Lecture 25: Wed, 5 Mar 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. T/F: In an interferometer, if you measure the interference pattern as a function of τ, $I(\tau)$, you can deduce the power spectrum $I(\omega)$ of the incident light from your result.

$V = |\gamma|$

$= \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$

Q2. "Fringe visibility" refers to

a. closeness/separation of fringe peaks

b. height/depth of fringe peaks

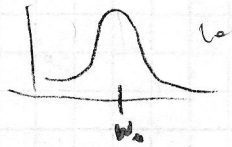
c. the wavelengths contributing to fringes

Q3. In a typical interferometer, the connection between coherence *time* and coherence *length* is:

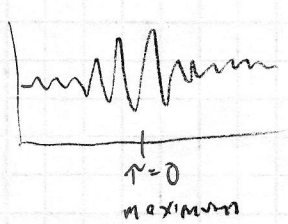
a. $x_c = c\, t_c$

b. $x_c = v_{group}\, t_c$

c. $x_c = v_{phase}\, t_c$

HW problem: $I(\omega) =$  Lorentzian shape

What does intensity do as you send this (cw) light through interferometer and vary τ?

Answer: 

$\tau = 0$ maximum

note $\mathrm{Re}(\gamma)$ looks very similar since $I \sim 1 + \mathrm{Re}(\gamma)$

$|\gamma|$ = envelope of oscillations

Some terms

Coherence time: the amount of delay (τ) needed to cause I to stop oscillating

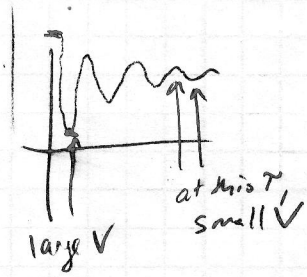
one measure: $\tau_c = \int_{-\infty}^{\infty} |\gamma(\tau)|^2 \, d\tau$

i.e. the more area, the longer you'll have to wait to have no oscillations

Coherence length: $l_c = c\,\tau_c$ distance associated w/ τ_c clear enough.

Fringe visibility: $V = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$ of nearby oscillations

(or integrated intensity, if pulses)



HW problem: $V = |\gamma(\tau)|$

Monday - Stephen

Now - begin here

cw light

For $\boxed{\text{Gaussian wave packet}}$

$E(t) = E_0 e^{-t^2/2\tau^2} e^{-i\omega_0 t}$

$E(\omega) = E_0 \tau \, e^{-\tau^2(\omega-\omega_0)^2/2}$

can't use τ! already using for delay

$\tau \rightarrow \frac{1}{\Delta\omega}$

$I(\omega) = \frac{n}{2\mu_0 c} |E(\omega)|^2 = I_0 e^{-\tau^2(\omega-\omega_0)^2}$

$= I_0 e^{-\frac{(\omega-\omega_0)^2}{\Delta\omega^2}}$

Can calculate $\gamma(\tau)$:

$$\gamma(\tau) = \frac{\int_{-\infty}^{\infty} I(\omega) e^{-i\omega\tau} d\omega}{\int_{-\infty}^{\infty} I(\omega) d\omega}$$

$$\text{Num} = I_0 \int_{-\infty}^{\infty} e^{-\frac{(\omega-\omega_0)^2}{\Delta\omega^2}} e^{-i\omega\tau} d\omega = \Delta\omega\sqrt{\pi}\, e^{-i\omega_0\tau} e^{-\frac{1}{4}\tau^2\Delta\omega^2}$$

$$\text{Denom} = I_0 \int_{-\infty}^{\infty} e^{-\frac{(\omega-\omega_0)^2}{\Delta\omega^2}} = \Delta\omega\sqrt{\pi}$$

Int. done w/ Mathematica

$$\gamma = e^{-i\omega_0\tau} e^{-\frac{1}{4}\tau^2\Delta\omega^2}$$

I looks the same as Re γ but going from 0 to $2\mathcal{E}_0$

$$\mathcal{E}_0 = \int_{-\infty}^{\infty} I(\omega) d\omega$$

= total energy (per area) of pulse

What's coherence time?

