Lecture 11: Fri, 1 Feb 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. In polaroid films, polymer chains are aligned. The polarization axis that transmits light is oriented

a. along the chains
b. perpendicular to the chains.
c. at 45 degrees to the planes

Q2. When multiple elements are in a beam path, the Jones matrices to find the final optical polarization are written

a. with the first element's matrix on the right
b. with the first element's matrix on the left
c. in any order.

Q3. The element that is typically used to change linearly polarized light into circularly polarized light is the

a. linear polarizer
b. quarter waveplate
c. half waveplate

[sidenote: unpolarized light? Can't be described w/ Jones vectors.]

Jones Matrices

Certain optical components can affect the polarization state of light

linear polarizer → force into x, y, or other linear polarization

in y NIR polarizer: 

represent mathematically w/ matrix:

$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ for x-direction

matrices to change value of a vector

proof: new polar $= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix}$ ✓

current polar

leaves only x-polar

similarly $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ = forces into y-polar.

$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ = forces to 45°

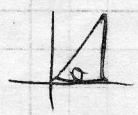
proof: $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \frac{1}{2}E_x + \frac{1}{2}E_y \\ \frac{1}{2}E_x + \frac{1}{2}E_y \end{pmatrix}$

→ x + y components now equal → 45°!

could have overall phase shift

$\begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$ = forces to linear @ angle θ from x-axis

proof $\begin{pmatrix} \cos^2\theta & \sin\cos \\ \sin\cos & \sin^2\theta \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \cos^2\theta A + \sin\cos B \\ \sin\theta\cos\theta A + \sin^2\theta B \end{pmatrix}$



$\tan\theta \overset{?}{=} \dfrac{\sin\theta\cos\theta A + \sin^2\theta B}{\cos^2\theta A + \sin\theta\cos\theta B}$

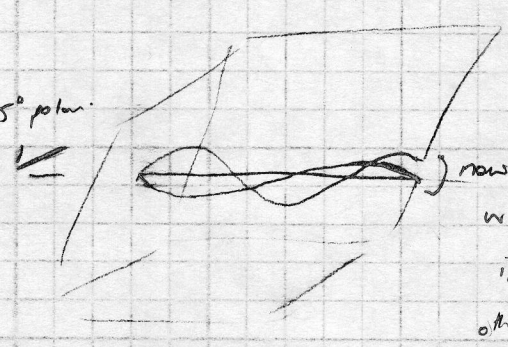
$\overset{?}{=} \dfrac{\sin\theta\,(\cos\theta \cancel{A + \sin\theta B})}{\cos\theta\,(\cos\theta \cancel{A + \sin\theta B})}$ ✓

Waveplate: As EM wave goes through material, the index of refraction is larger for one polar than the other

aka "Optical Retarder"

→ different λ inside material

ex: 45° polar.



now out of phase with each other!

if 90° → circular polarization!

otherwise, elliptical.

Such materials called **bifringent**

happen in materials (esp. crystals) that don't look the same in all directions

example: calcite $n_o = 1.658$, $n_e = 1.486$

or even ice $n_o = 1.309$, $n_e = 1.313$

$\boxed{\text{Calcite Structure}}$

o = ordinary = slow

e = extraordinary = fast

= ⊥ to axis of anisotropy

= ∥ to axis of anisotropy aka "optical axis"

⇓

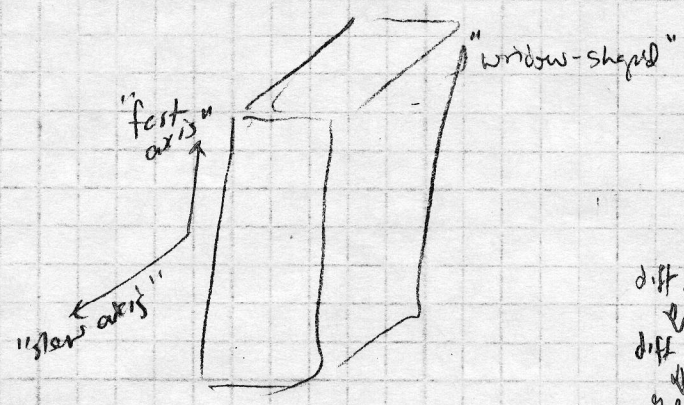
different n

⇓

diff Snell's law angle

⇓

different paths



diff λ's inside

⇓

diff phase by end

⇓

a phase shift!

