

Lecture 37: Wed, 2 Apr 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. The smallest radius of a laser beam is known as the

- a. ankle
- b. knee
- c. neck
- d. pinky
- e.** waist
- f. wrist

Q2. The distance over which the beam radius near a focus stays about constant is the _____ range

- a. Fresnel
- b. Gaussian
- c. Helmholtz
- d. Home_on_the
- e.** Rayleigh

Q3. The ABCD law for Gaussian beams is used for:

- a. Finding the ABCD matrices for a Gaussian laser beam
- b. Finding the diffraction pattern in the near-field
- c.** Finding the Gaussian beam parameters after an optical element

Ch. 11: More Applications

Sectn 11.2. Diffract off Gaussian field profile



laser beams typically brightest in middle, often have Gaussian fall-off

$$E(x'y') = E_0 e^{-\frac{r'^2}{w_0^2}}$$

$$r' = \sqrt{x'^2 + y'^2}$$

w_0 = "beam waist"

\downarrow
(distance
not omega)

measure of width

→ radius at which Intensity = $I_0 \left(\frac{r'}{r}\right)^2$

$$= 13.5\% I_0$$

How does it propagate? → diffracts through
"infinitely large aperture"



Back to Fresnel Approximation

$$E(x, y, z) = -\frac{i e^{ikz}}{\lambda^2} e^{\frac{i k}{2z}(x^2 + y^2)} \int_0^{2\pi} \int_{-\infty}^{\infty} \left(E_0 e^{-\frac{r'^2}{w_0^2}} \right) e^{\frac{i k}{2z} r'^2} e^{-i \frac{k}{2} (x'y' + yz')} r' dr' d\theta'$$

$r \cos \theta \quad r' \cos \theta' \\ \text{g. milia for } y \\ -i \frac{k}{2} R' (\cos \theta \cos \theta' - \sin \theta \sin \theta')$

$\underbrace{e}_{\cos(\theta + \theta')}$

As before, $\int \text{integral} \rightarrow 2\pi J_0\left(\frac{kr'}{2z}\right)$

$$= -\frac{i}{\lambda^2} 2\pi e^{ikz} e^{\frac{i kr^2}{2z}} E_0 \underbrace{\int_0^\infty r' dr' e^{-(\frac{1}{w_0^2} - \frac{ikr^2}{2z})} J_0\left(\frac{kr'}{2z}\right)}$$

$$\downarrow \frac{2\pi}{\lambda} = k$$

$$\int_0^\infty e^{-\frac{x^2}{a^2}} J_0(bx) x dx = e^{-\frac{b^2}{4a}}$$

$$= e^{-\frac{(kr)^2}{4(w_0^2 - \frac{k^2}{2z})}}$$

"normalize" denominator

$$\frac{2(w_0^2 - \frac{k^2}{2z})}{2(w_0^2 - \frac{kr^2}{2z})}$$

$$= 2 \frac{k}{2z} \left(\frac{2z}{kw_0^2} - \frac{1}{k} \right)$$

$$\text{put in polar form} \\ \text{and } \tan^{-1}\left(\frac{-1}{x}\right) = \tan^{-1}(-1)$$

end Monday

start Wednesday

introduce new symbols

$$E(x, y, z) = E_0 \frac{w_0}{W} e^{-\frac{r^2}{W^2}} e^{ikz + i\frac{kr^2}{2R} - i\tan^{-1}\left(\frac{R}{w_0}\right)}$$

$$\text{with } \begin{cases} z_0 = \frac{kw_0^2}{2} \\ R = z + \frac{z_0^2}{z} \\ W = w_0 \sqrt{1 + \frac{z^2}{z_0^2}} \end{cases}$$

} factors of z

Anciently, I think you
can show this
satisfies Helmholtz eqn

$$I \propto |E|^2$$

$$I = I_0 \left(\frac{w_0}{w}\right)^2 e^{-2r^2/w^2}$$

$$= \frac{I_0}{1 + 2^2/z_0^2} e^{-2r^2/w^2}$$

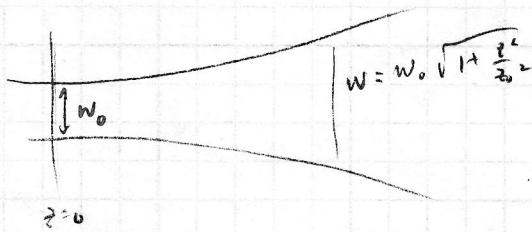
z_0 = "Rayleigh range"

