Lecture 38: Fri, 4 Apr 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. Creating a hologram involves how many light waves striking the film?

a. 1
b. 2
c. 3

Q2. What nationality was Dennis Gabor (by birth)?

a. American
b. British
c. German
d. Hungarian
e. Norwegian

Q3. What was one application of interferograms mentioned in the chapter?

a. designing lasers
b. digital image recording
c. measuring coherence times
d. testing optical components

Lecture 39: Mon, 7 Apr 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. Who theoretically derived the correct blackbody radiation formula (energy emitted at various frequencies)?

a. Colton
b. Curie & Weiss
c. Planck
d. Rayleigh & Jeans
e. Sommerfeldt
f. Stefan & Boltzmann

Q2. What critical assumption did he/she/they make to produce the correct formula?

a. continuous energy
b. quantized energy
c. long-λ limit
d. short-λ limit

Q3. ~~Who's~~ whose law says that the amount of emitted blackbody radiation is proportional to T^4?

a. Colton's
b. Curie & Weiss's
c. Planck's
d. Rayleigh & Jeans'
e. Sommerfeldt's
f. Stefan & Boltzmann's

Blackbody "Hot objects glow"

Boltzmann factor: Prob of an energy state $\propto e^{-E/k_BT}$

Plank: light at a particular frequency f, only available in integral multiples of hf

$$\begin{array}{ll} & \text{—} \\ & \text{—} \\ & \text{— } 3hf \\ n=2 & \text{— } 2hf \\ n=1 & \text{— } hf \\ & \text{— } 0 \end{array}$$

Today: hf = photon energy

Plank: "act of desperation" to fit experimental data

$$\text{Prob of being in } n^{th} \text{ state} = \frac{e^{-E_n/k_BT}}{\sum \text{all B.F.s}}$$

$$P_n = \frac{e^{-nhf/kT}}{e^0 + e^{-hf/kT} + e^{-2hf/kT} + \cdots}$$

$$= e^{-nhf/kT}\underbrace{\left(1 - e^{-hf/kT}\right)}_{\text{result of infinite sum}}$$

actual energy added with n^{th} state $= (nhf) \times P_n$

Total energy of all f-related states

$$E = \sum_{n=0}^{\infty} nhf\, e^{-nhf/kT}\left(1 - e^{-hf/kT}\right)$$

$$= hf\left(1 - e^{-hf/kT}\right) \sum_{n=0}^{\infty} n\, e^{-nhf/kT}$$

$$\text{Trick: } \sum_{x=0}^{\infty} x e^{-ax} = -\sum_{x=0}^{\infty} \frac{\partial}{\partial a}\left(e^{-ax}\right)$$

$$= -\frac{\partial}{\partial a} \underbrace{\sum_{x=0}^{\infty} e^{-ax}}_{\frac{1}{1-e^{-a}}}$$

or sum with Mathematica

$$= hf\left(\cancel{1 - e^{-hf/kT}}\right)\left[\frac{e^{hf/kT}}{\left(1 - e^{hf/kT}\right)^2}\right]$$

$$\frac{e^{a}}{(1-e^{a})(1-e^{a})} \quad \frac{e^{-a}}{e^{-a}}$$

Cancel, with (-1) $\longrightarrow$ $\dfrac{1}{\cancel{(e^{-a}-1)}(1-e^{a})}$

$$= \frac{hf}{e^{a}-1}$$

$$\boxed{E = \frac{hf}{e^{hf/kT}-1}}$$

energy associated with all photons frequency f or plane waves with that f $\sim\sim$

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"Box" (Could let $L \to \infty$)

radiation inside → which freqs are possible?

Bndy cond.

