

Lecture 16: Wed, 13 Feb 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. What is the difference between a Fabry-Perot etalon and a Fabry-Perot interferometer?

- a. An etalon has a variable spacing between the two surfaces
- b. An interferometer has a variable spacing between the two surfaces

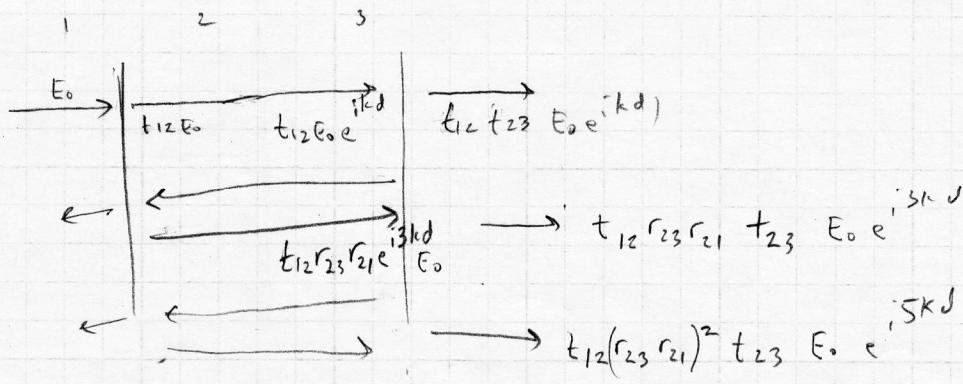
Q2. True/False – In P&W's treatment of the double boundary problem, (for the middle region) all reflections that end up traveling to the right are lumped into a single “net forward-moving field”.

Q3. The electric field at the right side of the middle layer is connected to the electric field at the left side of the middle layer via:

- a. the Fresnel coefficients
- b. a phase factor
- c. the quadratic formula
- d. transforming s- to p-polarization, and vice-versa.

Hecht's method

(Sort of)

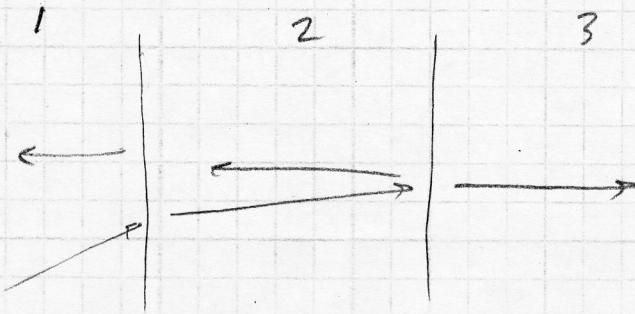


$$t_{\text{out}}^{1 \rightarrow 3} = t_{12}t_{23}e^{ikd} \left(1 + r_{23}r_{21}e^{i2kd} + (r_{23}r_{21})^2 e^{i4kd} + \dots \right)$$

$\underbrace{1 + x + x^2 + x^3 + \dots}_{\frac{1}{1-x}}$

$$t^{1 \rightarrow 3} = \frac{t_{12}t_{23}e^{ikd}}{1 - r_{23}r_{21}e^{i2kd}}$$

$p+1$ = multiple parallel interfaces



What is transmission?

You would

$$\text{think} \dots 1. \frac{E_{1\leftarrow}}{E_{1\rightarrow}} = r^{12}$$

$$3. \frac{E_{2\leftarrow}}{E_{2\rightarrow}} = r^{23}$$

$$2. \frac{E_{2\rightarrow}}{E_{1\rightarrow}} = t^{12}$$

$$4. \frac{E_{3\rightarrow}}{E_{2\rightarrow}} = t^{23}$$

Fresnel coefficients,
could be s_i ,
could be p

But no... several problems!

1. (Not helpful; probably not quite correct either)

2. Not quite correct

$$\text{fixed: } E_{2\rightarrow} = t^{12} E_{1\rightarrow} + \underbrace{r^{21} E_{2\leftarrow}}_{\text{as guessed}}$$

left moving moving wave
reflecting off of 1-2 interface
will add to right moving wave

What about multiple (infinite)
reflections?

