

Lecture 30: Mon, 17 Mar 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. A complex imaging system can be reduced to an equivalent

- a. thin lens
- b. thick lens
- c. thin plate
- d. thick plate

Q2. The effective beginning and end of the equivalent element are called the _____ planes.

- a. dominant
- b. primary
- c. principal
- d. secondary

Q3. For a laser cavity to be stable, the rays

- a. must repeat after one round trip
- b. must repeat after finite number of round trips
- c. must remain finite height after large numbers of round trips

ABCD matrices summary, operating on ray vector (y , θ)

$$\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$$

Straight line, distance d

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Reflection from flat mirror

$$\begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix}$$

Refraction at flat surface (Snell's Law)

$$\begin{pmatrix} 1 & 0 \\ \frac{1}{R} \left(\frac{n_1}{n_2} - 1 \right) & \frac{n_1}{n_2} \end{pmatrix}$$

Refraction at curved surface
 $R =$ positive for convex;
 negative for concave

$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

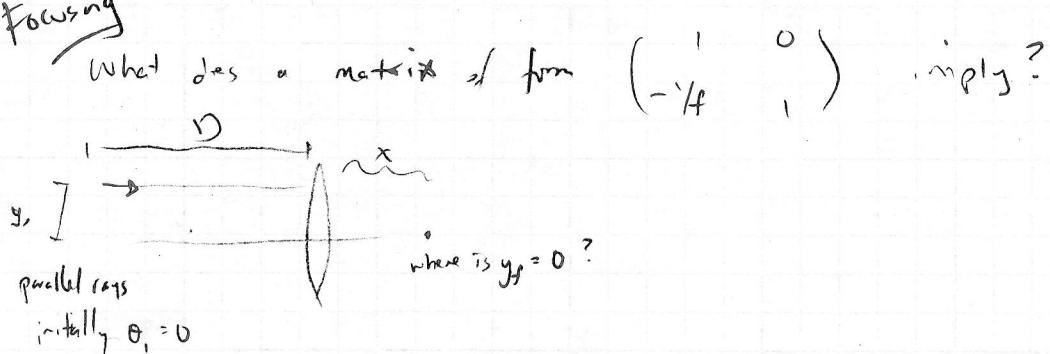
Spherical mirror & Thin lens

$$f_{lens} = \left[\left(\frac{n_2}{n_1} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \right]^{-1}$$

$$f_{mirror} = \frac{R}{2}$$

Lens: $R =$ positive for curving away; negative for curving towards
 Mirror: $R =$ positive for concave

Focusing



$$\begin{pmatrix} y_f \\ 0_f \end{pmatrix} = (\text{transl. of } x) \underbrace{\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}}_{\text{thin lens or focusing mirror}} (\text{transl. of } D) \begin{pmatrix} y_1 \\ 0_1 \end{pmatrix}$$

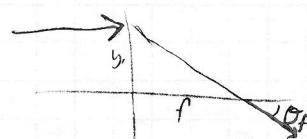
$$\begin{pmatrix} 0 \\ 0_f \end{pmatrix} = \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}}_{\begin{pmatrix} y_1 \\ 0 \end{pmatrix}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ -y_1/f \end{pmatrix}$$

top row: $0 = y_1 - \frac{xy_1}{f} \Rightarrow \boxed{x = f}$

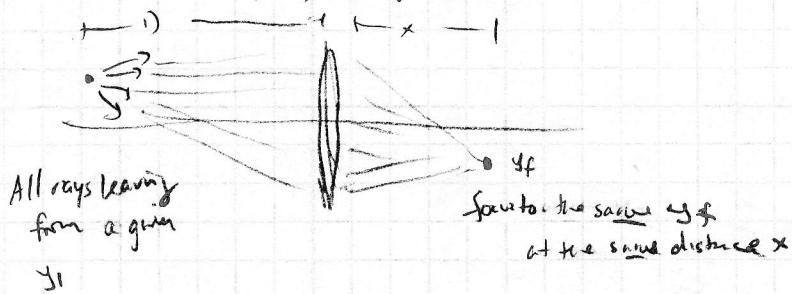
All parallel rays converge to axis at $x=f$
regardless of initial height!

They converge with angle of $-\frac{y_1}{f}$



makes perfect sense!

