Lecture 9: Mon, 28 Jan 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. Brewster's angle occurs when θ_i and θ_t

a. Are equal
b. Sum to 90°
c. Sum to 45°
d. Sum to 60°

Q2. During total internal reflection, the electric field in the air just outside the glass is called the

a. reduction wave
b. florescent wave
c. evanescent wave
d. vanishing wave
e. nothing…there can be no E-field there.

Q3. Total internal reflection happens for:

a. s-polarized light
b. p-polarized light
c. both polarizations

day 9

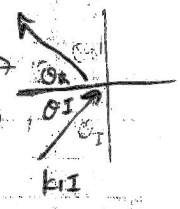
Also $k_{1Iy} = k_{1Ry}$

$k_{1Iy} = k_{1Ry}$ → incident and reflected have same y-component!

k_{1I} same magnitude, same y-component

only possible if

(optics only, not usual θ!)



$\boxed{\theta_R = \theta_I}$ 2nd Law!

"Law of Reflection"

Also $k_{1Iy} = k_{2Ty}$

$k_1 \sin\theta_1 = k_2 \sin\theta_2$ $\qquad \frac{\omega}{k} = v = \frac{c}{n}$

$v = \frac{\omega}{k}$

$k = \frac{\omega}{v}$

$k = \frac{\omega n}{c}$

$\frac{\omega n_1}{c} \sin\theta_1 = \frac{\omega n_2}{c} \sin\theta_2$

$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$ 3rd Law

"Law of Refraction", "Snell's Law"

(also critical angle → can't get any transmission)

3 Laws of Geometrical Optics!

Gives deeper explanation: a beam of light bends because its fields propagate at a different speed (like driving on shoulder)

Explains focusing, image formation, ray tracing, etc, etc

→ only used most basic boundary condition, not really any advanced Maxwell Eqn

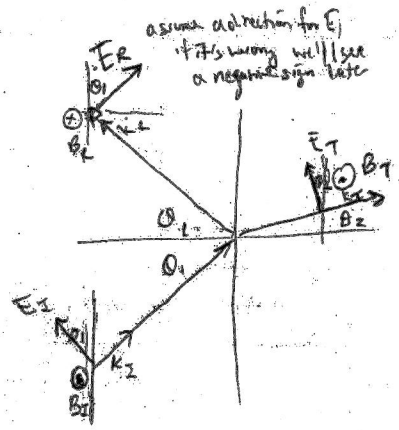
So these are true for basically any waves!

→ you're starting to realize why it was so hard to accept Einstein's photon idea!

Back to Basic Problem

Now let's assume a particular polarization: p - polarization

$\vec{E}$ = // to plane of incidence



BC $E_{\parallel} = E_{\parallel}$ $\quad \tilde{E}_{0I}\cos\theta_1 + \tilde{E}_{0R}\cos\theta_1 = \tilde{E}_{0T}\cos\theta_2$ (lost all exponential factors because they're equal)

BC $B_{\parallel} = B_{\parallel}$ $\quad \frac{1}{\mu_1 v_1}\tilde{E}_{0I} - \frac{1}{\mu_1 v_1}\tilde{E}_{0R} = \frac{1}{\mu_2 v_2}\tilde{E}_{0T}$

$\vec{B}$ in opp. direction

let $\boxed{\beta = \frac{\mu_1 v_1}{\mu_2 v_2}}$ as before $\boxed{\frac{n_2}{n_1}}$

then $\underline{\tilde{E}_{0I} - \tilde{E}_{0R} = \beta\,\tilde{E}_{0T}}$

let $\boxed{\alpha = \frac{\cos\theta_2}{\cos\theta_1}}$ then $\underline{\tilde{E}_{0I} + \tilde{E}_{0R} = \alpha\,\tilde{E}_{0T}}$

Solve simult. eqns with a little algebra

$$\boxed{\begin{aligned}\tilde{E}_{0R} &= \left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0I}\\ \tilde{E}_{0T} &= \frac{2}{\alpha+\beta}\,\tilde{E}_{0I}\end{aligned}}$$

