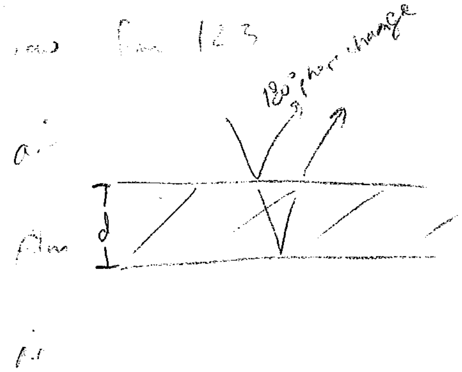
day 10 pg 1

Chap 4 · Multiple Parallel Interfaces

Reviews from 123



Normal incidence: constructive interference

$$2d = \left(m + \tfrac{1}{2}\right) \lambda_{\text{in film}}$$

$\lambda_{\text{in film}} = \lambda/n$

to match 180° phase shift on surface

ie. "optical path length" $= (m+\frac{1}{2})\lambda$

$OPL = (P.L.) \times n$

Where does OPL come from in our eqns?

$$e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

phase shift between two pts at same time

$$= e^{i\vec{k}\cdot(\Delta\vec{r})}$$

eg between 0 and $d\hat{z}$

$$= e^{ikd}$$

$$= e^{i 2\pi \frac{1}{\lambda} n d}$$

$$= e^{i 2\pi \, nd/\lambda}$$

$$\frac{\omega}{k} = \frac{c}{n}$$

$$k = \frac{\omega}{c} \cdot n$$

$$= 2\pi \frac{f}{c} n$$

$$= 2\pi \frac{1}{\lambda} n$$

when nd/λ = integer

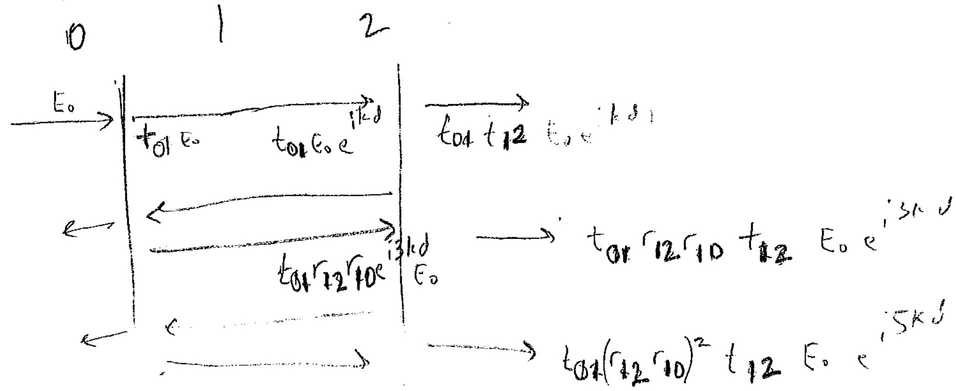
have constructive interference

(or destructive, if 180° phase shift)

Reading quiz

Day 10 pg 2

Hecht's method (sort of)



$$t_{total}^{0\to 2} = t_{01}t_{12}e^{ikd}\Big(\underbrace{1 + r_{12}r_{10}e^{i2kd} + (r_{12}r_{10})^2 e^{i4kd} + \dots}_{\substack{1 + x + x^2 + x^3 + \dots \\ = \frac{1}{1-x}}}\Big)$$

$$\boxed{t^{0\to 2} = \frac{t_{01}t_{12}e^{ikd}}{1 - r_{12}r_{10}e^{i2kd}}}$$

Note: "k" $= k_1 = \frac{2\pi}{\lambda_1}$

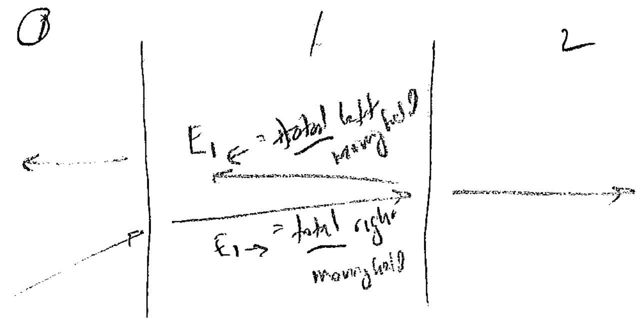
At an angle? $d \to d\cos\theta_1$

because $\vec{k}\cdot\vec{r} = kd\cos\theta_1$ phase factor

day 10 pg 3

Reading quiz

P+W = multiple parallel interfaces



What is transmission?

common "going to left" notation

You would think... 1. $\dfrac{E_{0\leftarrow}}{E_{0\rightarrow}} = r^{01}$ 3. $\dfrac{E_{1\leftarrow}}{E_{1\rightarrow}} = r^{12}$

2. $\dfrac{E_{1\rightarrow}}{E_{0\rightarrow}} = t^{01}$ 4. $\dfrac{E_{2\rightarrow}}{E_{1\rightarrow}} = t^{12}$

Fresnel coefficients, could be s, could be p

But no... several problems!

