Lecture 18: Tues, 19 Feb 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. Which of the following was *not* mentioned in the book as something that could be varied to allow you to see Fabry-Perot fringes (transmission peaks)?

a. angle going through the etalon
b. diameter of the light beam
c. spacing between partial reflectors
d. wavelength of the light

Q2. The maximum $\Delta\lambda$ that can be accurately measured is called the free _____ range

a. experiment
b. resolution
c. spectral
d. wavelength

Q3. The etalons with the narrowest transmission peaks are those with:

a. low R
b. high R

Now Section 6.3

Case 1: No evanescent waves, θ_2 = real; $\cos\theta_2$ = real

$$\text{denom} = \left| e^{-ik_2 d\cos\theta_2} - r^{21} r^{23} e^{ik_2 d\cos\theta_2} \right|^2 = ?$$

$$= \left(e^{-ik_2 d\cos\theta_2} - r^{21} r^{23} e^{ik_2 d\cos\theta_2}\right)\left(e^{+ik_2 d\cos\theta_2} - r^{21*} r^{23*} e^{-ik_2 d\cos\theta_2}\right)$$

$$= 1 - \underbrace{\left(e^{+2ik_2 d\cos\theta_2} r^{21} r^{23} + r^{21*} r^{23*} e^{-2ik_2 d\cos\theta_2}\right)}_{x \;+\; x^*} + \left|r^{21}\right|^2 \left|r^{23}\right|^2$$

$$= 2\,\text{Real}(x)$$

write $r^{21} = |r^{21}| e^{i\delta_{21}}$

$r^{23} = |r^{23}| e^{i\delta_{23}}$

$$\text{denom} = 1 + |r^{21}|^2 |r^{23}|^2 - 2\,\text{Real}\left\{ |r^{21}| e^{i\delta_{21}} |r^{23}| e^{i\delta_{23}} e^{i2k_2 d\cos\theta_2} \right\}$$

$$\boxed{\delta = 2k_2 d\cos\theta_2}$$

$$= 1 + |r^{21}|^2 |r^{23}|^2 - 2|r^{21}||r^{23}| \underbrace{\cos(\delta + \delta_{21} + \delta_{23})}$$

trick: $= 1 - 2\sin^2 \frac{\delta + \delta_{21} + \delta_{23}}{2}$

from halfangle formula

$$= \left(1 - |r^{21}||r^{23}|\right)^2 + 4|r^{21}||r^{23}| \sin^2 \frac{\delta + \delta_{21} + \delta_{23}}{2}$$

Then

$$T^{13} = \frac{n_3}{n_1} \frac{\cos\theta_3}{\cos\theta_1} \frac{|t^{12}|^2 |t^{23}|^2}{\left(1 - |r^{21}||r^{23}|\right)^2 + 4|r^{21}||r^{23}| \sin^2 \frac{\Phi}{2}} \times \frac{\frac{1}{(1-(\,))^2}}{\frac{1}{(1-(\,))^2}}$$

$\Phi = \delta + \delta_{21} + \delta_{23}$

end of Friday

start at Monday

$$\boxed{T = \frac{T^{max}}{1 + F\sin^2 \frac{\Phi}{2}}}$$

with

$$T_{max} = \frac{n_3}{n_1} \frac{\cos\theta_3}{\cos\theta_1} \frac{|t^{12}|^2 |t^{23}|^2}{\left(1 - |r^{21}||r^{23}|\right)^2} = \frac{(T^{12})(T^{23})}{\left(1 - \sqrt{R^{21}}\sqrt{R^{23}}\right)^2}$$

multiply numerator by $\frac{n_2\cos\theta_2}{n_2\cos\theta_2}$

$$F = \frac{4|r^{21}||r^{23}|}{\left(1 - |r^{21}||r^{23}|\right)^2}$$

T_{max} because T is largest when $\Phi = 0$

coeff of Finesse or Finesse coeff

not reflecting finesse of what a finesse

section 6.5 Fabry Perot – picks up where Finesse left off

consider symmetric material 1 = 3

then

$$T_{max} = \frac{(T_{12})_1 \, (T_{23})_1}{\left(1 - \sqrt{R^{21}}\sqrt{R^{23}}\right)^2}$$

equal

$$= \frac{T_{12} \; T_{23}}{(1-R)^2} = \frac{T^2}{(1-R)^2}$$

is not $T = 1 - R$?
• Not if absorption

is $T_{12} = T_{23}$? yes

prove for p-polar

$$T = 1 - R = 1 - \left(\frac{\alpha - \beta}{\alpha + \beta}\right)^2$$

$$= \frac{4\alpha\beta}{(\alpha+\beta)^2} \times \frac{(1/\alpha\beta)^2}{\left(\frac{1}{\alpha}\right)^2}$$

