Lecture 11: Fri, 1 Feb 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. In polaroid films, polymer chains are aligned. The polarization axis that transmits light is oriented
   a. along the chains
   b. perpendicular to the chains.
   c. at 45 degrees to the planes

Q2. When multiple elements are in a beam path, the Jones matrices to find the final optical polarization are written
   a. with the first element’s matrix on the right
   b. with the first element’s matrix on the left
   c. in any order.

Q3. The element that is typically used to change linearly polarized light into circularly polarized light is the
   a. linear polarizer
   b. quarter waveplate
   c. half waveplate
Certain optical components can affect the polarized state of light. Linear polarizers rotate into x, y, or other linear polarized magnetic planes. Represent mathematically with matrices:

\[
\begin{pmatrix}
1 & 0 \\
0 & 0 \\
\end{pmatrix}
\]

for x-directed polarized light.

Transform: \[
\begin{pmatrix}
x' \\
y'
\end{pmatrix} = \begin{pmatrix}
1 & 0 \\
0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix}
\]

leaves only x-polarized light.

Similarly, \[
\begin{pmatrix}
z' \\
y'
\end{pmatrix} = \begin{pmatrix}
0 & 1 \\
0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix}
\]

for y-polarized light.

\[
\begin{pmatrix}
\frac{1}{2} & \frac{i}{2} \\
\frac{1}{2} & \frac{-i}{2}
\end{pmatrix}
\]

for a 90° transformation.

\[
\begin{pmatrix}
\cos\theta & -\sin\theta \\
\sin\theta & \cos\theta
\end{pmatrix}
\]

for a linear transformation at an angle \(\theta\).

\[
\begin{pmatrix}
\cos\phi & \sin\phi \\
-\sin\phi & \cos\phi
\end{pmatrix}
\]

Transformation matrix for \(\phi\) degrees.

\[
\begin{pmatrix}
\cos^2\phi + \sin^2\phi & \cos\phi \sin\phi + \cos\phi \sin\phi \\
\cos\phi \sin\phi + \cos\phi \sin\phi & \cos^2\phi + \sin^2\phi
\end{pmatrix}
\]

Overall transformation matrix.

\[
\begin{pmatrix}
z' \\
y'
\end{pmatrix} = \begin{pmatrix}
\cos^2\phi + \sin^2\phi & \cos\phi \sin\phi + \cos\phi \sin\phi \\
\cos\phi \sin\phi + \cos\phi \sin\phi & \cos^2\phi + \sin^2\phi
\end{pmatrix}
\begin{pmatrix}
z \\
y
\end{pmatrix}
\]

\[
\begin{pmatrix}
\cos\phi & \sin\phi \\
-\sin\phi & \cos\phi
\end{pmatrix}
\begin{pmatrix}
z \\
y
\end{pmatrix}
\]

Transformations and rotations of polarized light.
Waeghele & As EM waves pass through material, the index of refraction is larger for one polar than the other.

"Optic (Birefringent) -- different inside material.

\[
e_n = 45^\circ
e\]

Row out of phase with each other!
If 90° -> circular polarization!

Said materials called Birefringent

Example: calcite \( n_o = 1.658 \)
\( n_e = 1.486 \)

Water ice \( n_o = 1.309 \)
\( n_e = 1.313 \)

C (ordinary = slow)
\( \epsilon = \text{extraordinary = fast} \)

\( \Delta \) different paths

diff. speeds wave prop

\( \Delta \phi = \frac{2\pi}{n_e} - \frac{2\pi}{n_o} \)

Typical applications:

Quarter wave plate \( \Delta \phi = \frac{\pi}{2} + 2\pi n \)

Half wave plate \( \Delta \phi = \pi + 2\pi n \)
Calcite Structure (Calcium Carbonate)

http://www.uwgb.edu/DutchS/PETROLGY/Calcite%20Structure.HTM
by Steven Dutch, UW Green Bay

- Has a "modified NaCl" structure (in about the same sense that an antique that has had every part replaced is still an antique)
- Left: NaCl—Na atoms = purple, Cl atoms = green
- Right: calcite—Ca = yellow, O = blue, C = gray
  - Note the rows of alternating Ca and CO₃ units, just like Na and Cl alternate in halite.
- Calcite is not cubic. The carbonate groups break up the cubic symmetry in several ways:
  - Their three-fold symmetry axes line up with only one of the symmetry axes of the cube (in red).
  - They alternate in orientation (shown by the two shades of gray).
  - The wide spacing of the carbonate groups stretches the atomic planes and distorts the cube into a rhombohedron.
Birefringence of Calcite Crystal
quarter wave plate - used for transmitting light to cire (as via versa)
needs linear plo @ 45° to fast axis.