~~X) (not helpful, partially not quite correct either)~~

1) Not quite correct

fixed: $E_{1\rightarrow} = \underbrace{r^{01} E_{0\rightarrow}}_{\text{as guessed}} + \underbrace{t^{10} E_{1\leftarrow}}$

left moving wave reflecting off of 0-1 interface will add to right moving wave

What about multiple (infinite) reflections?
Not needed. These are total fields

2) corrected $\dfrac{E_{1\rightarrow LHS}}{E_{0\rightarrow LHS}} = t^{01}$

3) partially corrected version $\dfrac{E_{1\leftarrow, RHS}}{E_{1\rightarrow RHS}} = r^{12}$

4) partially corrected version $\dfrac{E_{2\rightarrow}}{E_{1\rightarrow RHS}} = t^{12}$

How to connect RHS to LHS? Phase factors for $\rightarrow$ $e^{i\vec{k}\cdot\vec{r}} = e^{ik_1(\sin\theta_1\hat{y}+\cos\theta_1\hat{z})\cdot d\hat{z}}$ $\leftarrow$ right hand edge relative to left hand edge.

$= e^{ik_1 d\cos\theta_1}$

So $\underline{E_{1\rightarrow RHS} = E_{1\rightarrow LHS}\, e^{ik_1 d\cos\theta_1}}$

similarly $\underline{E_{1\leftarrow RHS} = E_{1\leftarrow LHS}\, e^{-ik_1 d\cos\theta_1}}$ (negative since $\hat{k} = \sin\theta\hat{y} - \cos\theta\hat{z}$ for $\leftarrow$)

Adjusted Eqn 3: $\dfrac{E_{1\leftarrow}\, e^{-ik_1 d\cos\theta_1}}{E_{1\rightarrow}\, e^{+ik_1 d\cos\theta_1}} = r^{12}$ $\left(E_{1\leftarrow} : E_{1\leftarrow LHS}\right)$ etc

Adjusted eqn 4: $\dfrac{E_{2\rightarrow}}{E_{1\rightarrow}\, e^{ik_1 d\cos\theta_1}} = t^{12}$

A little algebra with eqns 2, 3, 4   4 unknowns: $E_{0\rightarrow}$, $E_{1\rightarrow}$, $E_{1\leftarrow}$, $E_{2\rightarrow}$

Skip?

From Eqn 4: $E_{1\rightarrow} = \dfrac{E_{2\rightarrow}}{t^{12}}\, e^{-ik_1 d\cos\theta_1}$

From Eqn 3: $E_{1\leftarrow} = E_{1\rightarrow}\, r^{12}\, e^{2ik_1 d\cos\theta_1}$

$= \left(\dfrac{E_{2\rightarrow}}{t^{12}}\, e^{-ik_1 d\cos\theta_1}\right) r^{12}\, e^{2ik_1 d\cos\theta_1}$

$= E_{2\rightarrow}\dfrac{r^{12}}{t^{12}}\, e^{ik_1 d\cos\theta_1}$

Plug into Eqn 2

$$\left(\frac{E_{2\rightarrow}}{t^{12}}\, e^{-ik_1 d\cos\theta_1}\right) = t^{01} E_{0\rightarrow} + r^{10}\left(E_{2\rightarrow}\frac{r^{12}}{t^{12}}\, e^{ik_1 d\cos\theta_1}\right)$$

$$E_{2\rightarrow} = E_{0\rightarrow}\frac{t^{01}}{\frac{e^{-ik_1 d\cos\theta_1}}{t^{12}} - \frac{r^{10} r^{12}}{t^{12}}\, e^{ik_1 d\cos\theta_1}} \times \frac{t^{12}}{t^{12}}$$

$$\boxed{\frac{E_{2\rightarrow}}{E_{0\rightarrow}} = \frac{t^{01} t^{12}}{e^{-ik_1 d\cos\theta_1} - r^{10} r^{12} e^{ik_1 d\cos\theta_1}}} = \boxed{\frac{t^{01}\, e^{ik_1 d\cos\theta_1}\, t^{12}}{1 - r^{10} r^{12}\, e^{2ik_1 d\cos\theta_1}}}$$

matches Hecht!