$$= \frac{4 \frac{1}{\alpha}\frac{1}{\beta}}{\left(\frac{1}{\beta} + \frac{1}{\alpha}\right)^2}$$

so $T_{12} = T_{21} = T$

$$F = \frac{4 \, |r^{21}||r^{23}|}{\left(1 - |r^{21}||r^{23}|\right)^2}$$

$$= \frac{4 \, R}{(1-R)^2}$$

Plot of $T = \dfrac{T_{max}}{1 + F \sin^2 \frac{\Phi}{2}}$

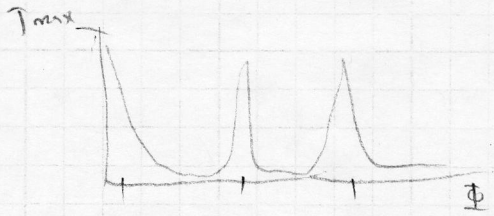
$$\Phi = \delta + \underbrace{\delta_{21} + \delta_{23}}_{\text{typically less important than } \delta \text{ (unchanging)}}$$

recall $\delta = 2 k_2 d \cos\theta_2$

$$= \frac{4\pi}{\lambda_2} d \cos\theta_2$$

$$= \frac{4\pi n_2}{\lambda_{vac}} d \cos\theta_2$$

$\boxed{\text{Can vary } \Phi \text{ via } ① \, d, \; ② \, \theta_2, \text{ or } ③ \, \lambda_{vac}}$



To get large F → want large R (R = .9 → F = 360)

(But large R → small T, so Tmax is reduced)

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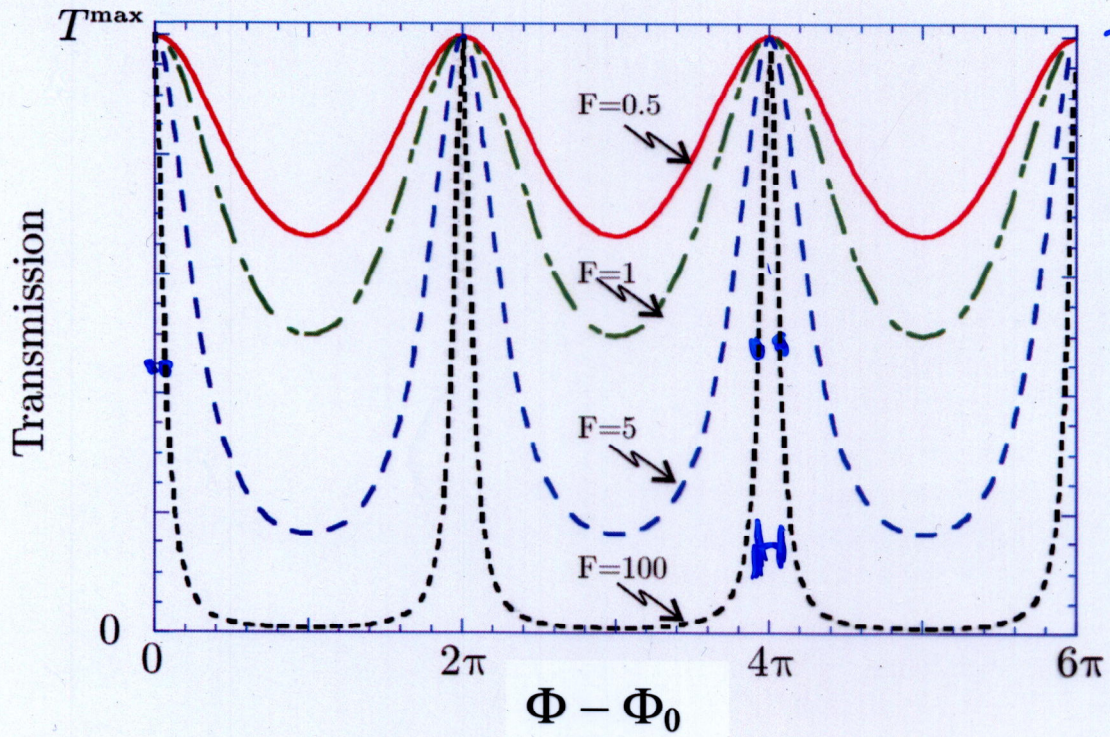
Fabry-Perot Interferometers/Etalons