Another view of focusing



$$\begin{pmatrix} y_f \\ o_f \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} y_i \\ o_i \end{pmatrix}$$

\uparrow
dist/frac/dist

$$= \begin{pmatrix} Ay_i + Bo_i \\ Cy_i + Do_i \end{pmatrix}$$

If $y_f = \text{constant}$ for all o_i , B must = 0

(Because $y_f = Ay_i + Bo_i$ depends on o_i)

then $y_f = Ay_i$,

TA = magnification

let's compute the matrix

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 1 & D \\ 0 & 1 \end{pmatrix}}_{\left(\begin{array}{cc} 1 & D \\ -\frac{1}{f} & -\frac{D}{f} + 1 \end{array} \right)}$$

$$\left(\begin{array}{cc} 1 - \frac{x}{f} & D - \frac{xf}{f} + x \\ -\frac{1}{f} & 1 - \frac{D}{f} \end{array} \right)$$

$$B = 0 \rightarrow D + x \left(1 - \frac{D}{f} \right) = 0$$

$$x = \frac{D}{D-1} \cdot \frac{f}{f} = \frac{Df}{D-f} \rightarrow \frac{1}{x} = \frac{1}{f} - \frac{1}{D}$$

$$M = - \frac{xD}{x+D} \times \frac{x+D}{x+D}$$

$$= - \frac{xD}{xD+D^2-xD}$$

$$= - \frac{x}{D}$$

$$A = \text{mag} = 1 - \frac{x}{f} = 1 - \left(\frac{D-f}{D-f} \right) \frac{1}{f}$$

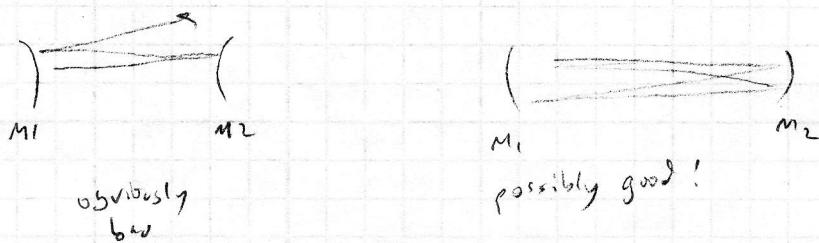
$$= 1 - \frac{D}{D-f} = \frac{D-f-D}{D-f}$$

$$M = \frac{-f}{D-f} \quad M = -\frac{x}{D}$$

$$\boxed{\frac{1}{f} = \frac{1}{x} + \frac{1}{D}}$$

9.8 Stability of laser cavities

Light rays need to get trapped, so they can continually interact with laser medium



Analysis

~~Matrix multiplication representation (A₁₁) X₁₁~~

This multiple bounces, is like this

(light path) $M_1 \rightarrow L \rightarrow M_2 \rightarrow L \rightarrow M_1 \rightarrow L \rightarrow M_2 \rightarrow \dots$

We have an infinite # of repetitions of this basic structure

$$M_1 - L - M_2 - L$$

... matrices go in opposite order

$$M = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{pmatrix}$$

Multiply together

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$f_1 = \frac{R_1}{2}$$

$$f_2 = \frac{R_2}{2}$$

Sylvester's Thm: if $\det M = 1$ (it is!)

then $M^N = \text{complicated formula}$

$$\begin{pmatrix} A_N & B_N \\ C_N & D_N \end{pmatrix}$$

stable quantity: need matrix elements to remain finite as $N \rightarrow \infty$

Matrix elements like $\sin(N\theta)$ $\theta = \cos^{-1} \frac{A+D}{2}$ \leftarrow of original matrix

- \rightarrow can blow up if $\theta = \text{imaginary}$

stability condition: $\theta = \text{real}$

or $\cos \theta$ between -1 and 1

or $-1 < \frac{A+D}{2} < 1$

HW problem: this \Leftrightarrow do matrix multiplying $\xrightarrow{\text{for 2 rows}} \text{matrix}$, then apply this condition

rearrange:

$$\text{stable if } 0 < \left(1 - \frac{L}{R_1}\right) \left(1 - \frac{L}{R_2}\right) < 1$$

Lecture 31: Wed, 19 Mar 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. Which of the following was *not* an aberration discussed in the textbook:

- a. astigmatism
- b. chromatic
- c. coma
- d. curvature of the field
- e. distortion
- f.** scalar
- g. spherical

Q2. To correct aberrations, it is most common to use:

- a. elliptical lenses
- b. hyperbolic lenses
- c. parabolic lenses
- d.** spherical lenses

Q3. Spherical aberration can be partially corrected by placing a plano-convex lens:

- a. so the light strikes the flat side
- b. so the light strikes the curved side