Fresnel eqns! 3.15 & 3.16 [same as P&W eqns on pg 60 multiply num & denom by $n_1\cos\theta_1$ to get right most form]

Notice: • a 180° phase shift in reflected beam if $\beta > \alpha$

• $\beta = \alpha$ special case, "Brewster's Angle" $\rightarrow$ $\underline{E_{0R} = 0}$! no reflection

$$\frac{\cos\theta_2}{\cos\theta_1} = \frac{n_2}{n_1}$$

$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$

$$\frac{\sqrt{1-\sin^2\theta_2}}{\cos\theta_1} = \frac{n_2}{n_1}$$

$$\sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta_1} = \frac{n_2}{n_1}\cos\theta_1$$

$$1-\frac{n_1^2}{n_2^2}\sin^2\theta_1 = \frac{n_2^2}{n_1^2}\cos^2\theta_1$$

$$1 - \frac{1}{\beta^2}\sin^2\theta_1 = \beta^2\cos^2\theta_1 \qquad (\times\ \beta^2)$$

$$\beta^2 - \sin^2\theta_1 = \beta^4(1-\sin^2\theta_1)$$

$$\beta^4\sin^2\theta_1 - \sin^2\theta_1 = \beta^4 - \beta^2$$

$$\sin^2\theta_1(\beta^4-1) = \beta^4-\beta^2$$

$$\sin^2\theta_1 = \frac{\beta^2(\beta^2-1)}{\beta^4-1} = \frac{\beta^2\cancel{(\beta^2-1)}}{(\beta^2+1)\cancel{(\beta^2-1)}}$$

$$\sin^2\theta_1 = \frac{\beta^2}{\beta^2+1}$$

consider $\cos^2\theta_1 = 1-\sin^2\theta_1 = 1 - \frac{\beta^2}{\beta^2+1}$

$$= \frac{\cancel{\beta^2}+1-\cancel{\beta^2}}{\beta^2+1} = \frac{1}{\beta^2+1}$$

so $\tan^2\theta_1 = \dfrac{\frac{\beta^2}{\cancel{\beta^2+1}}}{\frac{1}{\cancel{\beta^2+1}}} = \beta^2$

can skip unless needed

$$\boxed{\tan\theta_B = \beta = \frac{n_2}{n_1}}$$ = Brewster angle condition!

only a 180° phase shift for certain angles ($\theta < \theta_B$)

Consider Boot Figs

→ phase shift in Refl. for $\theta \le \theta_B$ (if $n_2 > n_1$??)

P7.391 and 3a2

- For $\theta_I = 90°$ (grazing), $\alpha = \frac{\cos\theta_2}{\theta} = \infty$

→ all reflected, none transmitted!

- For $\theta_I = 0°$ (normal) ⇒ $\alpha = 0$, recover previous Fresnel Eqns

Reflected intensities: $I = \frac{1}{2}\epsilon_1 v_1 E_{01}^2$ → not good enough, we need $\vec{S}\cdot\hat{z}$ ($\times \cos\theta$)

Power unitary of transmission

$$R = \frac{E_{0R}^2}{E_{0I}^2} = \boxed{\left(\frac{\alpha-\beta}{\alpha+\beta}\right)^2}$$ ϵ's, v's, θ's equal

$$T = \underbrace{\frac{n_2}{n_1}}_{\beta}\underbrace{\left(\frac{E_{0T}}{E_{0I}}\right)^2}_{\left(\frac{2}{\alpha+\beta}\right)^2}\underbrace{\frac{\cos\theta_T}{\cos\theta_I}}_{\alpha} = \boxed{\frac{4\alpha\beta}{(\alpha+\beta)^2}}$$