Not needed. These are to 4/1 fields

3. partially
corrected version

$$\frac{E_{2\leftarrow, \text{RHS}}}{E_{2\rightarrow, \text{RHS}}} = r^{23}$$

(Eqn 2 $E_{2\rightarrow}$ and $E_{2\leftarrow}$
are $E_{2\rightarrow, \text{LHS}}$ $E_{2\leftarrow, \text{LHS}}$)

4. partially
corrected version

$$\frac{E_{3\rightarrow}}{E_{2\rightarrow, \text{RHS}}} = t^{23}$$

How to connect RHS to LHS? Phase factors

$$\text{for } \rightarrow e^{ik_2 \cdot r} = e^{ik_2 (\sin \theta_2 \hat{y} + \cos \theta_2 \hat{z}) \cdot d\hat{z}} \xrightarrow{\text{right hand edge relative left hand edge.}}$$

$$= e^{ik_2 d \cos \theta_2}$$

$$\text{So } E_{2 \rightarrow \text{RHS}} = \underbrace{E_{2 \rightarrow \text{LHS}} e^{ik_2 d \cos \theta_2}}$$

$$\text{similarly } E_{2 \leftarrow \text{RHS}} = \underbrace{E_{2 \leftarrow \text{LHS}} e^{-ik_2 d \cos \theta_2}}_{(\text{negative since } k = \sin \theta \hat{y} - \cos \theta \hat{z} \text{ for } \leftarrow)}$$

$$\text{Adjusted Eqn 3: } \frac{E_{2 \leftarrow} e^{-ik_2 d \cos \theta_2}}{E_{2 \rightarrow} e^{ik_2 d \cos \theta_2}} = r^{23} \quad (\underbrace{E_{2 \leftarrow} = E_{2 \rightarrow \text{LHS}}}_{\text{etc}})$$

$$\text{Adjusted eqn 4: } \frac{E_{3 \rightarrow}}{E_{2 \rightarrow} e^{ik_2 d \cos \theta_2}} = r^{23}$$

A little algebra with eqns 2, 3, 4 4 unknowns: $E_{1 \rightarrow}$
 $E_{2 \rightarrow}$
 $E_{2 \leftarrow}$
 $E_{3 \rightarrow}$

$$\text{From Eqn 4: } E_{2 \rightarrow} = \underbrace{\frac{E_{3 \rightarrow}}{r^{23}} e^{-ik_2 d \cos \theta_2}}$$

$$\text{From Eqn 3: } E_{2 \leftarrow} = \underbrace{E_{2 \rightarrow} r^{23} e^{ik_2 d \cos \theta_2}}$$

$$= \left(\frac{E_{3 \rightarrow}}{r^{23}} e^{-ik_2 d \cos \theta_2} \right) r^{23} e^{2ik_2 d \cos \theta_2}$$

$$= \underbrace{E_{3 \rightarrow} \frac{r^{23}}{t^{23}} e^{ik_2 d \cos \theta_2}}$$

Plug into Eqn 2.

$$\left(\frac{E_{3 \rightarrow}}{t^{23}} e^{-ik_2 d \cos \theta_2} \right) = t^{12} E_{1 \rightarrow} + r^{21} \left(E_{3 \rightarrow} \frac{r^{23}}{t^{23}} e^{ik_2 d \cos \theta_2} \right)$$

$$E_{3 \rightarrow} = E_{1 \rightarrow} \frac{t^{12}}{\frac{e^{-ik_2 d \cos \theta_2}}{t^{23}}} - \frac{r^{21}}{t^{12}} \frac{e^{ik_2 d \cos \theta_2}}{r^{23}} \times \frac{t^{23}}{t^{23}}$$

$$\boxed{\frac{E_{3 \rightarrow}}{E_{1 \rightarrow}} = \frac{t^{12} + t^{23}}{e^{-ik_2 d \cos \theta_2} - r^{21} r^{23} e^{ik_2 d \cos \theta_2}}} \quad (\text{still looks complex})$$

check: normal incidence $\theta_1 = \theta_3 = 0$

$$t^{1 \rightarrow 3} = \frac{t_1^{1 \rightarrow 2} t_2^{2 \rightarrow 3}}{\frac{-ik_2 d}{c - r_{21} r_{23}}} \times e^{\frac{ik_2 d}{c}}$$
$$= \boxed{\frac{e^{ik_2 d} t_1^{1 \rightarrow 2} t_2^{2 \rightarrow 3}}{1 - r_{21} r_{23} e^{2ik_2 d}}}$$

matches Hecht method ✓

$$\frac{I_3}{I_1} = T^{13} = \frac{n_3}{n_1} \frac{\cos \theta_3}{\cos \theta_1} |T^{13}|^2$$

like with previous
Fresnel coeff

from Rayoptics from geometrical
 $\langle S \rangle = \frac{1}{2} E \frac{E}{c_m}$ factor

$$T^{13} = \frac{n_3}{n_1} \frac{\cos \theta_3}{\cos \theta_1} \frac{|T^{12}|^2 |T^{23}|^2}{|e^{-ik_2 d \cos \theta_2} - e^{ik_2 d \cos \theta_2}|^2}$$