Integral # of half wavelengths, $L_x = \frac{n\lambda_x}{2}$ $\rightarrow \lambda_x = \frac{2L}{n}$

Allowed k's: $k_x = \frac{2\pi}{\lambda_x}$

$k_x = \frac{\pi}{L} \times n = k_0 n$ with $k_0 = \frac{\pi}{L}$ factor of 2 different from P+W

similarly $k_y = k_0 m$

$k_z = k_0 l$

$k = \sqrt{k_x^2 + k_y^2 + k_z^2} = k_0\sqrt{n^2+m^2+l^2}$

$= k_0 s$ $s = \sqrt{n^2+m^2+l^2}$

(like $|\vec{r}|$ for normal space)

k also $= \frac{\omega}{c} = \frac{2\pi f}{c}$

so $f = \frac{c}{2\pi} k_0 s$

$$U_{tot} = \underset{\text{polar.}}{2} \sum_{n,m,l} \frac{hf}{e^{hf/kT}-1}$$

$$= 2 \sum_{n,m,l=0}^{\infty} \frac{h\left(\frac{c}{2\pi}k_0 s\right)}{e^{h\left(\frac{c}{2\pi}\frac{k_0 s}{kT}\right)}-1}$$

$$= 2\,\frac{hc}{2\pi}k_0 \underset{\substack{\text{only positive}\\ n,m,l\\ (\frac{1}{2})^3 \text{ diff from P+W}}}{\frac{1}{8}} \int_0^\infty \frac{s}{e^{\frac{hc}{2\pi}\frac{k_0 s}{kT}}-1} \times 4\pi s^2\, ds$$

$x = \frac{hc\,k_0\,s}{2\pi kT}$ $s = \frac{2\pi kT}{hck_0}x$

$ds = (\quad)\,dx$

$$2\,\frac{hc}{2\pi}k_0\,\frac{4\pi}{8}\left(\frac{2\pi kT}{hck_0}\right)^4 \int_0^\infty \frac{x^3\,dx}{e^x-1}$$

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$$\frac{1}{2} \; K \cancel{c} \cancel{k_0} \; \frac{16\,\pi^4 k^4 T^4}{h^{\cancel{3}\cancel{4}}\, c^{\cancel{3}\cancel{4}}\, k_0^{\cancel{4}3}} \quad \frac{\pi^4}{15}$$

$$\frac{8}{15} \; \frac{\pi^8}{h^3 c^3} \; \frac{k^4 T^4}{\pi^3/L^3}$$

$$\frac{8}{15} \; \frac{\pi^5 L^3 k^4 T^4}{h^3 c^3}$$

$\hbar = \frac{h}{2\pi}$

$h = 2\pi\hbar$

$$u = \frac{U}{L^3} = \underline{\frac{8}{15} \; \frac{\pi^5 k^4}{h^3 c^3} \; T^4}$$

book (13.1) $I = \frac{c}{4} \cdot u$ $\quad$ I = total intensity of radiation flowing outward

c from speed of light moving away

$\frac{1}{2}$: half energy travelling away, half towards

$\frac{1}{2}$ energy from (eg x-dir) is distributed like

So

$$I = \underbrace{\frac{2}{15} \; \frac{\pi^5 k_B^4}{h^3 c^2}} \; T^4$$

$\sigma = 5.67 \cdot 10^{-8} \; \frac{W}{m^2 K^4}$ Stefan-Boltzmann constant

correction factor emissivity (from backwards surface reflection)

$I = \epsilon \sigma T^4$ Stefan-B. Law

How much of u is at each frequency? ρ_f energy density in f

ie $u = \int \rho_f \, df$.

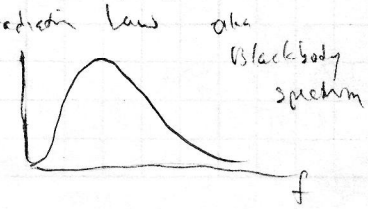
Go back to integral

$$u = 2 \frac{hc}{2\pi} k_0 \; \frac{1}{8} \int_0^\infty \frac{s}{e^{\frac{hck_0 s}{2\pi kT}} - 1} \, 4\pi s^2 \, ds$$

Go back to find in f: substitute in $f = \frac{c}{2\pi} k_0 s$

Simplify ... $u = \int_0^\infty \rho_f \, df$

with $\boxed{\rho_f = \frac{8\pi h f^3}{c^3 (e^{hf/kT} - 1)}}$ "Plank radiation Law" aka Blackbody spectrum