$\Delta\Phi = \frac{4}{\sqrt{F}}$

P&W Fig 6.3



$T = \frac{T_{max}}{1 + F\sin^2\Phi/2}$

drops to 1/2 when

$F\sin^2\frac{\Phi}{2} = 1$

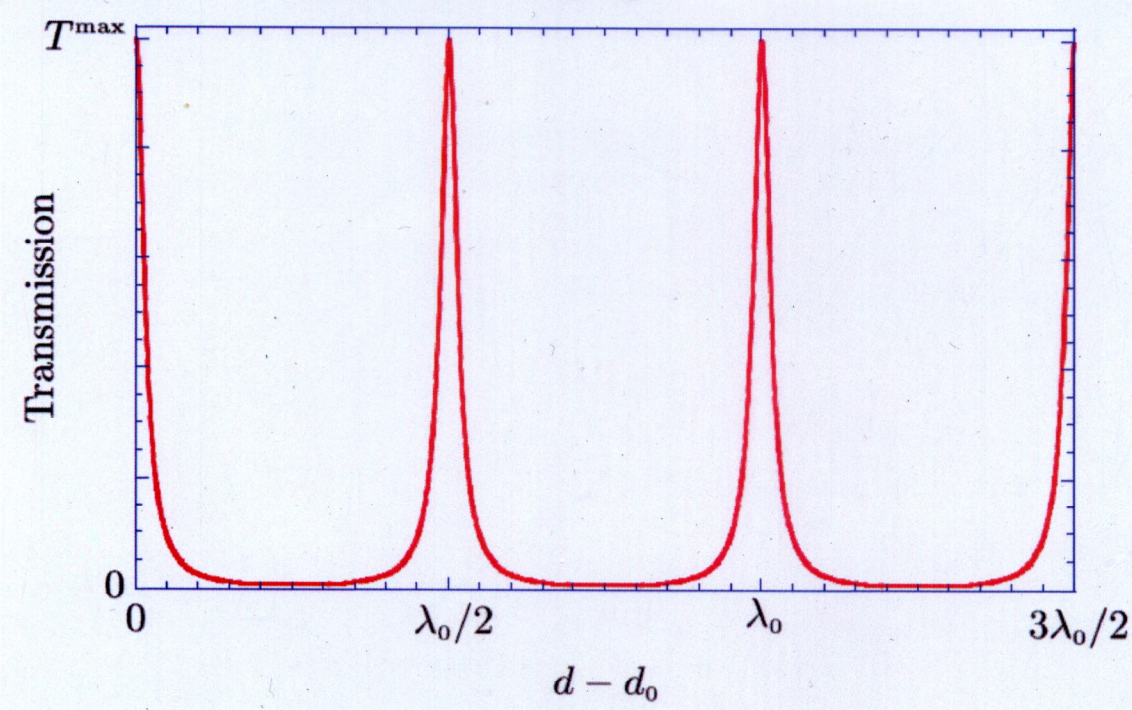
$\sin\frac{\Phi}{2} = \sqrt{\frac{1}{F}}$