= aka "confocal parameter"

$w \approx$ width of

beam, expands with z

Also notice it's a measure of intensity decrease



Maybe
widths should
be $2w_0$ and $2w$
No - same as used for
original width
on last page

z_0 = measure of
how fast beam expands

$$\text{At } z = z_0 \quad I = \frac{I_0}{2} e^{-2r^2/(w_0/2)^2} \quad \text{width up by } \sqrt{2}$$

$w = r\sqrt{2}z_0$ $\text{and intensity down by } 2$

$$\text{At } z = 2z_0 \quad I = \frac{I_0}{5} e^{-2r^2/(w_0\sqrt{5})^2} \quad \text{width up by } \sqrt{5}$$

center intensity down by $\sqrt{5}$

$$\text{Large } z \Rightarrow w = \frac{w_0 z}{z_0} \quad \text{growing straight like } y = mx \quad \tan \theta = \frac{w}{z} = \frac{w_0 z}{z_0} \rightarrow \theta = \tan^{-1} \frac{w_0}{z_0} = \frac{\pi}{2} - \frac{w_0}{z_0}$$

Small w_0 (tight focus) \rightarrow small z_0 (fast diverging)

$$\theta = \frac{z}{w_0}$$

$$S = \frac{\lambda}{\pi w_0}$$

$$(\lambda = \frac{\lambda_{\text{max}}}{n})$$



Note: Run light backwards in time, get focusing (and then diverging beam after it focuses to w_0)

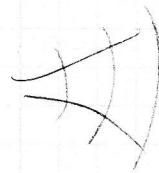
"depth of focus"
 $= 2z_0$

Coming into, and out of, focus

phase factors of E

A) e^{ikz} : obvious

B) $e^{ikr^2/2R}$ curved wavefront



(note: $z \rightarrow 0$)

$R = \infty$ this term goes away

2) $z \rightarrow \text{large}$

$R = z$

$$\text{then } e^{ik(z + r^2/2z)} = e^{ik(z + \frac{r^2}{2z})} \\ = e^{ik\sqrt{z^2 + r^2}} \quad (z \gg r) \\ = e^{ikr} \quad (\text{real } n!)$$

spherical wavefront!

C) $e^{-i\pi n^2(\frac{r^2}{2z_0})}$ Jackson's
phase shift between actual wave + pure plane wave

$= -\frac{\pi}{2}$ when $z = -\infty$

$= +\frac{\pi}{2}$ when $z = +\infty$

$= \pm \frac{\pi}{4}$ when $z = \pm z_0$

also →

→ when light goes through a lens,

it has an overall phase shift of π

"Gauge shift" (book: see for other profiles, not just Gaussian)