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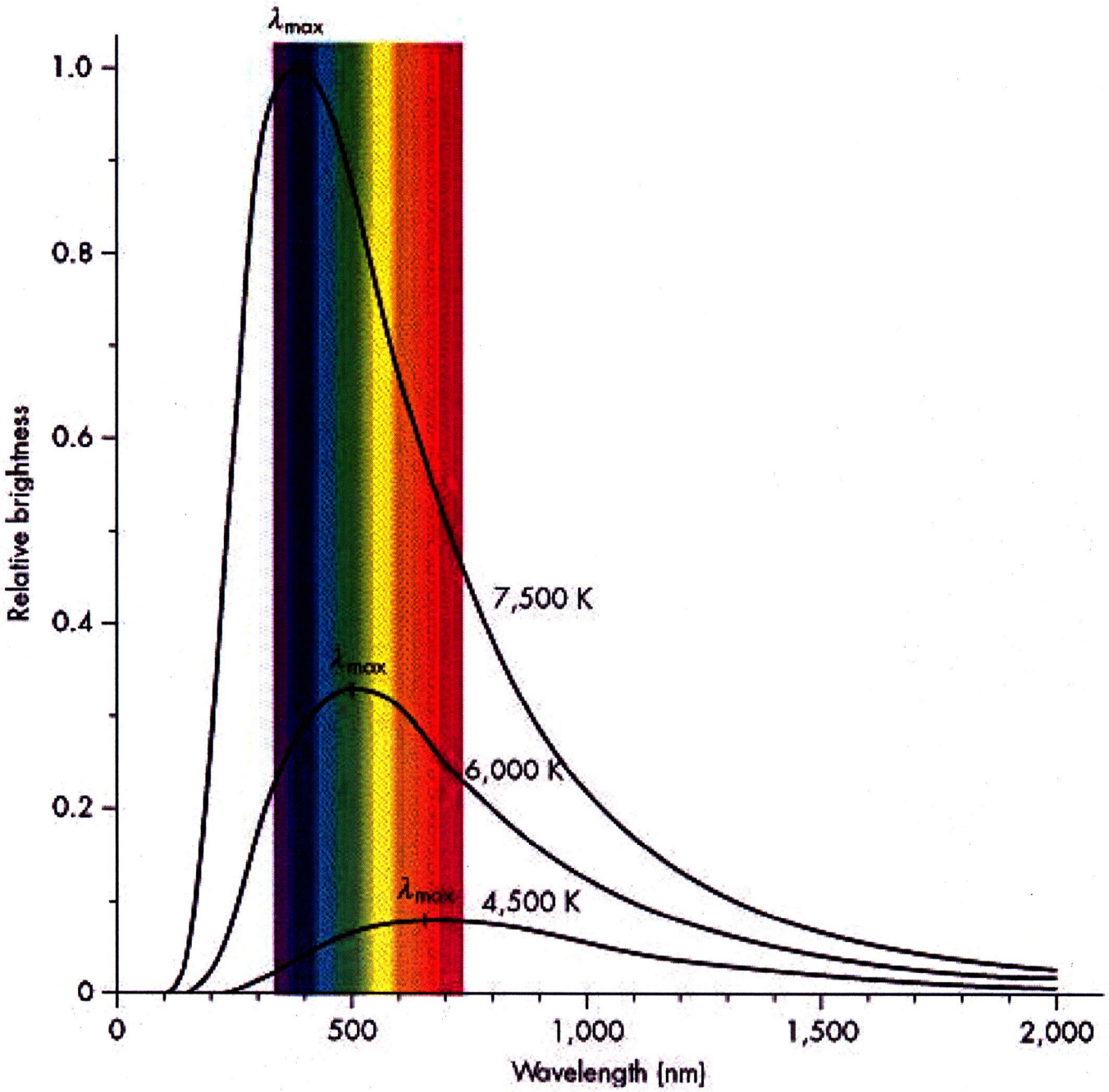
Plank Radiation Law, in terms of wavelength