$\frac{\Phi}{2} = \sqrt{\frac{1}{F}}$

$\Phi = \frac{2}{\sqrt{F}}$

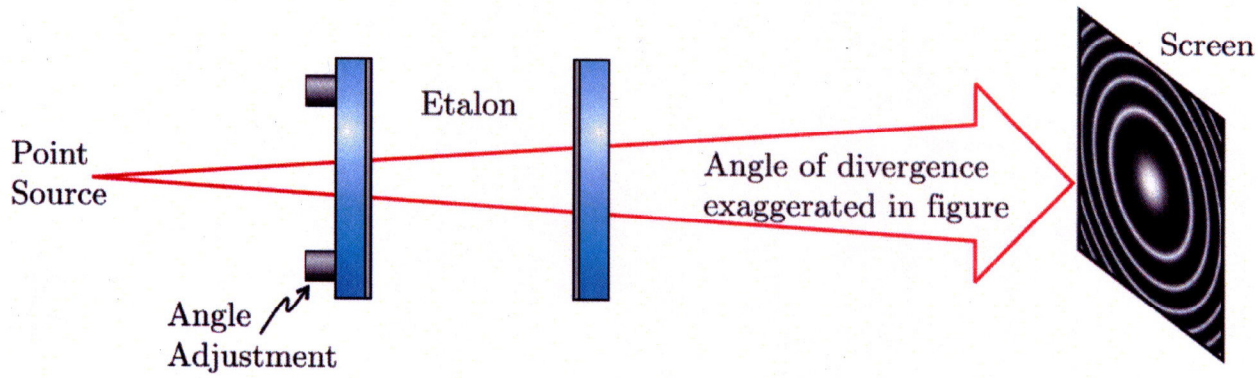
FWHM → double $\frac{4}{\sqrt{F}}$

P&W Fig 6.6 - **Φ changed by changing *d***

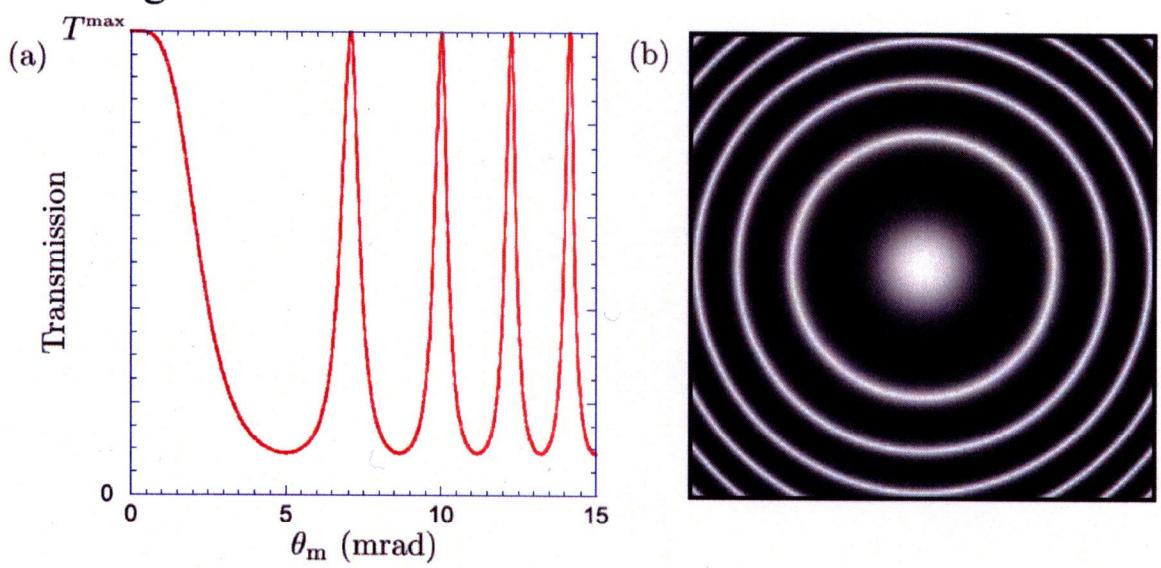


Φ changed by changing θ_2

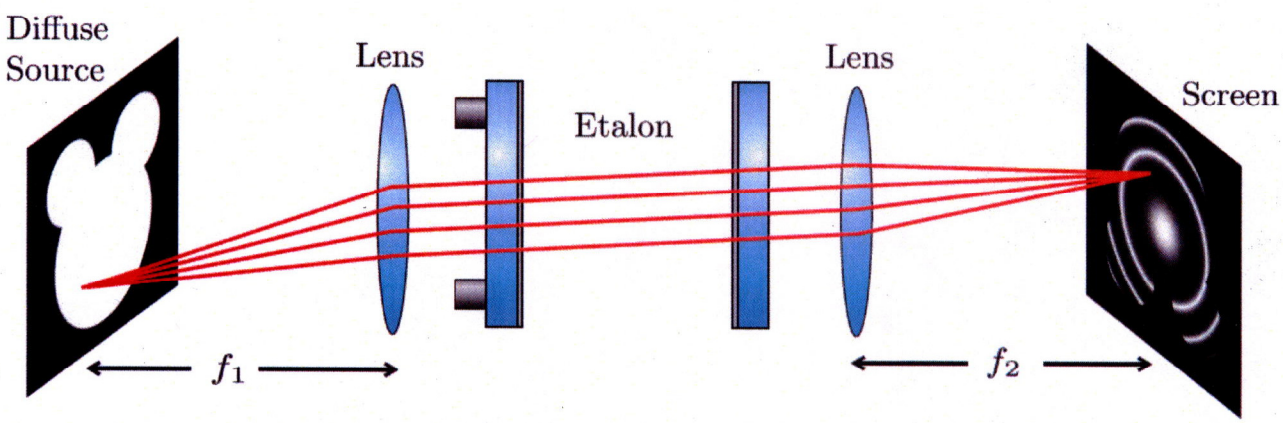
P&W Fig 6.7



P&W Fig 6.8

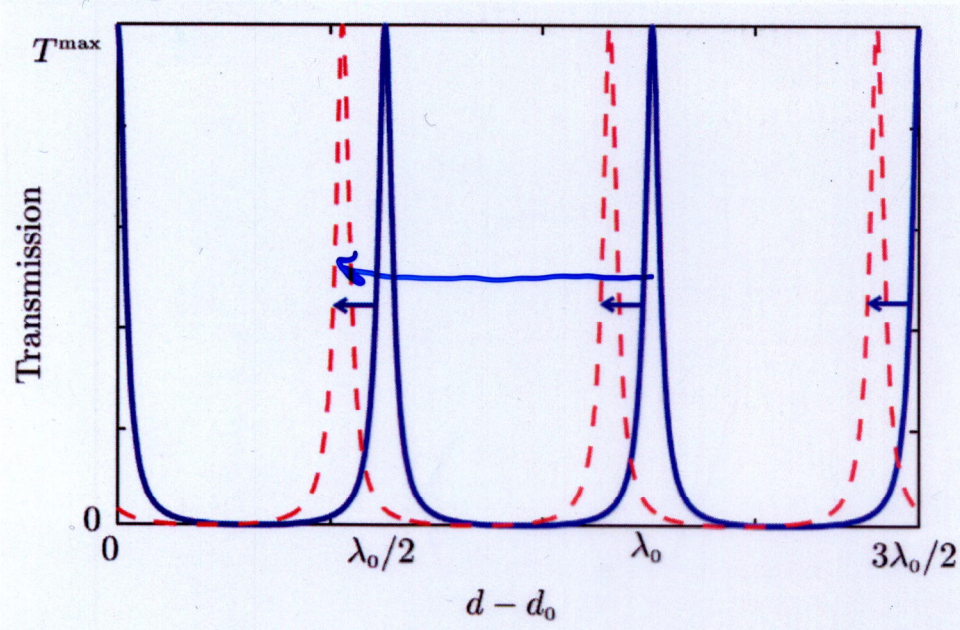


P&W Fig 6.9



Φ changed by changing λ

P&W Fig 6.10



P&W Fig 6.11

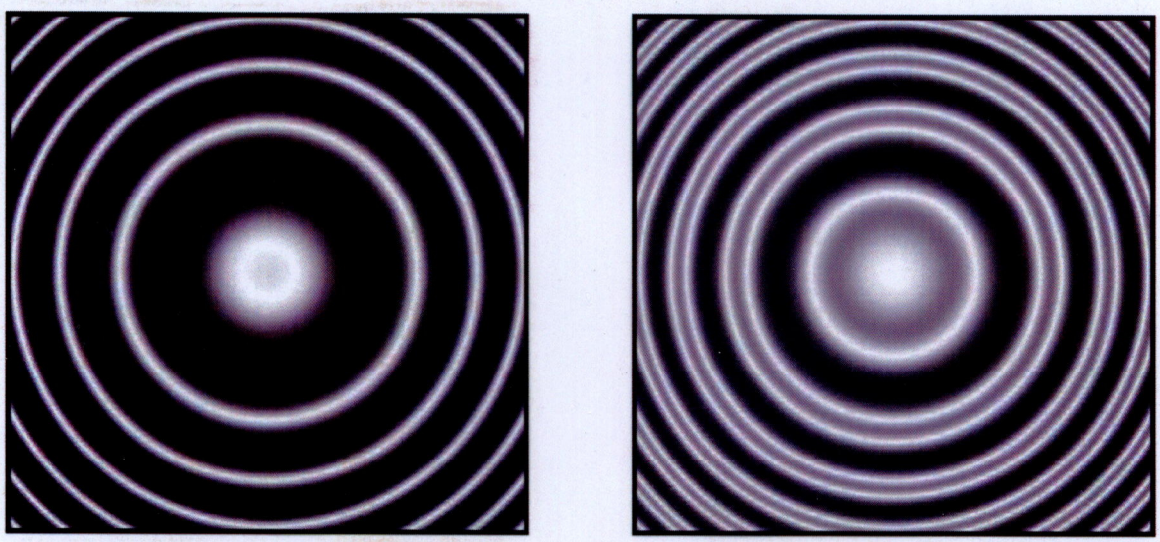


Figure 6.11 Transmittance as function of angle through a Fabry-Perot etalon. Two nearby wavelengths are sent through the instrument simultaneously, (left) barely resolved and (right) easily resolved.

Misc odds + ends

Wavelength discrimination

$$\Phi + \Delta\Phi = \frac{4\pi n_2 d \cos\theta_2}{\lambda + \Delta\lambda} = \frac{4\pi n_2 d \cos\theta_2}{\lambda}\underbrace{\left(1 + \frac{\Delta\lambda}{\lambda}\right)^{-1}}_{1 - \frac{\Delta\lambda}{\lambda}}$$

$$\Delta\Phi = 4\pi n_2 d \cos\theta_2 \frac{\Delta\lambda}{\lambda^2}$$ (drop negative sign)

