Optics | Winter 08

day 1
- take roll
- Go over syllabus
- Report results of prerequisite survey
- type up notes each day?

↑ discuss

Adam's office hour

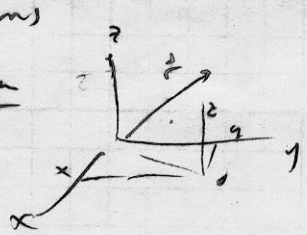
CAMPAD

Multivariable Calc review

Vector: ordered triple

Coordinate Systems

Cartesian

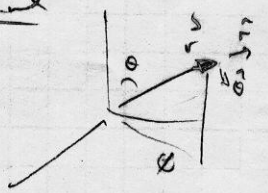


$pt = (x, y, z)$

$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

tiny volume element $dv = dx\,dy\,dz$ (not $d\vec{r}$)

Spherical

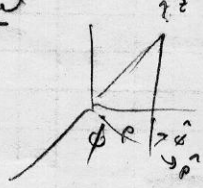


$pt = (r, \theta, \phi)$ (Poll: θ vs φ)

$\vec{r} = r\hat{r}$

$dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$

cylindrical

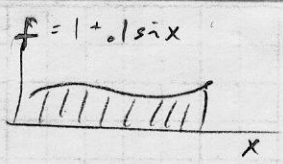


$pt = (\rho, \phi, z)$

$\vec{r} = \rho\hat{\rho} + z\hat{z}$

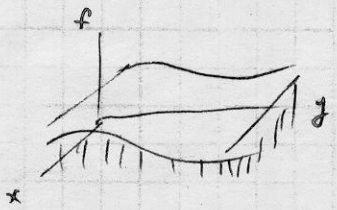
$dv = \rho\, d\rho\, d\phi\, dz$

1D integral: $f = 1 + .1\sin x$



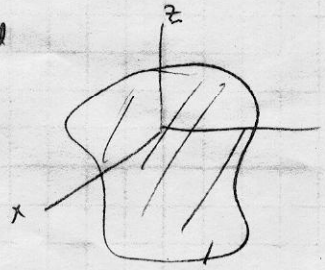
$\int f\,dx$ = area under curve

2D integral



$\int f\,dx\,dy$
Volume under the surface defined by limits of integration

3D integral



$\int f\,dx\,dy\,dz$

if $f = 1$, $= \int dv$ = total volume inside

if f = function (like a weighted average)

$\int f\,dx\,dy\,dz$

simplest example f = charge density

$\int f\,dv$ would give you total charge

Scalars vs Vectors

functions can be "scalar fields" like charge density, temperature, etc

or "vector fields" where each pt in space has magnitude + direction

like eg wind or electric fields

Given a vector symbol: $\vec{E} = E_x(x,y,z)\,\hat{x} + E_y(x,y,z)\,\hat{y} + E_z(x,y,z)\,\hat{z}$

E_x, E_y, E_z scalar fields (but not independent)

Derivatives of fields

Scalar: gradient of a scalar field $\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$

- partial derivatives
- gradient is a vector field
- pts in "downhill" direction. direction of eg. heat flow

Lecture 2: Thurs, 10 Jan 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. How many "Maxwell Equations" are there?

Q2. Coulomb's Law is:

a. $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{Vol}\rho(\vec{r}')\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}dv'$

b. $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{Vol}\rho(\vec{r}')\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^2}dv'$

c. $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{Vol}\rho(\vec{r}')\frac{1}{|\vec{r}-\vec{r}'|^2}dv'$

d. $\vec{E}(\vec{r}) = \frac{1}{4\pi\mu_0}\int_{Vol}\rho(\vec{r}')\frac{1}{|\vec{r}-\vec{r}'|^2}dv'$

e. $\vec{E}(\vec{r}) = \frac{1}{4\pi\mu_0}\int_{Vol}\rho(\vec{r}')\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^2}dv'$

Q3. What is the divergence of the magnetic field? $\vec{\nabla}\cdot\vec{B} = \ldots$

a. ρ/ε_0
b. $-d\rho/dt$
c. 0
d. $\mu_0\mathbf{J}$

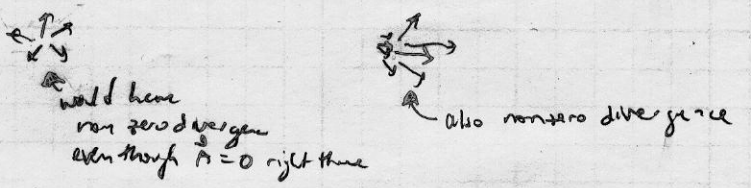
day 2

CAMPAD

vector fields derivatives of $\vec{A}$ (could be wind)

divergence of a vector field $\vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