$$\tau_c = \int_{-\infty}^{\infty} |\gamma|^2 d\tau$$

$$= \int_{-\infty}^{\infty} e^{-\tau^2\Delta\omega^2/2} d\tau$$

$$\tau_c = \frac{\sqrt{2\pi}}{\Delta\omega}$$

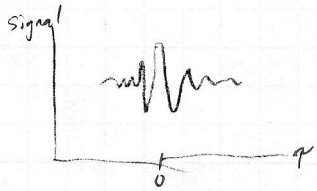
Recall $\frac{1}{\Delta\omega}$ was approx. a width of the Gaussian pulse

$\tau_c \sim$ width of pulse

Section 8.5 Fourier Spectroscopy

$$\text{signal}(\tau) \sim \left[2\int_{-\infty}^{\infty} I(\nu)\,d\nu\right]\left[1 + \mathrm{Re}(\gamma(\tau))\right] = 2\mathcal{E}_0\left(1 + \frac{\mathrm{Re}\int I(\omega)e^{-i\omega\tau}d\omega}{\mathcal{E}_0}\right)$$

If we measure



can we deduce the frequency spectrum?

Yes!

$$NS(\tau) = \frac{\text{signal}}{\text{proportionality factor}} = 2\mathcal{E}_0 + 2\,\mathrm{Re}\int I(\omega)e^{-i\omega\tau}d\omega$$

$NS(\tau)$ → normalized signal

Take FT of $NS(\tau)$:

$$\underbrace{FT(NS(\tau))}_{\text{experimental}} = \underbrace{FT(2\mathcal{E}_0)}_{2\mathcal{E}_0\,\underbrace{FT(1)}_{\sqrt{2\pi}\delta(\omega)}} + \underbrace{FT\Big(2\,\underbrace{\mathrm{Re}\int I(\omega)e^{-i\omega\tau}d\omega}_{\mathrm{Re}\,x = \frac{1}{2}(x + x^*)}\Big)}_{\text{can do with computer}}$$

$$\sqrt{2\pi}\,FT\left(\frac{1}{\sqrt{2\pi}}\int I(\omega)e^{-i\omega\tau}d\omega + \frac{1}{\sqrt{2\pi}}\int I(\omega)e^{+i\omega\tau}d\omega\right)$$

$\omega' = -\omega$

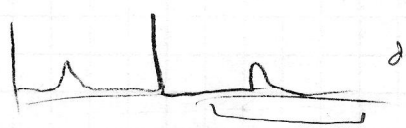
$$FT\left\{IFT(I(\omega)) + IFT(I(-\omega))\right\}$$

$$= 2\sqrt{2\pi}\,\mathcal{E}_0\,\delta(\omega) + \sqrt{2\pi}\,I(\omega) + \sqrt{2\pi}\,I(-\omega)$$

$$\frac{FT(NS(\tau))}{\sqrt{2\pi}} = 2\mathcal{E}_0\,\delta(\omega) + I(\omega) + I(-\omega)$$

experimental graph

Typically



don't overlap

$I(\omega)$ identified!

Lecture 26: Fri, 7 Mar 2008

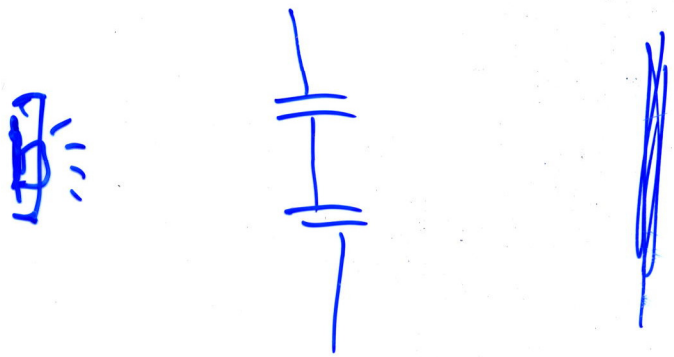
Reading quizzes: no talking, no looking in your books/notes