(2) "Free spectral range" $\Delta\lambda$ that will cause $\Delta\Phi = 2\pi$ = "largest permissible $\Delta\lambda$ while avoiding confusion"

$$2\pi = 4\pi n_2 d \cos\theta_2 \frac{\Delta\lambda_{FSR}}{\lambda^2}$$

$$\boxed{\Delta\lambda_{FSR} = \frac{\lambda^2}{2 n_2 d \cos\theta_2}}$$

numbers: $\lambda = 500\,nm$ (green)
$d = 1\,cm$
$\theta_2 = 0°$
$n_2 = 1$

then $\Delta\lambda_{FSR} = .013\,nm$

we can't tell diff between 500 and 500.013

if $\lambda' > \lambda + \Delta\lambda_{FSR}$ you will underestimate the change

reflecting finesse aka finesse

(4) next page

$$f = \frac{2\pi}{\Delta\Phi_{FWHM}} \simeq \frac{2\pi}{4/\sqrt{F}} = \boxed{\frac{\pi\sqrt{F}}{2}}$$

$f \simeq \frac{\text{fraction down}}{\text{fraction up}}$

(1) $\Delta\lambda_{FWHM}$: $$\Delta\Phi_{FWHM} = \frac{4\pi n_2 d \cos\theta_2}{\lambda^2}\Delta\lambda_{FWHM}$$

$$\frac{4}{\sqrt{F}} = \frac{4\pi n_2 d \cos\theta_2}{\lambda^2}\Delta\lambda_{FWHM}$$

$$\boxed{\Delta\lambda_{FWHM} = \frac{\lambda^2}{\pi n_2 d \cos\theta_2 \sqrt{F}}}$$

minimum resolvable wavelength change
i.e. 500 and 500.0008 look the same

numbers: $\lambda = 500\,nm$
$d = 1\,cm$
$F = 100$
$\theta = 0$
$n_2 = 1$

$\rightarrow \Delta\lambda_{FWHM} = .0008\,nm$

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(3) resolving power $RP = \frac{\lambda_0}{\Delta\lambda} = \frac{500\text{ nm}}{.006\text{ nm}} = \underline{\underline{600{,}000}}$

my grating spectrometer $RP \approx \frac{500\text{nm}}{.05\text{ nm}} = \underline{\underline{10000}}$

but spectrometer can see much wider range of wavelengths at once (not limited to $\Delta\lambda_{FSR}$)

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Lecture 19: Wed, 20 Feb 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. The treatment of multilayers used

a. Fresnel coefficients

b. E and B field boundary conditions

Q2. The symbol used for the matrix for the inner layers, that was multiplied over and over, was __

a. H

b. L

c. M

d. Q

Q3. For high reflectivity we often use multiple layers, each having thickness:

a. λ

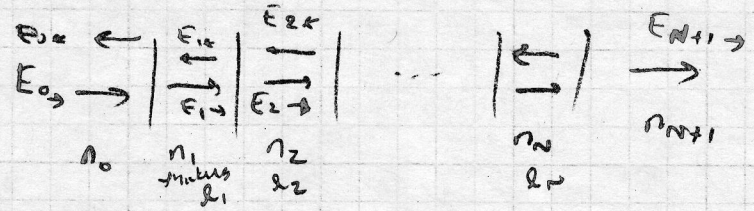
b. $\lambda/2$

c. $\lambda/4$

d. $\lambda/8$

e. $\lambda/16$

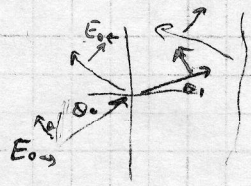
Multilayers



angles given by Snell's Law

$$n_0 \sin\theta_0 = n_1 \sin\theta_1 = \cdots = n_N \sin\theta_N = n_{N+1} \sin\theta_{N+1}$$

bdy conds ① $(E_{\parallel})_0 = (E_{\parallel})_1$ at interface



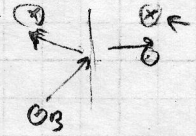
p-pol specifically

$\Rightarrow$ s pol = homework

$$E_{0\rightarrow}\cos\theta_0 + E_{0\leftarrow}\cos\theta_0 = E_{1\rightarrow}\cos\theta_1 + E_{1\leftarrow}\cos\theta_1$$

$$(E_{0\rightarrow} + E_{0\leftarrow})\cos\theta_0 = (E_{1\rightarrow} + E_{1\leftarrow})\cos\theta_1$$

② $(B_{\parallel})_0 = (B_{\parallel})_1$ at interface



$$\frac{E_{0\rightarrow}}{v_0} - \frac{E_{0\leftarrow}}{v_0} = \frac{E_{1\rightarrow}}{v_1} - \frac{E_{1\leftarrow}}{v_1}$$

eqn for first bndy

$$n_0 (E_{0\rightarrow} - E_{0\leftarrow}) = n_1 (E_{1\rightarrow} - E_{1\leftarrow})$$

matrix form: $\begin{pmatrix} \cos\theta_0 & \cos\theta_0 \\ n_0 & -n_0 \end{pmatrix}\begin{pmatrix} E_{0\rightarrow} \\ E_{0\leftarrow} \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & \cos\theta_1 \\ n_1 & -n_1 \end{pmatrix}\begin{pmatrix} E_{1\rightarrow} \\ E_{1\leftarrow} \end{pmatrix}$

remember phase shifts going from LHS to RHS

in general ① $\left(E_{j\rightarrow} e^{ik_j l_j \cos\theta_j} + E_{j\leftarrow} e^{-ik_j l_j \cos\theta_j}\right)\cos\theta_j = \left(E_{j+1\rightarrow} - E_{j+1\leftarrow}\right)\cos\theta_{j+1}$