$\frac{1}{\sqrt{\mu\epsilon}} = v$

From *Griffiths*

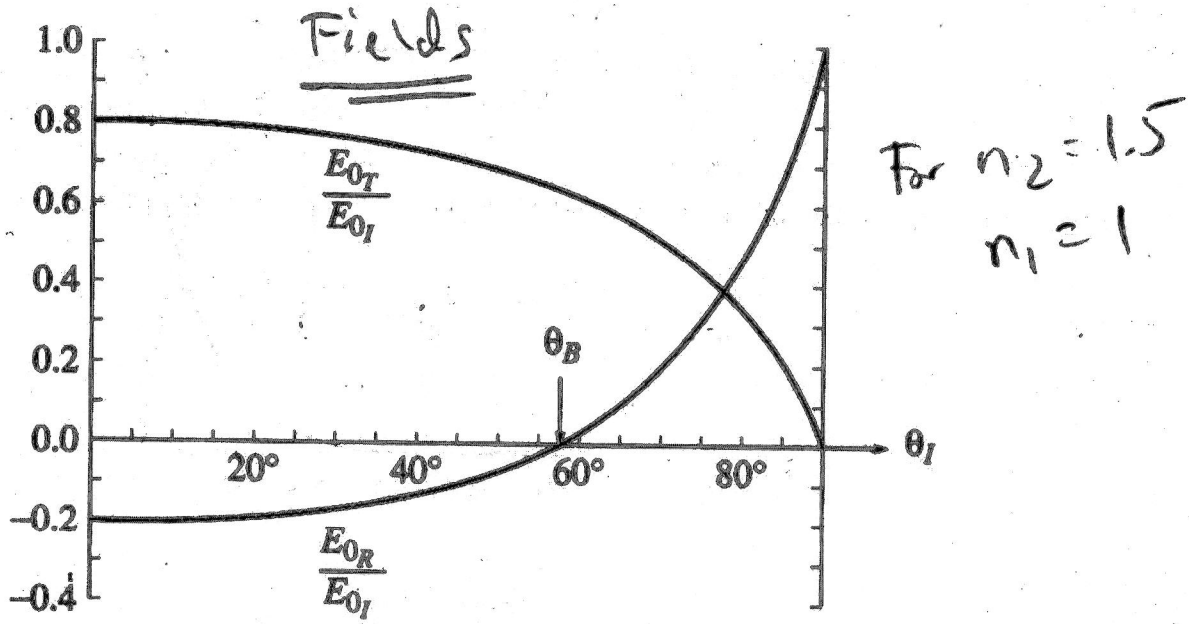


Figure 9.16

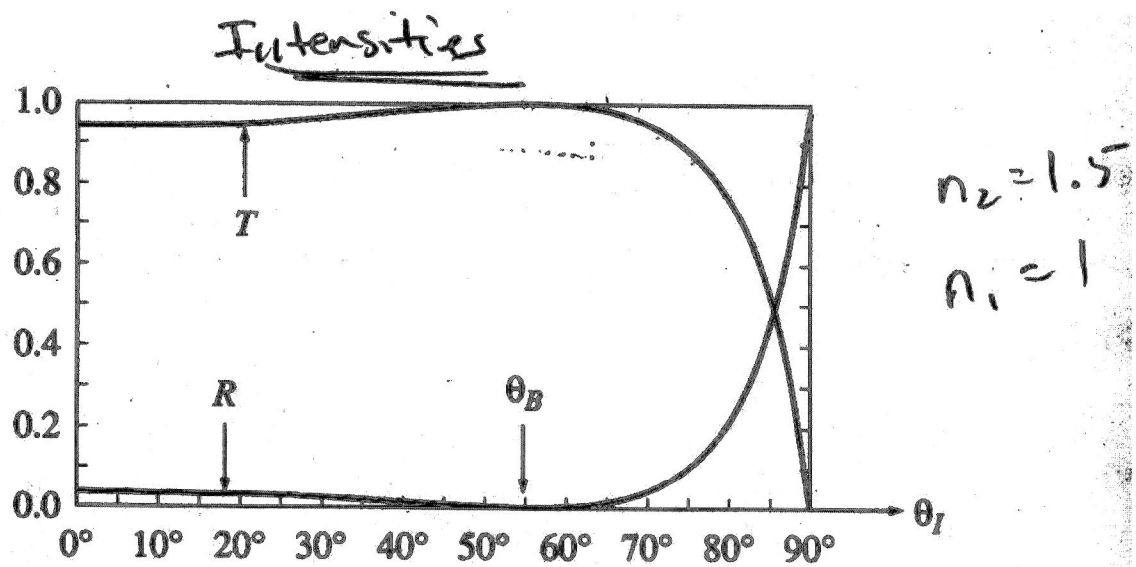
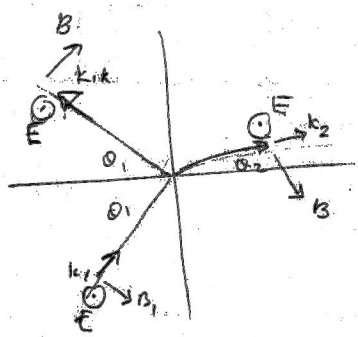