Lecture 17: Fri, 15 Feb 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. The quantity F (labeled F_s in the book's equation) is called the:

- a. coefficient of Fabry-Perot
- b. coefficient of finesse
- c. coefficient of Fred
- d. coefficient of Fresnel
- e. coefficient of fringes

not to be confused

$f = \text{"finesse"}$

$\approx \text{"reflecting finesse"}$

Q2. In the example of frustrated total internal reflection, the book used two:

- a. triangular prisms
- b. rectangular slabs
- c. mirrors

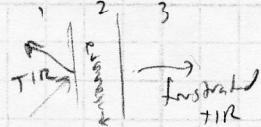
Q3. As the separation between the above two becomes much smaller than a wavelength, the transmission goes to

- a. 0
- b. $\frac{1}{2}$
- c. 1

Do Case 2 first! Section 6.4

Case 2: evanescent wave, $\theta_2 = \text{complex}$ $\cos\theta_2 < \text{imaginary}$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \text{TIR: } n_1 \sin\theta_1 = n_2 \sin 90^\circ$$

$$\sin\theta_1 = \frac{n_2}{n_1} \quad \text{critical}$$

$$\theta_1 > \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \text{still true (comes from complex plane wave phases)}$$

$$\sin\theta_2 = \frac{n_1}{n_2} \sin\theta_1 \quad \text{this is } > 1$$

$$\sin^2 + \cos^2 = 1 \quad \text{still even if complex}$$

$$\begin{aligned} \cos\theta_2 &= \sqrt{1 - \left(\frac{n_1 \sin\theta_1}{n_2}\right)^2} &= \sqrt{\text{negative number}} \\ &\quad \underbrace{\qquad\qquad\qquad}_{>1} && \leftarrow \text{purely imaginary} \\ &= i \sqrt{\left(\frac{n_1 \sin\theta_1}{n_2}\right)^2 - 1} \\ &\quad \underbrace{\qquad\qquad\qquad}_{\text{real number}} \end{aligned}$$

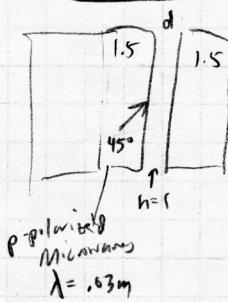
Sidemore: $n_1 \sin\theta_1 = n_3 \sin\theta_3$ also

θ_3 must still be real in order
to get light propagation into
material 3

$$\text{decomposition} = \sqrt{k_1^2 d^2 \cos^2\theta_2} e^{i k_1 d \sin\theta_2 / 2}$$

E_{fe}

Specific Application



$$n_1 = 1.5 \quad \theta_1 = 45^\circ \quad \sin \theta_1 = \frac{1}{\sqrt{2}} \quad \cos \theta_1 = \frac{1}{\sqrt{2}}$$

$$n_2 = 1$$

$$n_3 = 1.5$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$1.5 \left(\frac{1}{\sqrt{2}} \right) = 1 \sin \theta_2 \rightarrow \theta_2 \approx 1.5708 + -34.66i$$

$$\underline{\cos \theta_2 = .35355i} \quad - \frac{i}{2\sqrt{2}}$$

$$k_2 = \frac{2\pi}{\lambda}$$

$$\theta_3 = \text{some as } \theta_1 \rightarrow \underline{\cos \theta_3 = \frac{1}{\sqrt{2}}}$$

To use big eqn, still need t'^2 , t^{23} , r^{21} , and r^{23}

$$t'^2 = \frac{2}{\alpha + \beta} = \frac{2}{\frac{n_2}{n_1} + \frac{\cos \theta_2}{\cos \theta_1}} = \frac{2}{\frac{1}{1.5} + \frac{i/2\sqrt{2}}{1/\sqrt{2}}} = 1.92 - 1.44i \quad |t'^2|^2 = 5.76$$

$$t^{23} = \frac{2}{\alpha + \beta} = \frac{2}{\frac{n_3}{n_2} + \frac{\cos \theta_3}{\cos \theta_2}} = \frac{2}{\frac{1.5}{1} + \frac{1/\sqrt{2}}{i/2\sqrt{2}}} = .48 + .64i \quad |t^{23}|^2 = .64$$