- is a scalar field!
- represents how much vector associated w/ $\vec{A}$ are "spreading" at each pt in space



curl of a vector field $\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$

$= \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - \hat{y}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$

- is a vector field!
- represents whether vectors associated w/ $\vec{A}$ curl around your pt of interest

Integrals of fields specifically what do you get when you integrate a derivative?

Fundamental Thm of Calc. $\int_a^b \frac{df}{dx}\, dx = f(b) - f(a)$

Geometric: add up small height changes, get total height change
pic. on Griffiths pg 29

↓ in 3-D, we have 3 types of derivatives

Gradient Thm $\int_a^b (\vec{\nabla} f) \cdot d\vec{\ell} = f(b) - f(a)$ Geometric: same
pic on Griffiths pg 29

corollary 1: path independent
corollary 2: $\oint = 0$

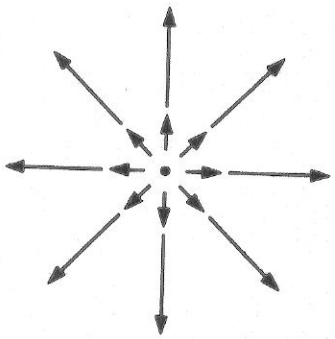
$\vec{\nabla} f$ is like "conservative force" $W = \int \vec{F} \cdot d\vec{\ell}$

Divergence Thm $\int_V (\vec{\nabla} \cdot \vec{A})\, d\tau = \oint_S \vec{A} \cdot d\vec{a}$

S = closed body of volume, $d\vec{a}$ pts outward ⊥ to S

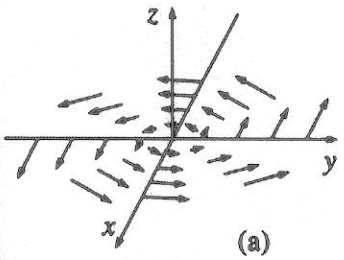
integral of derivative over a region = value of function at boundary
Geometric: source of flow like a faucet, 2 ways to measure how much produced
(1) count up faucets in volume
(2) count up flow across body

Figures from Griffiths, *Intro to Electrodynamics*



(a)