Figure 9.17

S-polarization

$$\tilde{E}_{0R} = \frac{1-\alpha\beta}{1+\alpha\beta}\tilde{E}_{0I}$$

$$\tilde{E}_{0T} = \frac{2}{1+\alpha\beta}\tilde{E}_{0I}$$



Result:

$$R = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)^2$$

$$T = \frac{4\alpha\beta}{(1+\alpha\beta)^2}$$

→ do analysis for $H_{\parallel}$!

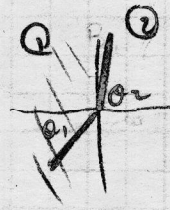
Notice: Always have 180° phase shift in reflected if $n_2 > n_1$

" " " " " " if $n_2 < n_1$

→ But no Brewster's angle (always get some reflection)

(unless weird μ's)

Total Internal Reflection



$n_1 > n_2$

Review

When Snell's law predicts $\theta_2 = 90°$
it can't bend anymore

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

critical $\sin\theta_1 = \frac{n_2}{n_1}$

$$\boxed{\theta_1 \geq \sin^{-1}\left(\frac{n_2}{n_1}\right) \quad \text{TIR occurs}}$$

Proof that it's total reflection

For p polar. $r = \frac{\tilde{E}_{0p}}{\tilde{E}_{0i}} = \frac{\alpha - \beta}{\alpha + \beta}$ $\quad \alpha = \frac{\cos\theta_2}{\cos\theta_1}$ $\quad \beta = \frac{n_2}{n_1}$

α is now imaginary! But that's OK with equations

proof: $\alpha = \frac{\sqrt{1-\sin^2\theta_2}}{\cos\theta_1} = \frac{\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin\theta_1}}{\cos\theta_1}$ → larger than 1

$= \frac{\sqrt{\text{negative}}}{\text{positive}}$

$=$ Imaginary!

$= ix$

What does that do to r?

$$r = \frac{ix - \beta}{ix + \beta}$$

What is magnitude of r? $|r| = \sqrt{r r^*}$

$= \sqrt{\left(\frac{ix-\beta}{ix+\beta}\right)\left(\frac{-ix-\beta}{-ix+\beta}\right)}$

$= \sqrt{(-1)(-1)}$

$= 1$ ✓

All is reflected!

Optics animation! animation 5

evanescent field → long λ β λ

Refl from conductors: n now can be complex $N = n + ik$
→ not only is α imaginary (complex?) but β is also!
but aside from that things are same

} $r \neq 1$ though

Lecture 10: Wed, 30 Jan 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. In circularly polarized light, the two components of the E-field differ by a phase of

a. 0
b. $\pi/3$
c. $\pi/2$
d. $2\pi/3$
e. π

Q2. The mathematical objects that describe polarization states are _____ vectors

a. Fred
b. Jones
c. Newton
d. Maxwell
e. Smith

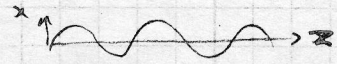
Q3. In the above vectors, the ones that use complex numbers represent

a. linear polarized light
b. elliptical light in general
c. circular only

Polarization

We've been studying things like $\vec{E} = \vec{E_0}\, e^{i(kz-\omega t)}$

$= (\text{say})\ E_0 \hat{x} \cos(kz-\omega t)$



This is a "linearly polarized" wave, because the electric field goes back + forth in a line in the x-y plane

(wave coming out of page)

What happens when we add two waves together?