Gaussian Wavefronts

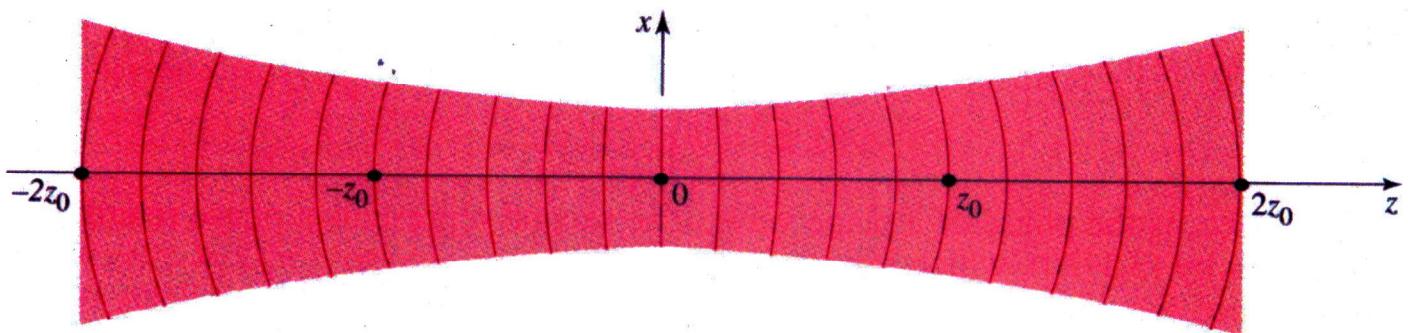
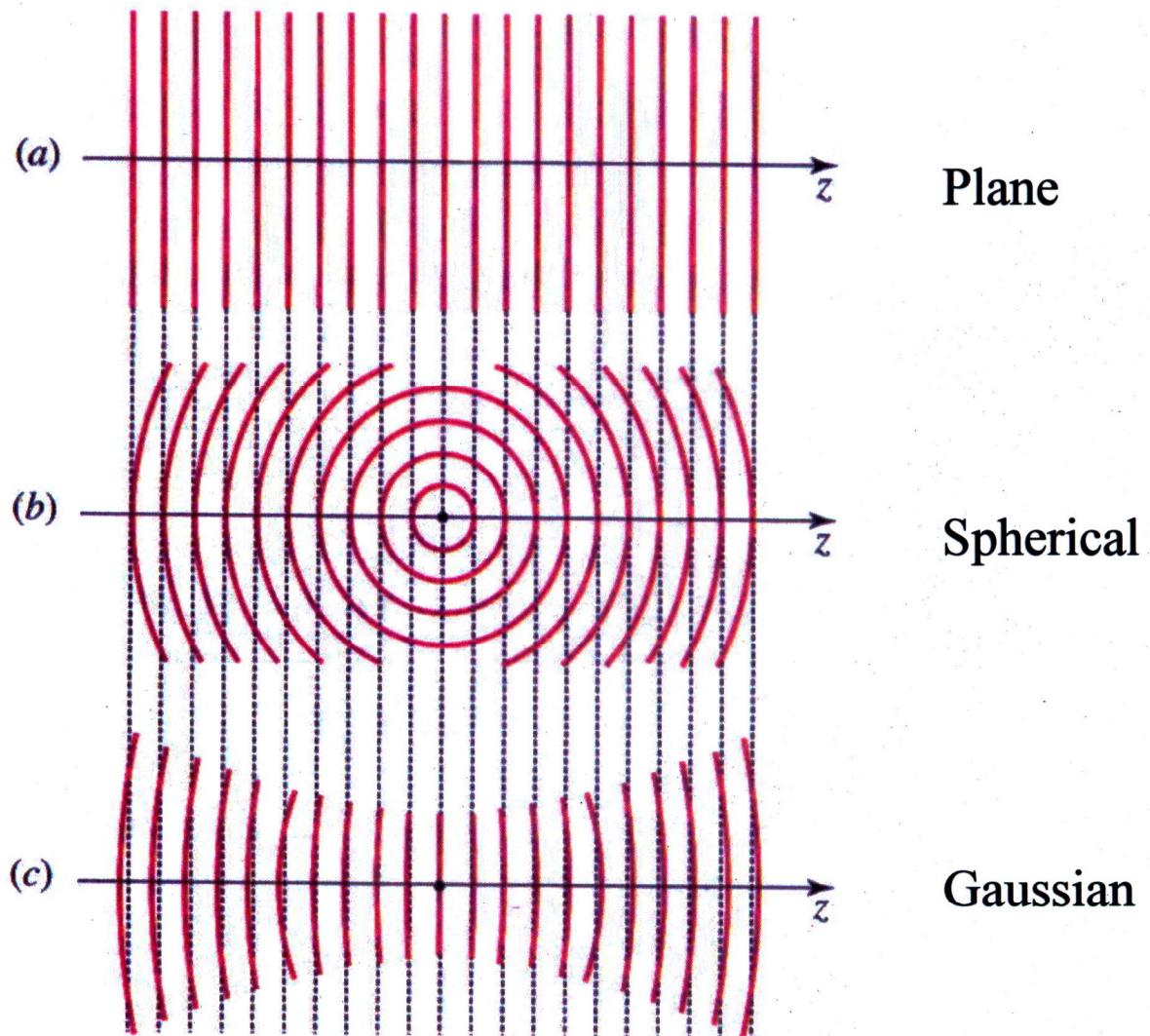