Divergence



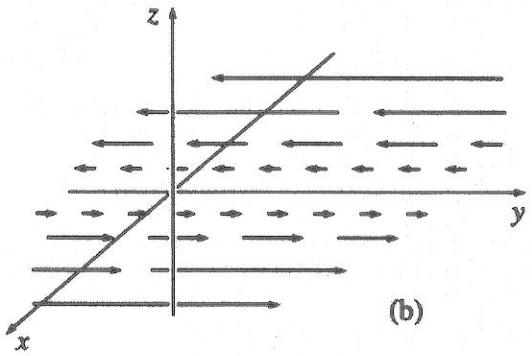


Figure 1.19

Curl

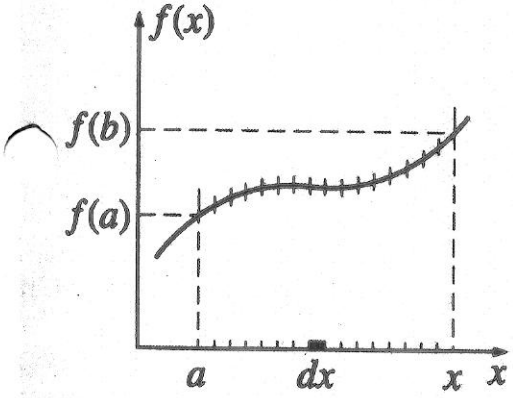


Figure 1.25

Fundamental Thm of Calc

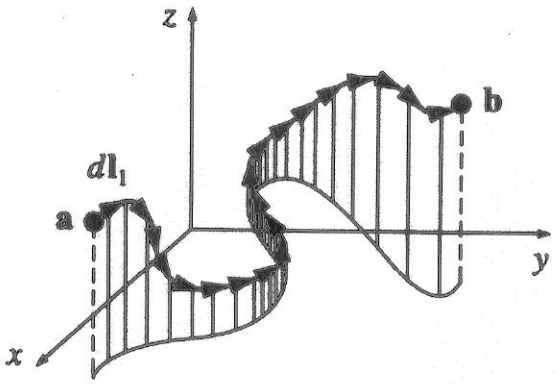


Figure 1.26

Gradient Theorem

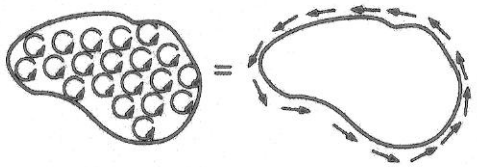


Figure 1.31

Curl Theorem (Stokes' Theorem)

Curl thm aka Stokes thm

$$\int_S (\nabla \times \vec{A}) \cdot d\vec{a} = \oint_P \vec{A} \cdot d\vec{\ell}$$

P = perimeter

integral of derivative over a region = value of function on body

Geometric: Griffiths, pg 35 figure

Infinitesimal of swirl:

(1) add up each swirl source on the surface
or (2) go along edge to find out how much the flow is following the body.

Maxwell Eqns $\vec{E}$ and $\vec{B}$ are fields with special properties

1. $\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$
2. $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
3. $\vec{\nabla} \cdot \vec{B} = 0$
4. $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$

Hence light is an electromagnetic wave!

- Helmholtz thm (?)

where do they come from? (Physics 441 in detail)

1. Coulomb's Law ←→ force from static charges

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \hat{r}$$

define $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$ then $\vec{F} = q\vec{E}$

more general

in Lagrange

$$\vec{E}_{\text{from } Q} = \frac{1}{4\pi\epsilon_0} \int \frac{dQ}{r^2} \hat{r}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0} \int \rho \frac{dv'}{r^2} \hat{r}}$$ ρ = charge density

✓ because you integrate by varying $\vec{r}'$, not $\vec{r}$

from this, we get

1. Gauss's Law $\oint_S \vec{E} \cdot d\vec{a} = q_{enc}/\epsilon_0$

✓ div thm ✓ defn of ρ

$$= \int_V (\nabla \cdot \vec{E})\, dv = \frac{1}{\epsilon_0} \int_V \rho\, dv$$

true for all arbitrary volumes

$$\rightarrow \boxed{\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0}$$

could also be obtained by taking divergence of ↗ directly

2. if you take curl of Coulomb's Law, $\nabla \times \vec{E} = 0$ true for statics

But Faraday found

Faraday's Law $\mathcal{E} = -\frac{d\Phi_B}{dt}$

defn $\mathcal{E}$ ↓

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a}$$

Stokes thm

$$\oint (\nabla \times \vec{E})\, da = \int \left(-\frac{\partial \vec{B}}{\partial t}\right) \cdot d\vec{a}$$

true for all arbitrary surfaces

$$\rightarrow \boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

3. No magnetic monopoles → "Gauss's Law for $\vec{B}$"

$$\oint_S \vec{B}\cdot d\vec{a} = 0$$

$$\downarrow$$

$$\int_V (\vec{\nabla}\cdot\vec{B})\, dv = 0$$

true for all volumes

$$\rightarrow \boxed{\nabla\cdot B = 0}$$

4. Biot Savart Law, force between a current + a moving charge

$$\vec{F} = q\,\vec{v}\times\vec{B}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int_l \frac{d\vec{I}\times\hat{r}}{r^2} \quad (dI = I\,d\vec{l}')$$ true for "static currents"

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}\times\hat{r}}{r^2}\, dv'}$$

$\vec{J}$ = current density, $\frac{\text{current}}{\text{area}}$

from that, we get

Ampere's Law

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$ (if J is not changing)

↓ Stoke's Thm

↳ defn J

$$\int_S (\nabla\times B)\cdot d\vec{a} = \int_S \mu_0 \vec{J}\cdot d\vec{a}$$

true for all surfaces

$$\rightarrow \vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$ true for statics

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day 3

Colton: if a changing B field can produce an E field (Faraday's Law) shouldn't a changing E field produce a B field?

Yes → something's missing

Review:
$\nabla\cdot E = \rho/\epsilon_0$
$\nabla\times E = -\partial B/\partial t$
$\nabla\cdot B = 0$
$\nabla\times B = \mu_0 J + \ldots$