Example: $E_0 \hat{x} \cos(kz-\omega t) + E_0 \hat{y} \cos(kz-\omega t)$

$= E_0\sqrt{2}\left(\frac{\hat{x}+\hat{y}}{2}\right) \cos(kz-\omega t)$

amplitude would be $E_0\sqrt{2}$

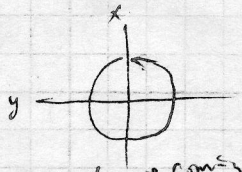
Still linear polarization!

Add a phase shift:

$$E_0 \hat{x} \cos(kz-\omega t) + E_0 \hat{y} \underbrace{\sin(kz-\omega t)}_{\cos(kz-\omega t-90^\circ)}$$

Circular pol! See picture!

("amplitude" = E_0, not $E_0\sqrt{2}$)

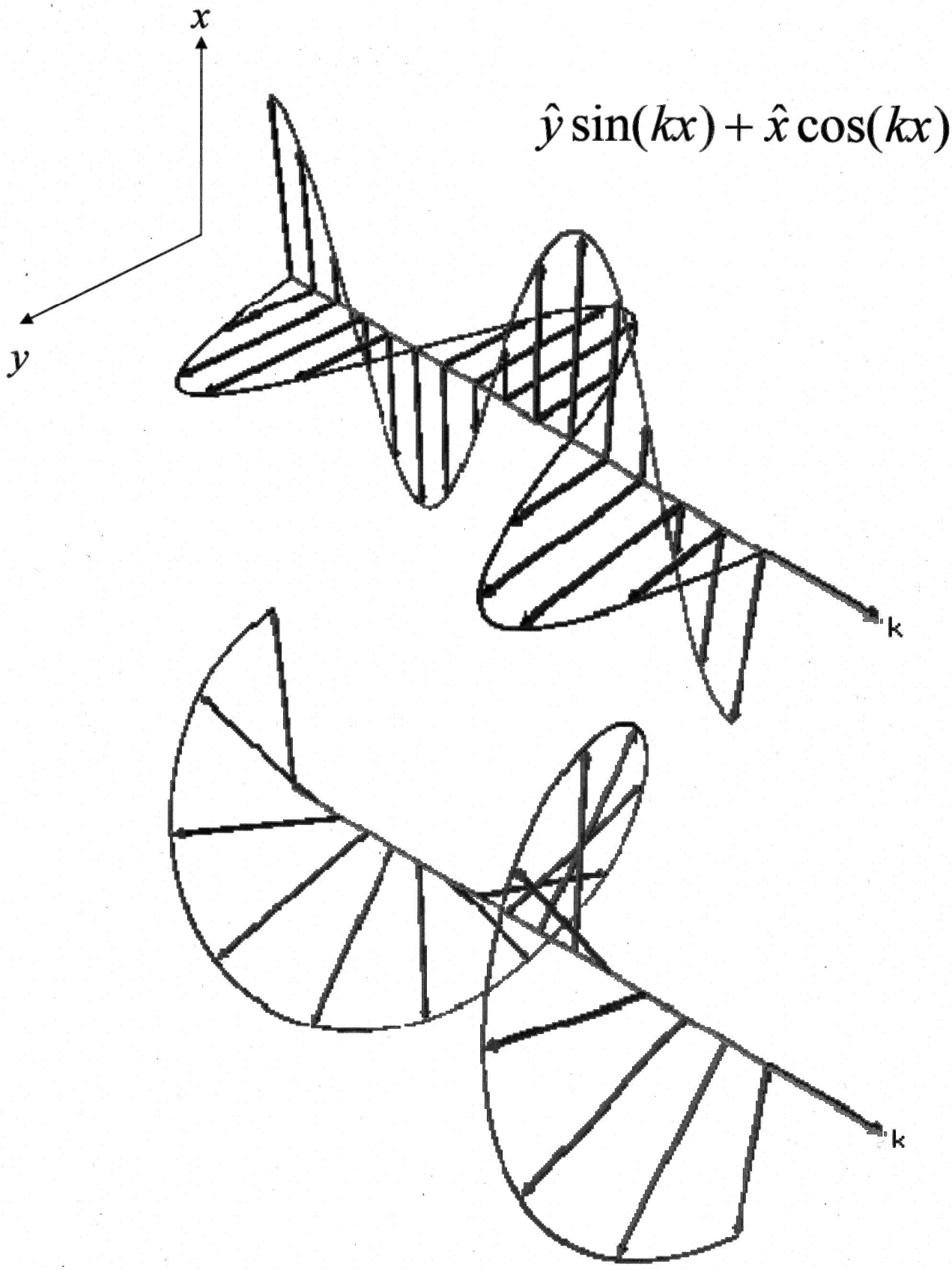


(wave coming out of page)

(right circular, to be precise)

→ stop time, look at screw in E field
same as regular screw → right handed
opposite → left handed

also: right handed: $\vec{E}$ rotates to right as seen from observer


x
y
$\hat{y}\sin(kx) + \hat{x}\cos(kx)$
k
k

writing circ. pol w/ complex numbers.

$$\vec{E}_0 = E_0\hat{x}\cos(kz-\omega t) + E_0\hat{y}\cos(kz-\omega t-90^\circ)$$