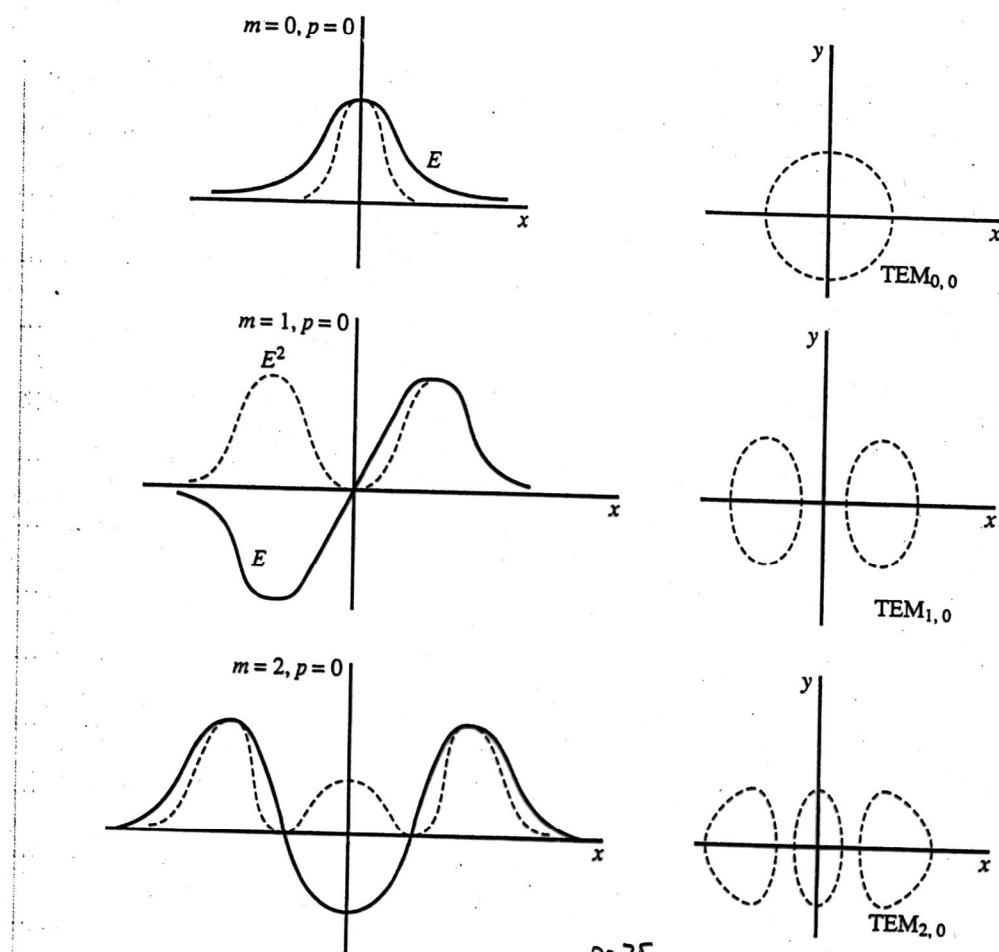
Figure 3.1-7 Wavefronts of a Gaussian beam.



(Figures from Saleh and Teich, *Photonics*, 2nd edition, pg 82)

Fundamentals of

Higher - Order Modes



pg 75

FIGURE 3.5. The field E , intensity E^2 , and "dot" pattern of various modes.

Laser Electronics, 3rd ed., J. T. Verdeyen

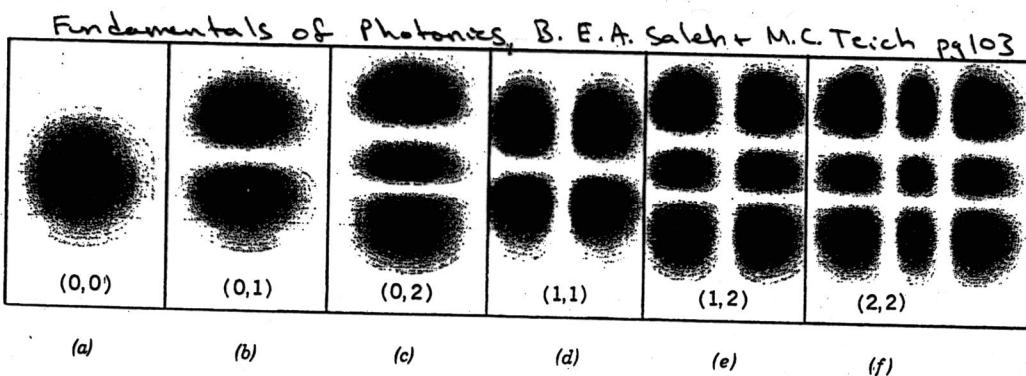


Figure 3.3-2 Intensity distributions of several low-order Hermite-Gaussian beams in the transverse plane. The order (l, m) is indicated in each case.