$$r^{21} = \frac{\alpha - \beta}{\alpha + \beta} = \frac{-\frac{n_1}{n_2} + \frac{\cos \theta_1}{\cos \theta_2}}{\frac{n_1}{n_2} + \frac{\cos \theta_1}{\cos \theta_2}} = \frac{-\frac{1.5}{1} + \frac{1/\sqrt{2}}{i/2\sqrt{2}}}{\frac{1.5}{1} + \frac{1/\sqrt{2}}{i/2\sqrt{2}}} = +.28 - .96i = e^{i(-1.287 \text{ radians})}$$

$$r^{23} = \text{some as } r^{21} \text{ since material 3 = material 1}$$

Put together:

$$T^{13} = \frac{n_3}{n_1} \frac{\cos \theta_3}{\cos \theta_1} \frac{|t'^2|^2 |t^{23}|^2}{|e^{-ik_2 d \cos \theta_2} - r^{21} r^{23} e^{ik_2 d \cos \theta_2}|^2}$$

$$= \frac{(5.76)(.64)}{|e^{-i(\frac{2\pi}{0.3})d(\frac{1}{2\sqrt{2}})} - (e^{i(-1.287)})(e^{i(-1.287)}) e^{i(\frac{2\pi}{0.3})d(\frac{1}{2\sqrt{2}})}|^2}$$

$$= \frac{3.6864}{|e^{+74.05d} - e^{-i2.574} e^{-74.05d}|^2}$$

$$= \frac{3.6864}{\left(e^{74.05d} - e^{-i2.574} e^{-74.05d} \right) \left(e^{-74.05d} - e^{+i2.574} e^{-74.05d} \right)}$$

$$T^{13} = \frac{3.6864}{e^{148.10} - \underbrace{\left(e^{i2.574} + e^{-i2.574} \right)}_{2 \cos 2.574} + e^{-148.10}}$$

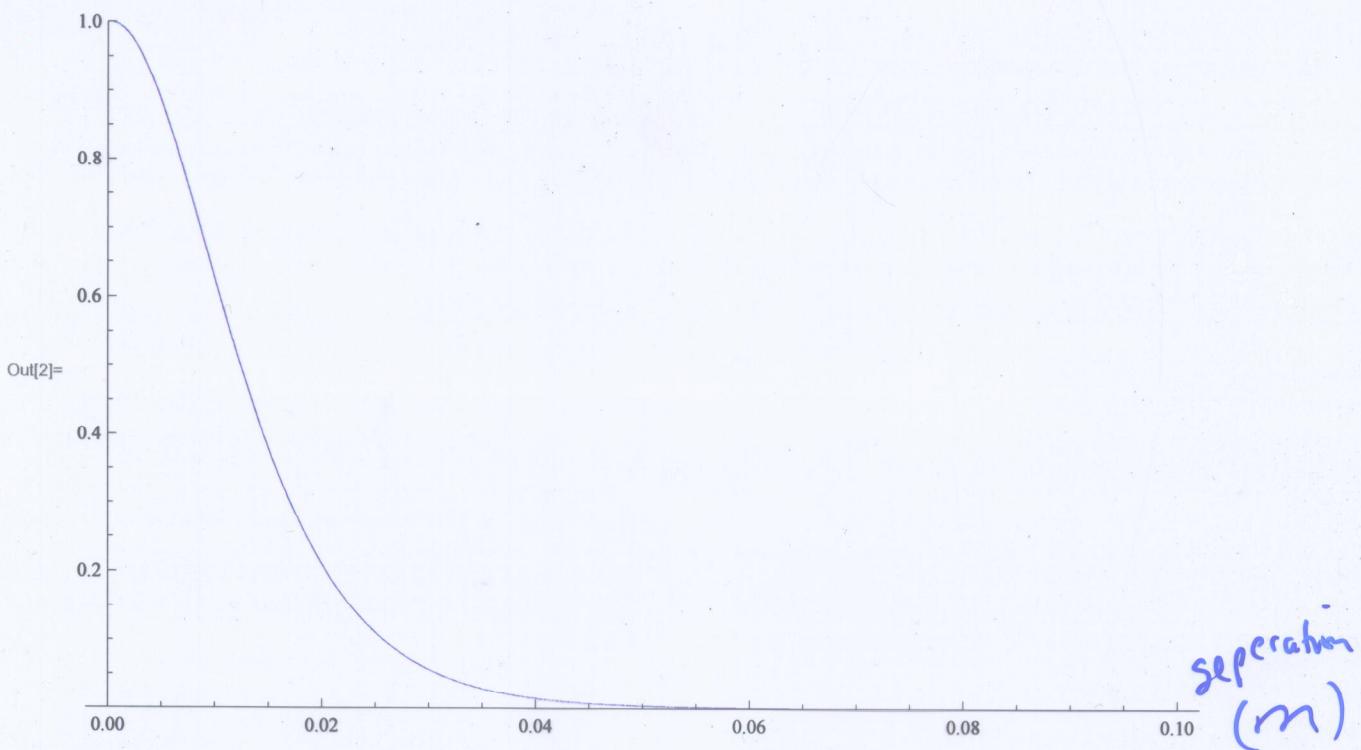
$$T^{13} = \frac{3.6864}{e^{148.10} + e^{-148.10} + 1.68638}$$