$$= E_0\hat{x}\,e^{i(kz-\omega t)} + E_0\hat{y}\,e^{i(kz-\omega t-90^\circ)}$$

$$= \left[E_0\hat{x} + E_0\hat{y}\,e^{i(-90^\circ)}\right] e^{i(kz-\omega t)}$$

$$E_0\begin{pmatrix}1\\ e^{i(-90^\circ)}\end{pmatrix} e^{i(kz-\omega t)}$$

$e^{i(-90^\circ)} = -i$

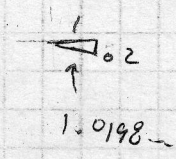
the "Jones vector"!

[Convention: vector has magnitude 1, so add $\sqrt{2}$]

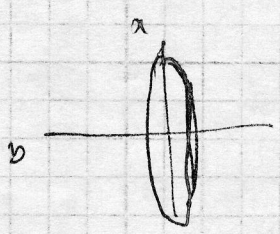
What if the y component not as strong as x-component?

$$E_0 = \left(E_0\hat{x} + .2E_0\hat{y}\,e^{i(-90^\circ)}\right) e^{i(kz-\omega t)}$$

$$\text{Jones} = \frac{1}{1.0198}\begin{pmatrix}1\\ .2e^{i(-90^\circ)}\end{pmatrix}$$



Omnator #3



Elliptically polarized

ellipticity $= \frac{E_{min}}{E_{max}}$

(like "eccentricity" of ellipse, but not quite the same mathematically)