② $n_j\left(E_{j\rightarrow} e^{ik_j l_j \cos\theta_j} + E_{j\leftarrow} e^{-ik_j l_j \cos\theta_j}\right) = n_{j+1}\left(E_{j+1\rightarrow} - E_{j+1\leftarrow}\right)$

Matrix Form: $\begin{pmatrix} \cos\theta_j e^{i(\ )} & \cos\theta_j e^{-i(\ )} \\ n_j e^{i(\ )} & -n_j e^{-i(\ )} \end{pmatrix}\begin{pmatrix} E_{j\rightarrow} \\ E_{j\leftarrow} \end{pmatrix} = \begin{pmatrix} \cos\theta_{j+1} & \cos\theta_{j+1} \\ n_{j+1} & -n_{j+1} \end{pmatrix}\begin{pmatrix} E_{j+1\rightarrow} \\ E_{j+1\leftarrow} \end{pmatrix}$

Final interface special (only $\rightarrow$ wave)

① $\left(E_{N\rightarrow} e^{ik_N l_N \cos\theta_N} + E_{N\leftarrow} e^{-ik_N l_N \cos\theta_N}\right)\cos\theta_N = E_{N+1\rightarrow}\cos\theta_{N+1}$

② $n_N\left(E_{N\rightarrow} e^{i(\ )} - E_{N\leftarrow} e^{-i(\ )}\right) = n_{N+1} E_{N+1\rightarrow}$

matrix form $\begin{pmatrix} e^{ik_N l_N \cos\theta_N} & e^{-ik_N l_N \cos\theta_N} \\ n_N e^{i(\ )} & -n_N e^{-i(\ )} \end{pmatrix}\begin{pmatrix} E_{N\rightarrow} \\ E_{N\leftarrow} \end{pmatrix} = \begin{pmatrix} \cos\theta_{N+1} & 0 \\ n_{N+1} & 0 \end{pmatrix}\begin{pmatrix} E_{N+1\rightarrow} \\ 0 \end{pmatrix}$

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$$\begin{pmatrix} E_0\rightarrow \\ E_0\leftarrow \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta_0 & \cos\theta_0 \\ n_0 & -n_0 \end{pmatrix}^{-1}}_{M_A} \underbrace{\begin{pmatrix} \cos\theta_1 & \cos\theta_1 \\ n_1 & -n_1 \end{pmatrix}}_{M_B} \begin{pmatrix} E_1\rightarrow \\ E_1\leftarrow \end{pmatrix}$$

E_1 to E_2, E_2 to E_3, etc.. all like this:

$$\begin{pmatrix} E_j\rightarrow \\ E_j\leftarrow \end{pmatrix} = \begin{pmatrix} \cos\theta_j e^{i(\ )} & \cos\theta_j e^{-i(\ )} \\ n_j e^{i(\ )} & -n_j e^{-i(\ )} \end{pmatrix}^{-1} \begin{pmatrix} \cos\theta_{j+1} & \cos\theta_{j+1} \\ n_{j+1} & -n_{j+1} \end{pmatrix} \begin{pmatrix} E_{j+1}\rightarrow \\ E_{j+1}\leftarrow \end{pmatrix}$$

then at last interface

$$\begin{pmatrix} E_N\rightarrow \\ E_N\leftarrow \end{pmatrix} = \begin{pmatrix} e^{i(\ )} & e^{-i(\ )} \\ n_N e^{i(\ )} & -n_N e^{-i(\ )} \end{pmatrix}^{-1} \begin{pmatrix} \cos\theta_{N+1} & 0 \\ n_{N+1} & 0 \end{pmatrix} \begin{pmatrix} E_{N+1}\rightarrow \\ 0 \end{pmatrix}$$

group slightly differently (2nd Matrix of $0\rightarrow 1$ with 1st Matrix of $1\rightarrow 2$, etc) contract all together. Let $\beta_j = k_j d_j \cos\theta_j$