Q1. In a "Young's experiment" setup, the light will produce better fringes if it comes from:

a. a small source
b. a large source
c. doesn't depend on source size

Q2. Young's experiment produces interference between how many slits?

a. 1
b. 2
c. 3
d. More than 3



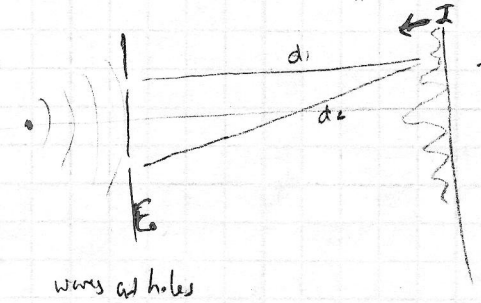
Q3. When did Young live?

a. Before Michelson
b. Contemporary with Michelson
c. After Michelson

Section 8.6 Young's 2-slit experiment - first conclusive demo that light = wave

quasi-monochromatic light

like Michelson: 2 beams of light travel different distances



phase factors from both waves

$$E_{tot} = E_0 e^{ikd_1} + E_0 e^{ikd_2}$$

waves at holes spatially coherent (ie come from same wave fronts

$$I \propto |E|^2$$

$$= |E_0|^2 \left(e^{ikd_1} + e^{ikd_2}\right)\left(e^{-ikd_1} + e^{-ikd_2}\right)$$

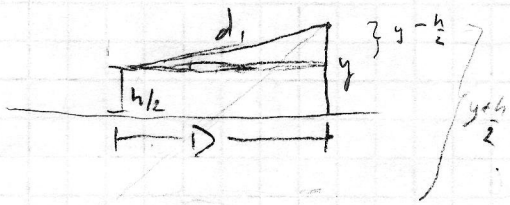
$$= |E_0|^2 \left(1 + 1 + \underbrace{e^{ik(d_2-d_1)} + e^{-ik(d_2-d_1)}}_{2\cos k(d_2-d_1)}\right)$$

$$\boxed{I = 2I_0\left[1 + \cos k(d_2-d_1)\right]}$$

if not completely symmetric, so different phases at each slit,

$$\boxed{I = 2I_0\left[1 + \cos\left(k(d_2-d_1) + \Delta\phi\right)\right]}$$

Some geometry



$$\sin\theta_1 = \frac{y - \frac{h}{2}}{d_1}$$

$$d_1 = \sqrt{D^2 + \left(y - \tfrac{h}{2}\right)^2}$$

$$= D\left(1 + \left(\frac{y - \frac{h}{2}}{D}\right)^2\right)^{1/2}$$

$$= D\left(1 + \tfrac{1}{2}\left(\frac{y - \frac{h}{2}}{D}\right)^2\right) \quad \text{for large } D$$

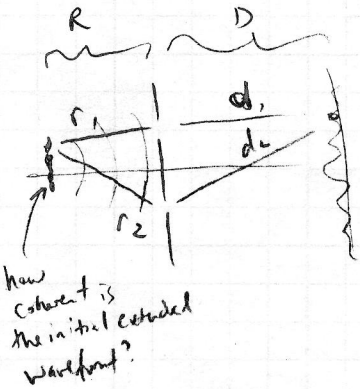
$$d_2 - d_1 = D\left(1 + \tfrac{1}{2}\left(\frac{y + \frac{h}{2}}{D}\right)^2\right) - D\left(1 + \tfrac{1}{2}\left(\frac{y - \frac{h}{2}}{D}\right)^2\right)$$

$$= \frac{1}{2D}\left[\cancel{y^2} + yh + \cancel{\tfrac{h^2}{4}} - \cancel{y^2} + yh - \cancel{\tfrac{h^2}{4}}\right]$$

$$= \frac{yh}{D}$$

$$\boxed{I = 2I_0\left[1 + \cos\left(\frac{kyh}{D} + \Delta\phi\right)\right]}$$

What if a finite source?



Not just a simple convolution if phase of source is varying (randomly?)