plotted w/ Mathematica

$$\text{In[1]:= } T13[d_] = 3.6864 / (E^{(138.1 d)} + E^{(-148.1 d)} + 1.68638)$$

$$\text{Out[1]= } \frac{3.6864}{1.68638 + e^{-148.1 d} + e^{138.1 d}}$$

In[2]:= Plot[T13[d], {d, 0, .1}, PlotRange -> {0, 1}]



Now Section 6.3

Case 1: No evanescent wave, $\theta_2 = \text{real}$, $c_0, \theta_2 = \text{real}$

$$\text{denom} = |e^{-ik_2 d \cos \theta_2} - r^{21} r^{23} e^{i k_2 d \cos \theta_2}|^2 = ?$$

$$\begin{aligned} &= (e^{-ik_2 d \cos \theta_2} - r^{21} r^{23} e^{ik_2 d \cos \theta_2})(e^{-ik_2 d \cos \theta_2} - r^{21} r^{23} e^{-ik_2 d \cos \theta_2}) \\ &= 1 - \underbrace{\left(e^{+2ik_2 d \cos \theta_2} r^{21} r^{23} + r^{21} r^{23} e^{2ik_2 d \cos \theta_2} \right)}_{+X} + \underbrace{\left(r^{21} r^{23} \right)^2}_{+X} \\ &= 2 \text{ Real}(X) \end{aligned}$$

$$\begin{aligned} \text{Write } r^{21} &= |r^{21}| e^{if_{21}} \\ r^{23} &= |r^{23}| e^{if_{23}} \end{aligned}$$

$$\begin{aligned} \text{denom} &= 1 + |r^{21}|^2 / |r^{23}|^2 - 2 \text{ Real} \left\{ |r^{21}| / e^{if_{21}} / |r^{23}| / e^{if_{23}} e^{i2k_2 d \cos \theta_2} \right\} \\ &\quad \boxed{f = 2k_2 d \cos \theta_2} \end{aligned}$$

$$= 1 + |r^{21}|^2 / |r^{23}|^2 - 2 |r^{21}| / |r^{23}| \cos(f + f_{21} + f_{23})$$

$$= \left(1 - |r^{21}| / |r^{23}| \right)^2 + 4 |r^{21}| / |r^{23}| \sin^2 \frac{f + f_{21} + f_{23}}{2}$$

trick: $= 1 - 2 \sin^2 \frac{f + f_{21} + f_{23}}{2}$
from half-angle formula

Then $T^{13} = \frac{n_3}{n_1} \frac{\cos \theta_3}{\cos \theta_1} \frac{|T^{12}|^2 |T^{23}|^2}{(1 + |r^{21}| / |r^{23}|)^2 + 4 |r^{21}| / |r^{23}| \sin^2 \frac{\Phi}{2}} \times \frac{\frac{1}{(1-f)^2}}{\frac{1}{(1-f)^2}}$

$\Phi = f + f_{21} + f_{23}$

$$T = \frac{T^{\max}}{1 + F \sin^2 \frac{\Phi}{2}}$$

with

$$T^{\max} = \frac{n_3 \cos \theta_3}{n_1 \cos \theta_1} \frac{|T^{12}|^2 / |T^{23}|^2}{(1 - |r^{21}| / |r^{23}|)^2}$$

$$= \frac{(T^{12}) (T^{23})}{(1 - \sqrt{|r^{21}| / |r^{23}|})^2}$$

multiply numerically by $\frac{n_2 \cos \theta_2}{n_0 \cos \theta_0}$

T^{\max} branch
 $T^{\text{largest}} \text{ when } f = 0$

$$F = \frac{4 |r^{21}| / |r^{23}|}{(1 - |r^{21}| / |r^{23}|)^2}$$

coeff of Fibesee
or Fibesee coeff
not reflecting fibese of
one fibese