$\boxed{\text{picture on wikipedia}}$

thickness: determines amount of phase shift +

Δ phase $= \Delta k \cdot d$

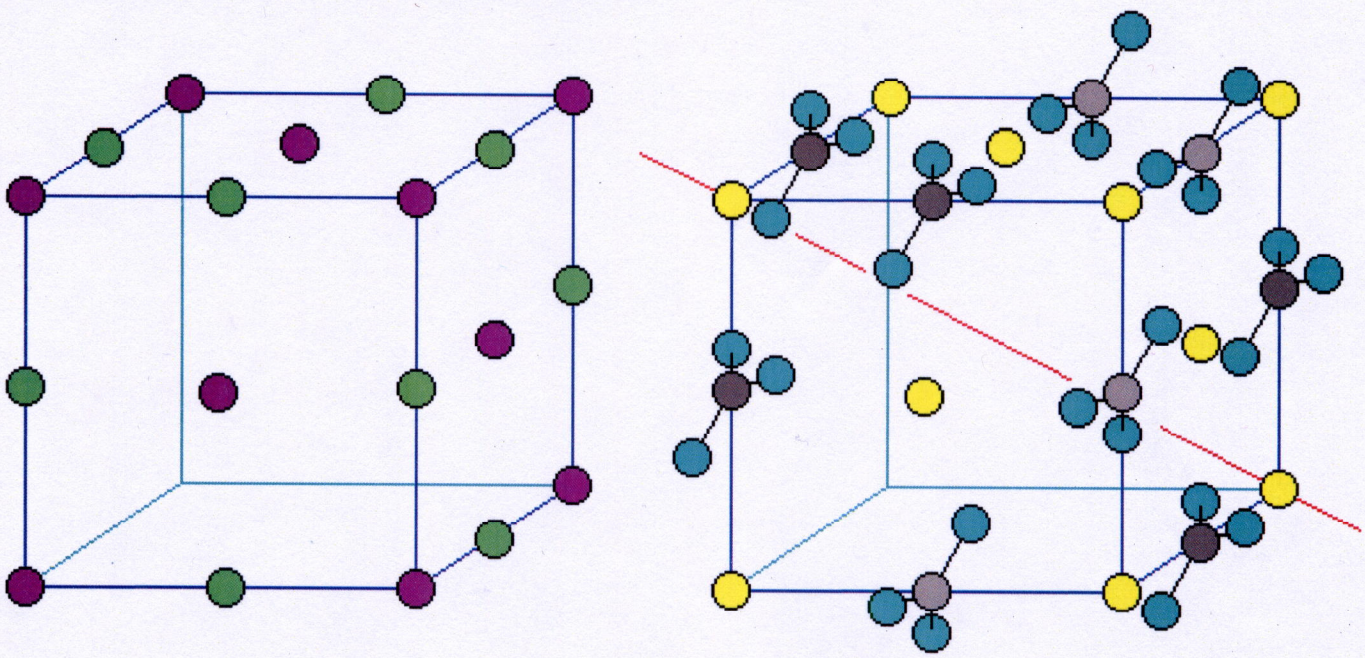
$= \frac{2\pi}{(\lambda_{vac}/n_{slow})} - \frac{2\pi}{(\lambda_{vac}/n_{fast})}$

typical applications:

quarter wave plate: Δphase $= \frac{\pi}{2} + 2\pi m$

half wave plate: Δphase $= \pi + 2\pi m$

Calcite Structure (Calcium Carbonate)
http://www.uwgb.edu/DutchS/PETROLGY/Calcite%20Structure.HTM
by Steven Dutch, UW Green Bay