Proof: Jones matrix for \(\frac{\pi}{4}\), fast axis @ 0° = \(
\begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
\)

Send in 45° light, \(
\begin{pmatrix}
\frac{\sqrt{2}}{2} \\
\frac{-\sqrt{2}}{2}
\end{pmatrix}
\)

Get out \(
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
\)
\begin{pmatrix}
\frac{\sqrt{2}}{2} \\
\frac{-\sqrt{2}}{2}
\end{pmatrix}
\frac{1}{\sqrt{2}} \begin{pmatrix}
1 & i \\
-i & 1
\end{pmatrix}
what would you get w/ -45° light? (I guess) \(
\begin{pmatrix}
0 & -1 \\
1 & 0
\end{pmatrix}
\)

Matrix for \(\frac{\pi}{4}\), fast axis @ 45°
\[
M = \frac{e^{i\frac{\pi}{4}}}{\sqrt{2}} \begin{pmatrix}
1 & -i \\
-i & 1
\end{pmatrix}
\]

Matrix for \(\frac{\pi}{4}\), fast axis @ 0°
\[
M = \begin{pmatrix}
\cos\frac{\pi}{4} + i\sin\frac{\pi}{4} & \sin\frac{\pi}{4} - i\cos\frac{\pi}{4} \\
\sin\frac{\pi}{4} + i\cos\frac{\pi}{4} & \cos\frac{\pi}{4} - i\sin\frac{\pi}{4}
\end{pmatrix}
\]

Good problem: send in RCP to \(\frac{\pi}{4}, 0°\). Cut out linear @ 45°? -45°? other?

If linear p/ not at 45° + fast axis, probably get some sort of elliptical out.
half wave plate - used mainly for rotating the polar.

proof: matrix for \( \frac{1}{2} \) twist arises as \( 0^\circ = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \) see why??

let's send in at 30° linear: \( \left( \frac{\sqrt{3}}{2}, \frac{1}{2} \right) \)

out = \( \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \left( \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix} \right) = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix} \)

= -30°

Rule: if you consider the light to be at 0° and \( \frac{1}{2} \) at 0° relative to theta then final light = 2θ

Did that just happen?

\[
\begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\]

matrix for 45°: \( M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \)

matrix for \( \frac{1}{2} \) θ°: \( M = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \)

\[
\begin{pmatrix}
\cos^2 \theta - \sin^2 \phi & 2 \cos \theta \sin \phi \\
2 \cos \theta \sin \phi & \sin^2 \phi - \cos^2 \phi
\end{pmatrix}
\]

\[
= \begin{pmatrix}
\frac{1+\cos 2\theta}{2} \sin 2\phi & \frac{1-\cos 2\theta}{2} \\
\frac{1-\cos 2\theta}{2} \sin 2\phi & \frac{1+\cos 2\theta}{2}
\end{pmatrix}
\]

[high school math]
Lecture 12: Mon, 4 Feb 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. Reflection generally
   a. changes handedness (left vs right light)
   b. conserves handedness

Q2. Which famous experimental physicist not only established a law of induction, but also discovered that magnetic fields can interact with light? Faraday

Q3. The technique of finding the optical constants $n$, $\kappa$ from polarization changes in reflectance is called
   a. circularometry
   b. ellipsometry
   c. hyperbolatry
   d. linearometry
   e. parabolitry
Figure 8.17  Atomic arrangement for calcite looking down the optical axis.
A calcite crystal (blunt corner on the bottom). The transmission axes of the two polarizers are parallel to their short edges. Where the image is doubled the lower, undeflected one is the ordinary image. Take a long look: there's a lot in this one. (Photo by E.H.)

**Figure 8.19** A light beam with two orthogonal field components traversing a calcite principal section.
Final Jones matrices

reflection: \[ m = \begin{pmatrix} r & 0 \\ 0 & r \end{pmatrix} \] if "r" means in x-direction
"s" in y-direction

why the negative sign?
"handshaking inversion"

- e.g., pair \[ \rightarrow \text{conservation of angular momentum} \]

transmission \[ m = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \]

Summary chart

end of chapter!
### Jones matrices summary

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{LP in x-dir}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \lambda/2, \text{ fast } = 0^\circ$$

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad \lambda/2, \text{ fast } = 45^\circ$$

$$\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \quad \text{LP at } 45^\circ$$

$$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \quad \lambda/2, \text{ fast } = \theta$$

$$\begin{pmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{pmatrix} \quad \text{LP at } \theta$$

$$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \quad \lambda/4, \text{ fast } = 0^\circ$$

$$\begin{pmatrix} t_p & 0 \\ 0 & t_s \end{pmatrix} \quad \text{trans, p in x-dir}$$

$$\begin{pmatrix} -r_p & 0 \\ 0 & r_s \end{pmatrix} \quad \text{reflect, p in x-dir}$$

$$e^{i\pi/4} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \quad \lambda/4, \text{ fast } = 45^\circ$$

$$\begin{pmatrix} \cos^2 \theta + i \sin^2 \theta & \sin \theta \cos \theta - i \sin \theta \cos \theta \\ \sin \theta \cos \theta - i \sin \theta \cos \theta & \sin^2 \theta + i \cos^2 \theta \end{pmatrix} \quad \lambda/4, \text{ fast } = \theta$$
Ellipsometry

Common use: find $d'$s knowing $n'$s, $\kappa'$s, or find $n'$s, $\kappa'$s, knowing $d'$s.