$$\begin{pmatrix} E_0\rightarrow \\ E_0\leftarrow \end{pmatrix} = \begin{pmatrix} \cos\theta_0 & \cos\theta_0 \\ n_0 & -n_0 \end{pmatrix}^{-1} \left(M_{B0-1} M_{A1-2}\right)\left(M_{B1-2} M_{A2-3}\right)\left(M_{B2-3}\cdots\right)\left(\cdots M_{AN-N+1}\right) M_{BN-N+1} \begin{pmatrix} E_{N+1}\rightarrow \\ 0 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} \cos\theta_0 & \cos\theta_0 \\ n_0 & -n_0 \end{pmatrix}^{-1} \left[\prod_{j=1}^{N} \underbrace{\begin{pmatrix} \cos\theta_j & \cos\theta_j \\ n_j & -n_j \end{pmatrix}\begin{pmatrix} \cos\theta_j e^{i\beta_j} & \cos\theta_j e^{-i\beta_j} \\ n_j e^{i\beta_j} & -n_j e^{-i\beta_j} \end{pmatrix}}_{M_j}\right] \begin{pmatrix} \cos\theta_{N+1} & 0 \\ n_{N+1} & 0 \end{pmatrix}}_{A} \begin{pmatrix} E_{N+1}\rightarrow \\ 0 \end{pmatrix}$$

<u>strategy</u>: compute all M_j's; compute A, then

$$\begin{pmatrix} E_0\rightarrow \\ E_0\leftarrow \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} E_{N+1}\rightarrow \\ 0 \end{pmatrix}$$

Divide by $E_{0\rightarrow}$:

$$\begin{pmatrix} 1 \\ r \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} t_{13} \\ 0 \end{pmatrix}$$

$$1 = a_{11} t \rightarrow \boxed{t_{13} = \frac{1}{a_{11}}}$$

$$r = a_{21} t = a_{21}\left(\frac{1}{a_{11}}\right)$$

$$\boxed{r = \frac{a_{21}}{a_{11}}}$$

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Simplifying M_j's: use $\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} = \frac{1}{\det M}\begin{pmatrix} D & -B \\ -C & A \end{pmatrix}$

$$M_j = \begin{pmatrix} \cos\theta_j & \cos\theta_j \\ n_j & -n_j \end{pmatrix} \frac{1}{-n_j\cos\theta_j - n_j\cos\theta_j} \begin{pmatrix} -n_j e^{-i\beta_j} & -\cos\theta_j e^{-i\beta_j} \\ -n_j e^{i\beta_j} & \cos\theta_j e^{i\beta_j} \end{pmatrix}$$

$$= \frac{1}{2n_j\cos\theta_j} \underbrace{\begin{pmatrix} \cos\theta_j & \cos\theta_j \\ n_j & -n_j \end{pmatrix} \begin{pmatrix} n_j e^{-i\beta_j} & \cos\theta_j e^{-i\beta_j} \\ n_j e^{i\beta_j} & -\cos\theta_j e^{i\beta_j} \end{pmatrix}}$$

$$= \frac{1}{2n_j\cos\theta_j} \begin{pmatrix} n_j\cos\theta_j\left(e^{i\beta_j} + e^{-i\beta_j}\right) & \\ & \text{etc.} \end{pmatrix}$$

$$\boxed{M_j = \begin{pmatrix} \cos\beta_j & -i\sin\beta_j\cos\theta_j \\ -\frac{in_j\sin\beta_j}{\cos\theta_j} & \cos\beta_j \end{pmatrix}}$$

Simplifying A: use same matrix inversion

$$A = \frac{-1}{-n_0\cos\theta_0 - n_0\cos\theta_0} \begin{pmatrix} -n_0 & -\cos\theta_0 \\ -n_0 & \cos\theta_0 \end{pmatrix} \left[\prod M_j\right] \begin{pmatrix} \cos\theta_{N+1} & 0 \\ n_{N+1} & 0 \end{pmatrix}$$

$$\boxed{A = \frac{1}{2n_0\cos\theta_0} \begin{pmatrix} n_0 & \cos\theta_0 \\ n_0 & -\cos\theta_0 \end{pmatrix} \left[\prod M_j\right] \begin{pmatrix} \cos\theta_{N+1} & 0 \\ n_{N+1} & 0 \end{pmatrix}}$$

S-polarization: HW problem

$$M_j = \begin{pmatrix} \cos\beta_j & -\frac{i\sin\beta_j}{n_j\cos\theta_j} \\ -in_j\cos\theta_j\sin\beta_j & \cos\beta_j \end{pmatrix}$$

and $$A = \frac{1}{2n_0\cos\theta_0} \begin{pmatrix} n_0\cos\theta_0 & 1 \\ n_0\cos\theta_0 & -1 \end{pmatrix} \left[\prod M_j\right] \begin{pmatrix} 1 & 0 \\ n_{N+1}\cos\theta_{N+1} & 0 \end{pmatrix}$$