- Has a "modified NaCl" structure (in about the same sense that an antique that has had every part replaced is still an antique)
- Left: NaCl—Na atoms = purple, Cl atoms = green
- Right: calcite—Ca = yellow, O = blue, C = gray
    - Note the rows of alternating Ca and CO_3 units, just like Na and Cl alternate in halite.
- Calcite is not cubic. The carbonate groups break up the cubic symmetry in several ways:
    - Their three-fold symmetry axes line up with only one of the symmetry axes of the cube (in red).
    - They alternate in orientation (shown by the two shades of gray).
    - The wide spacing of the carbonate groups stretches the atomic planes and distorts the cube into a rhombohedron.

Birefringence of Calcite Crystal
from http://en.wikipedia.org/wiki/Birefringence

quarter wave plate - used for transforming linear to circ. (and vice versa)
needs linear pol @ 45° to fast axis

proof:

Jones matrix for $\lambda/4$, fast axis @ 0° = $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

(see why??)

send in 45° light, $\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$

get out $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2}\, i \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\pi/2} \end{pmatrix}$

= RCP !

What would you get w/ -45° light? (1 guess!)

matrix for $\lambda/4$, fast axis @ 45°

$$M = \frac{e^{i\pi/4}}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$ (proved in book)

matrix for $\lambda/4$, fast axis @ θ

$$M = \begin{pmatrix} \cos^2\theta + i\sin^2\theta & \sin\theta\cos\theta - i\sin\theta\cos\theta \\ \sin\theta\cos\theta - i\sin\theta\cos\theta & \sin^2\theta + i\cos^2\theta \end{pmatrix}$$

Good problem: send in RCP to $\lambda/4$, 0°. Get out linear @ 45° ?
-45° ?
other ?

If linear pol not at 45° to fast axis, probably get some sort of elliptical out.

<u>half wave plate</u> - used mainly for <u>rotating</u> linear pol.

proof: matrix for $\frac{\lambda}{2}$, fast axis @ 0° = $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ see why??

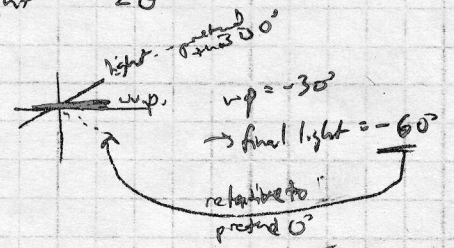
let's send in eg 30° linear: $\begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix}$

out = $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 \\ -1/2 \end{pmatrix}$

$= -30^\circ$!

Rule: if you consider the light to be at 0° and $\lambda/2$ at θ relative to that then final light = 2θ

Did that just happen?



✓

matrix for $\lambda/2$ fast axis @ 45°: $M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

matrix for $\frac{\lambda}{2}$ @ θ°: $M = \begin{pmatrix} \cos^2\theta - \sin^2\theta & 2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \sin^2 - \cos^2\theta \end{pmatrix} = \underbrace{\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}}_{\text{rotation matrix! (high school!?)}}$

Lecture 12: Mon, 4 Feb 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. Reflection generally

a. changes handedness (left vs right light)

b. conserves handedness

Q2. Which famous experimental physicist not only established a law of induction, but also discovered that magnetic fields can interact with light? Faraday