Figure

Summary table 4.1

$\begin{pmatrix}1\\0\end{pmatrix}$ x

$\begin{pmatrix}0\\1\end{pmatrix}$ y

$\begin{pmatrix}\cos\alpha\\ \sin\alpha\end{pmatrix}$ $\angle\alpha$ from x

$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -i\end{pmatrix}$ RCP

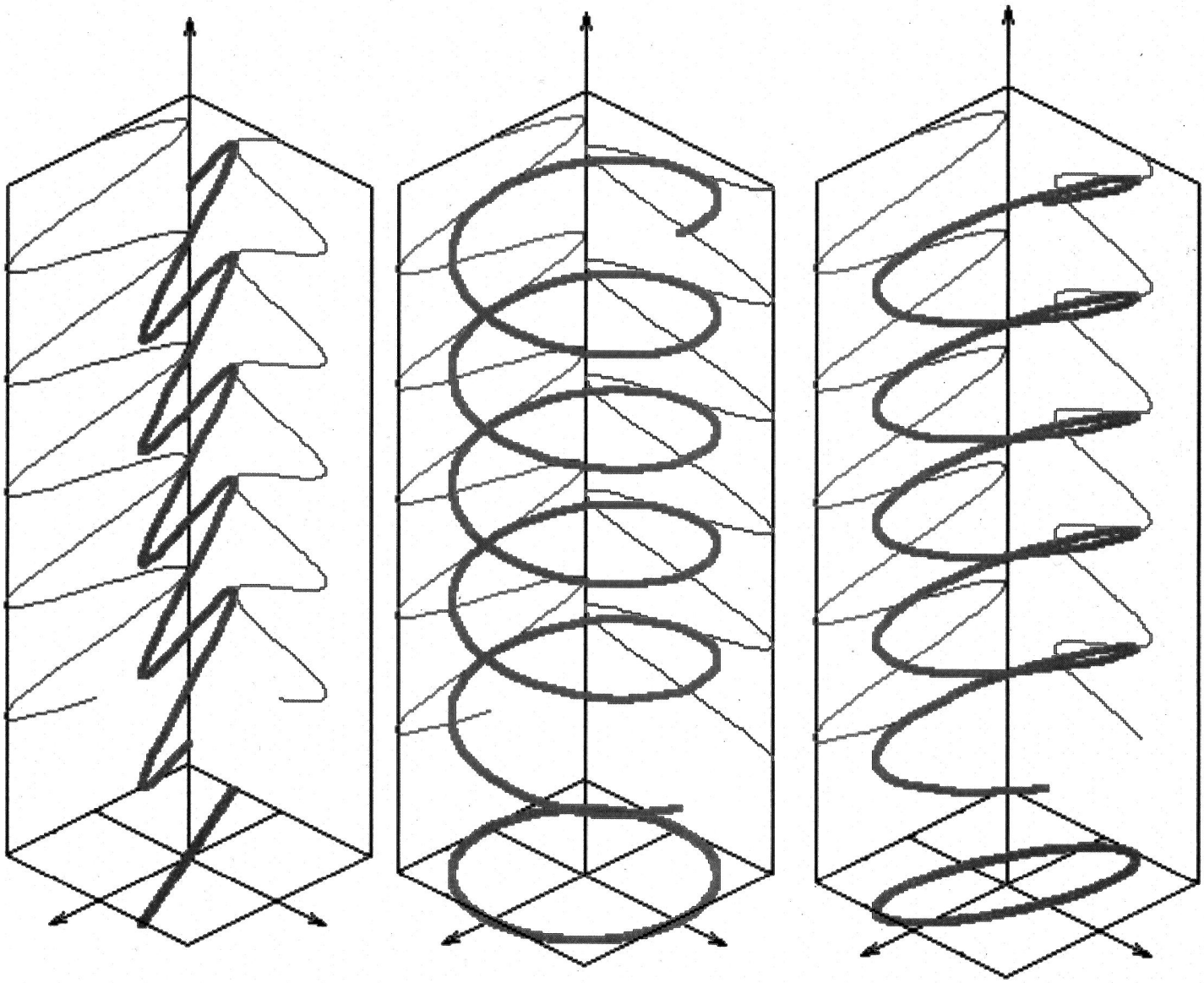
$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix}$ LCP

$\frac{1}{\sqrt{A^2+B^2}}\begin{pmatrix}A\\ Be^{i\delta}\end{pmatrix}$ RCP-like elliptic, long axis oriented off x or y axis

$\frac{1}{\sqrt{A^2+B^2}}\begin{pmatrix}A\\ Be^{i\delta}\end{pmatrix}$ elliptic at whatever axis (angle & δ given in text)

Elliptically polarized light

E still rotates at a given point in space. But E is strongest when it points along a certain line, and weakest when it points 90 degrees from that line.



linear *circular* *elliptical*

Jones vectors summary (similar to P&W table 4.1)

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	linearly polarized in x-direction
$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	linearly polarized in y-direction
$\begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$	linearly polarized at angle α from x-axis
$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i(-90°)} \end{pmatrix}$	RCP
$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i(+90°)} \end{pmatrix}$	LCP
$\begin{pmatrix} A \\ Be^{i(-90°)} \end{pmatrix}$	RCP-like elliptical, with ellipse oriented along x- and y-axes ($A^2 + B^2$ must $= 1$)
$\begin{pmatrix} A \\ Be^{i\delta} \end{pmatrix}$	elliptical oriented at angle α from x- or y-axis (long equation for α as a function of δ given in textbook; $A^2 + B^2$ must $= 1$)