}{\omega_{0j}^2-\omega^2-i\omega\gamma_j}$$

$\tilde{\chi} = \frac{\omega_p^2}{-i\omega\gamma - \omega^2}$ $\tilde{n} = \sqrt{1+\tilde{\chi}}$

Complex χ & n for conductors

Defn of DC conductivity $\sigma = \frac{Nq^2}{m\gamma}$

Energy stored in electric & magnetic fields $U = \int \left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) dV$

Poynting Theorem

$\nabla \cdot \vec{S} + \frac{\partial u_{field}}{\partial t} = -\frac{\partial u_{medium}}{\partial t}$

$\vec{S} = \frac{1}{\mu_0} E \times B$ $|B| = \frac{|E|}{v}$

Defn of Poynting vector

Directions of E, B, and $\hat{k}$ & S

$\hat{E} \times \hat{B} = \hat{k} = \hat{S}$

Relationship between I and E_0 $I = \langle S \rangle = \frac{1}{2} \frac{1}{\mu_0} \frac{E^2}{c/n}$

Snell's Law

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

Fresnel Equations for r and t

- p-polarization at an angle
- s-polarization at an angle

Defns of α and β

$\beta = \frac{n_2}{n_1}$ $\alpha = \frac{\cos\theta_2}{\cos\theta_1}$

$r = \frac{\alpha - \beta}{\alpha + \beta}$ $t = \frac{2}{\alpha + \beta}$

$r = \frac{1 - \alpha\beta}{1 + \alpha\beta}$ $t = \frac{2}{1 + \alpha\beta}$

Normal incidence formulas: subset of angle formulas

Reflectance, transmittance/transmission $R = |r|^2$ $T = 1 - R$

Brewster's angle condition $\tan\theta_B = \frac{n_2}{n_1}$

TIR condition

$n_1 \sin\theta_1 = n_2 \sin(90°)$

Complicated evanescent wave formula

Jones vectors for linear polarized light

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$

Jones vectors for:
- RCP
- LCP
- elliptical

Complicated eqn for α of elliptically polarized light

Jones matrices for:
- linear polarizer at θ
- $\lambda/4$ fast axis at θ
- $\lambda/2$ fast axis at θ

Jones matrices for:
- reflection $\begin{pmatrix} -r_p & 0 \\ 0 & r_s \end{pmatrix}$
- transmission $\begin{pmatrix} t_p & 0 \\ 0 & t_s \end{pmatrix}$