Q3. The technique of finding the optical constants n, κ from polarization changes in reflectance is called _____

a. circularometry

b. ellipsometry

c. hyperbolatry

d. linearometry

e. parabolitry

From *Hecht*

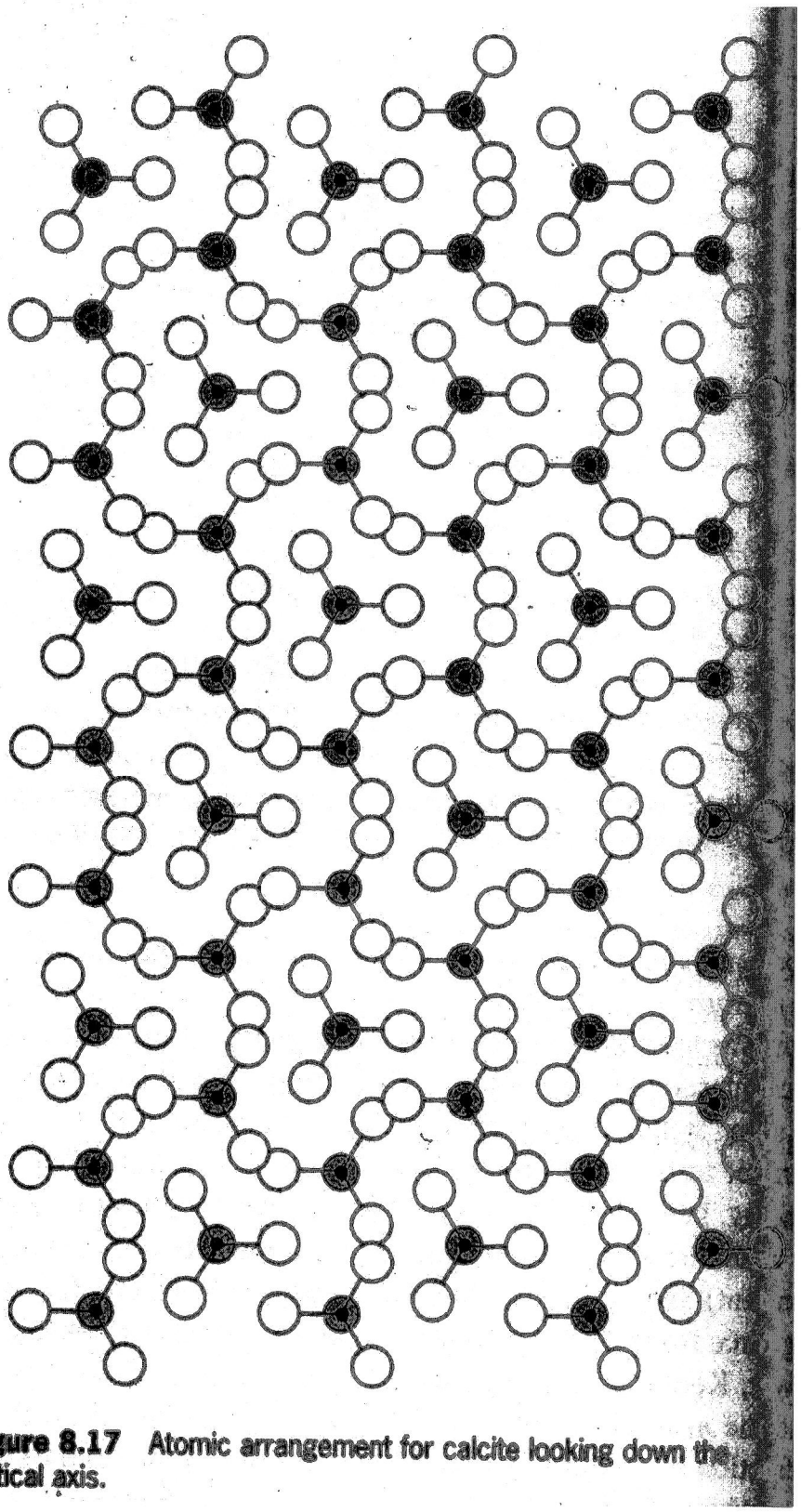
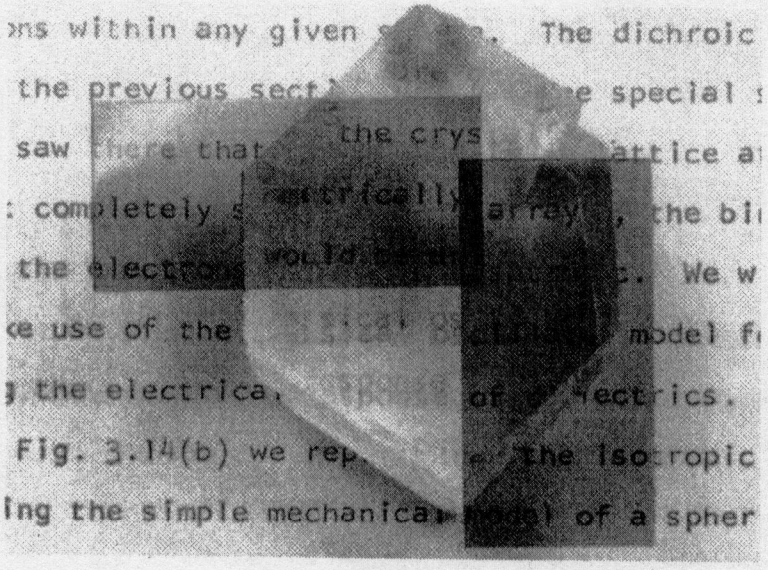