Gaussian beams \rightarrow stable resonator

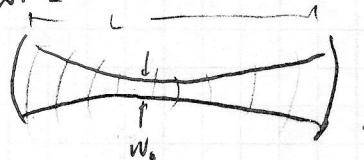
before



infinitely thin
rays bouncing
back + forth

$$\text{if not infinite} \rightarrow 0 < \left(1 - \frac{L}{R_1}\right) \left(1 - \frac{L}{R_2}\right) < 1 \quad \text{criterion}$$

Now, beam w/ width



Want mirror curvature to
match wave front curvature
at that point.

$$k = z + \frac{z^2}{z}$$

$$\textcircled{1} \quad \frac{z_1 + \frac{z_0^2}{z_1}}{z_1} = R_1 \quad \rightarrow z_1 = \frac{R_1}{2} + \frac{\sqrt{R_1^2 - 4z_0^2}}{2}$$

$$z^2 + z^2 = z_0^2$$

$$z^2 - zR - z_0^2 = 0$$

$$z = \frac{R}{2} \pm \frac{\sqrt{R^2 - 4z_0^2}}{2}$$

$$\textcircled{2} \quad \frac{z_2 + \frac{z_0^2}{z_2}}{z_2} = R_2 \quad \rightarrow z_2 = \text{similar}$$

take +

$$\textcircled{3} \quad z_1 + z_2 = L$$

$$\left(\frac{R_1}{2} + \frac{\sqrt{R_1^2 - 4z_0^2}}{2} \right) + \left(\frac{R_2}{2} + \frac{\sqrt{R_2^2 - 4z_0^2}}{2} \right) = L$$

$$z_0^2 = -L \frac{(L-R_1)(L-R_2)(L-R_1-R_2)}{(2L-R_1-R_2)^2} \quad (\text{done w/ } \text{Mathematica})$$

$$= +L \frac{(R_1-L)(R_2-L)(R_1+R_2-L)}{(R_1-L+R_2-L)^2} \quad R$$

$$\text{Num} = \lambda R_1 \left(1 - \frac{L}{R_1}\right) R_2 \left(1 - \frac{L}{R_2}\right) \underbrace{\left(\frac{R_1 R_2}{L} \left[1 - \left(1 - \frac{L}{R_1}\right) \left(1 - \frac{L}{R_2}\right)\right]\right)}_{\text{since this} = R_1 + R_2 - L}$$

$$= R_1^2 R_2^2 (x)(1-x)$$

$$\text{with } x = \left(1 - \frac{L}{R_1}\right) \left(1 - \frac{L}{R_2}\right)$$

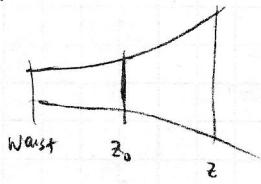
z_0^2 must be positive

$\rightarrow x$ must be between 0 and 1

$$0 < \left(1 - \frac{L}{R_1}\right) \left(1 - \frac{L}{R_2}\right) < 1$$

same condition as
before!

q parameter



All "input" info contained in

z_0 and z

↳ sets overall curvature

↳ sets where you are

Combine this info into one "q-parameter"

$$q = z + iz_0$$

How does q change as you pass through optical element?

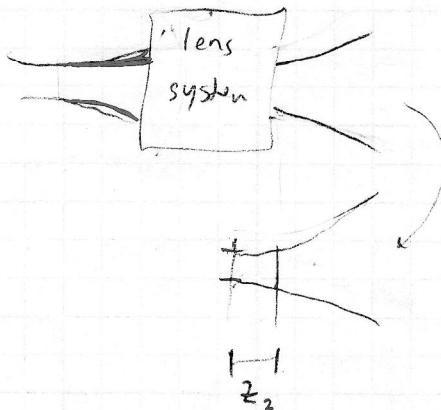
Then:

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}$$

when $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ = normal matrix (ray) for the element

z_{02} = new Rayleigh range
 z_2 = how far from "virtual waist"

"ABCD law for Gaussian beams"



new beam looks like it's coming from a new waist, with a new curvature

p+6
 a Books eqn defines $q = z - iz_0$] Note
 eqn still holds]

Proof in book (App. II, A)

- (1) free space
- (2) thin lenses
- (3) thick windows

} I won't prove