Treat the extended source as a series of pt sources, $j = 1 \text{ to } N$, each with slightly different phase ϕ_j

For each j $\quad E_j = |E_j| \left[e^{i(k(r_{1j}+d_1)-\omega t+\phi_j)} + e^{ik(r_{2j}+d_2)-\omega t+\phi_j} \right]$

$$E_{tot} = \sum E_j$$

$$I \propto |E_{tot}|^2 = \left[\sum_j |E_j| \left(e^{i(k(r_{1j}+d_1)-\omega t+\phi_j)} + e^{i(2)} \right) \right] \left[\sum_m \underbrace{e^{-i[k(r_{1m}+d_1)-\omega t+\phi_m]}}_{|E_m|} + e^{-i(2)} \right]$$

$$= \sum_{j,m} \Big[|E_j||E_m| \exp[i(k(r_{1j}-r_{1m}) + \phi_j - \phi_m)]$$
$$+ |E_j||E_m| \exp i[k(r_{2j}-r_{2m}) + \phi_j - \phi_m]$$
$$+ |E_j||E_m| \underbrace{\left[\exp i[k(r_{1j}-r_{2m}) + k(d_1-d_2) + \phi_j - \phi_m] + C.C. \right]}_{2\,\mathrm{Re}[\exp i(\text{stuff})]} \Big]$$

Assume: noncoherent source

like starlight or lightbulb, not laser

$\rightarrow$ $\phi_j - \phi_m$ causes oscillations which average out unless $j = m$

(and when $j = m$, all $r_{1j} - r_{1m}$ terms go away)

$$|E_{tot}|^2 = \sum_j \left[|E_j|^2 + |E_j|^2 + |E_j|^2\, 2\,\mathrm{Re}\left[\exp i\left[k(\underbrace{r_{1j}-r_{2j}}_{-\frac{y_j' h}{R}}) + k(\underbrace{d_1-d_2}_{-\frac{y h}{D}\ \text{already done}}) \right] \right] \right]$$

y_j' = distance upward for the jth source

$$|E_{tot}|^2 = 2\sum_j |E_j|^2 + 2\,\mathrm{Re}\sum_j |E_j|^2 e^{-i\left(k\frac{y_j' h}{R} + k\frac{yh}{D}\right)}$$

take out of sum

put in multiplicative constants

$$I = 2\sum_j I_j + 2\,\mathrm{Re}\, e^{-ik\frac{yh}{D}} \sum_j I_j e^{-ik\frac{hy_j'}{R}}$$

$$\boxed{I(h) = \left(2\sum_j I_j\right)\left[1 + \underbrace{e^{-ik\frac{yh}{D}} \frac{\sum_j I_j e^{-ik\frac{hy_j'}{R}}}{\sum_j I_j}}_{=\gamma(h)\ =\ \text{"degree of coherence"}}\right]}$$

Looks a lot like our time coherence formula!

Recall $\gamma(\tau) = \dfrac{\int I(\omega) e^{-i\omega\tau} d\omega}{\int I(\omega) d\omega}$

$\gamma = e^{-ik\frac{yh}{D}} \dfrac{\int I(y') e^{-ik\frac{hy'}{R}} dy'}{\int I(y') dy'}$

$I(y')$: spatial intensity distribution (units: watts/meter)

$\langle I(h)\rangle = 2\langle I_{one\ slit}\rangle (1 + \mathrm{Re}\,\gamma(h))$

main difference: • $\gamma(\tau)$ had continuous integral
→ could make $\sum_j$ into cont. integral

• $\gamma(h)$ has $e^{-ik\frac{yh}{D}}$ factor
→ defines position of fringes on screen

Notes: 1) when h large it's like when τ was large
→ amplitude of $\gamma \simeq 0$, no oscillations visible

2) like before $V = |\gamma|$

3) like coherence time

$$h_c = \int_{-\infty}^{\infty} |\gamma(h)|^2 dh$$ sets boundary between large h (no oscillations), small h (oscillations)

"coherence separation"

That's all for exam!