Figure 8.17 Atomic arrangement for calcite looking down the optical axis.

From *Hecht*



A calcite crystal (blunt corner on the bottom). The transmission axes of the two polarizers are parallel to their short edges. Where the image is doubled the lower, undeflected one is the ordinary image. Take a long look: there's a lot in this one. (Photo by E.H.)

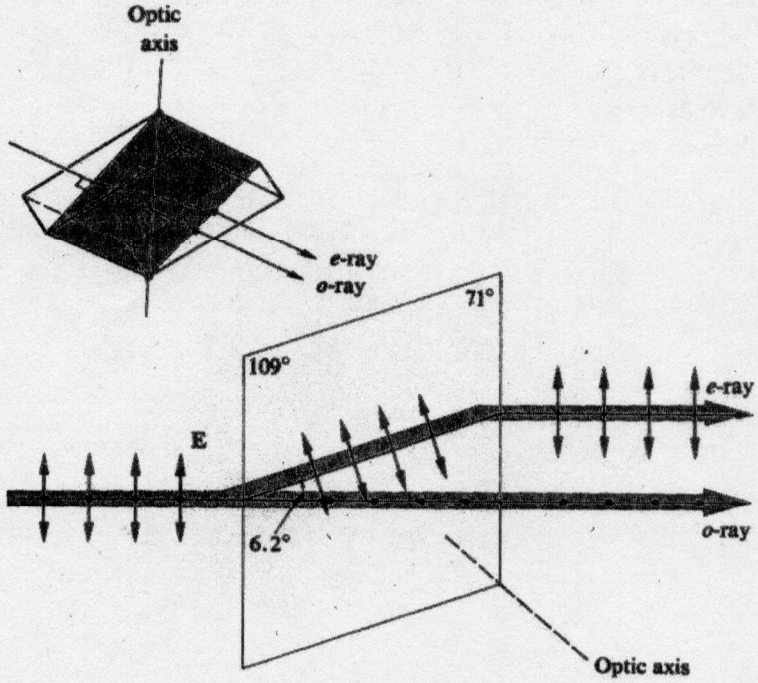


Figure 8.19 A light beam with two orthogonal field components traversing a calcite principal section.

Final Jones matrices

(have students guess)

reflection: $m = \begin{pmatrix} -r_p & 0 \\ 0 & r_s \end{pmatrix}$ if "p" means in x-direction, "s" in y direction

why the negative sign?

"handedness inversion"

→ circ. polar = conservation of angular momentum!

transmission $m = \begin{pmatrix} +t_p & 0 \\ 0 & +t_s \end{pmatrix}$

Summary chart

end of chapter!

Jones matrices summary

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$ LP in x-dir

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$ $\lambda/2$, fast = 0°

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$ LP in y-dir

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$ $\lambda/2$, fast = 45°

$$\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$ LP at 45°

$$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$$

$\lambda/2$, fast = θ

$$\begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$$

LP at θ

$$\begin{pmatrix} -r_p & 0 \\ 0 & r_s \end{pmatrix}$$ reflect, p in x-dir

$$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$ $\lambda/4$, fast = 0°

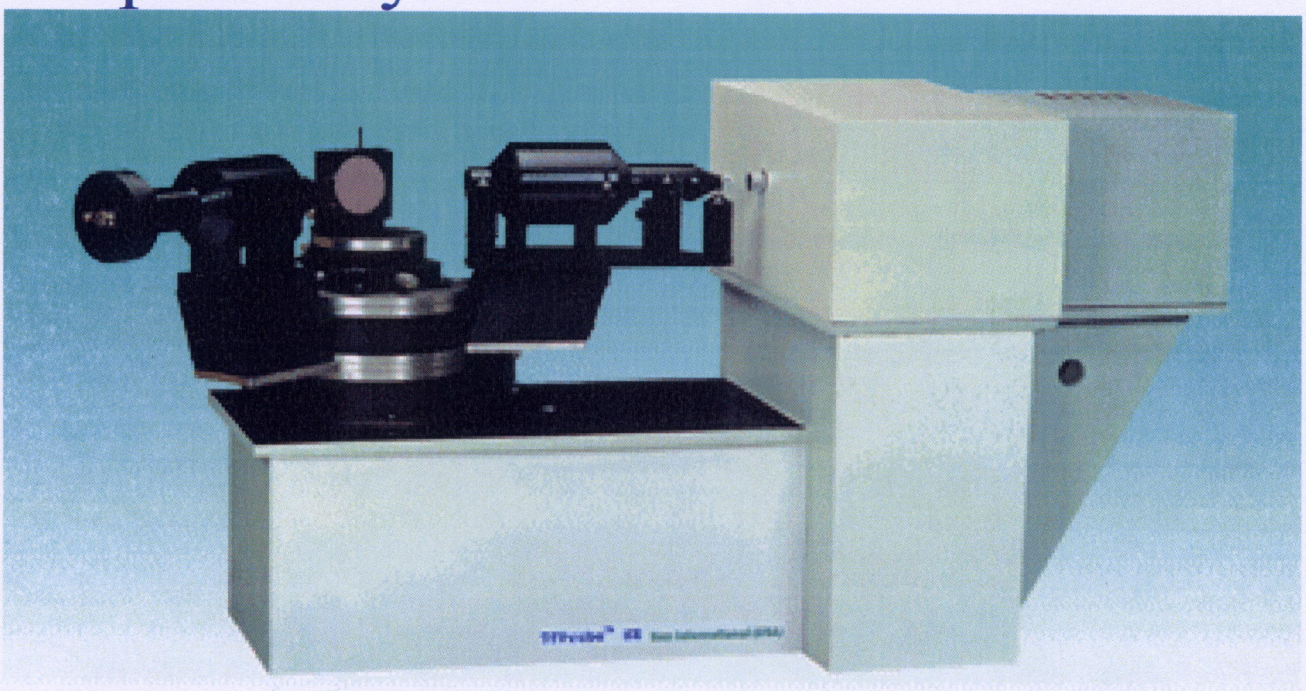
$$\begin{pmatrix} t_p & 0 \\ 0 & t_s \end{pmatrix}$$ trans, p in x-dir

$$\frac{e^{i\pi/4}}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

$\lambda/4$, fast = 45°

$$\begin{pmatrix} \cos^2\theta + i\sin^2\theta & \sin\theta\cos\theta - i\sin\theta\cos\theta \\ \sin\theta\cos\theta - i\sin\theta\cos\theta & \sin^2\theta + i\cos^2\theta \end{pmatrix}$$ $\lambda/4$, fast = θ

Ellipsometry



Common use: find *d*'s knowing *n*'s, κ's, or find *n*'s, κ's, knowing *d*'s.