still valid if θ = complex

day 10 pg 5

$$\frac{I_2}{I_0} = T^{02} = \frac{n_2}{n_0}\,\frac{\cos\theta_2}{\cos\theta_0}\,\left|t^{02}\right|^2$$

like with previous fresnel coeffs

AKA "T tot"

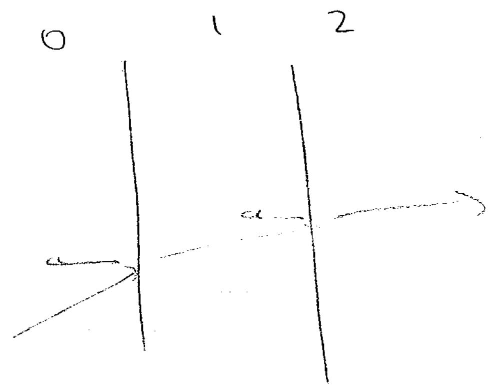
from Poynting $\langle S \rangle = \frac{1}{2}\epsilon_0 n c E$

from geometrical factor

$$T^{02} = \frac{n_2}{n_0}\,\frac{\cos\theta_2}{\cos\theta_0}\,\frac{\left|t^{01}\right|^2 \left|t^{12}\right|^2}{\left|e^{-ik_1 d\cos\theta_1} - r^{10} r^{12} e^{ik_1 d\cos\theta_1}\right|^2}$$

Eqn 4.14

Review



To find how much intensity transmits,

(a) find angles via Snell's Law

(b) find Fresnel coefficients t^{01}, t^{12}, r^{10}, r^{12} (which depend on angles, recall "α" and "β")

(c) use eqn

$$T^{02} = \frac{n_2}{n_0}\frac{\cos\theta_2}{\cos\theta_0}\frac{|t^{01}|^2\,|t^{12}|^2}{\left|e^{-ik_1 d\cos\theta_1} - r^{10}r^{12}e^{ik_1 d\cos\theta_1}\right|^2}$$

~~quiz~~

New: simplifies (?) to

$$T^{02} = \frac{T^{max}}{1 + F\sin^2\frac{\phi}{2}}$$ if θ_1 = real angle

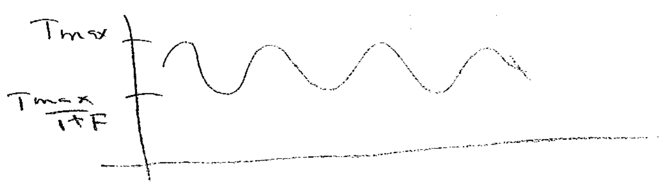
with T^{max}, F, and ϕ as defined on P4.3 solution handout

Note: • biggest T^{02} can be is when $\sin\frac{\phi}{2} = 0$

then $T^{02} = T^{max}$

• smallest T^{02} can be is when $\sin\frac{\phi}{2} = 1$ (or -1)

then $T^{02} = \frac{T^{max}}{1+F}$



We will return to this after a small detour

Problem P4.3 Solution - pg 1

Start with

$$T^{02} = \frac{n_2}{n_0}\frac{\cos\theta_2}{\cos\theta_0}\frac{|t^{01}|^2|t^{12}|^2}{|e^{-ik_1 d\cos\theta_1} - r^{10}r^{12}e^{ik_1 d\cos\theta_1}|^2} \qquad \text{Eqn 4.14}$$

$$\text{denominator} = \left(e^{-ik_1 d\cos\theta_1} - r^{10}r^{12}e^{ik_1 d\cos\theta_1}\right) \times \text{complex conjugate}$$

$$= \left(\quad " \qquad " \quad \right)\left(e^{+ik_1 d\cos\theta_1} - r^{10*}r^{12*}e^{-ik_1 d\cos\theta_1}\right)$$

$$(\text{FOIL}) \quad = 1 \underbrace{- e^{2ik_1 d\cos\theta_1}r^{10}r^{12} - e^{-2ik_1 d\cos\theta_1}r^{10*}r^{12*}}_{\substack{= -(\text{something} + \text{complex conjugate}) \\ = -2 \times \text{Real part}(\text{something})}} + |r^{10}|^2|r^{12}|^2$$

Write $r^{10} = |r^{10}|e^{i\delta_{10}}$ $\qquad \delta = 2k_1 d\cos\theta_1$

$r^{12} = |r^{12}|e^{i\delta_{12}}$

$$= 1 + |r^{10}|^2|r^{12}|^2 - 2\,\text{Real}\Big[\underbrace{|r^{10}|e^{i\delta_{10}}|r^{12}|e^{i\delta_{12}}e^{i\delta}}_{=|r^{10}||r^{12}|e^{i(\delta_{10}+\delta_{12}+\delta)}}\Big]$$

$$= 1 + |r^{10}|^2|r^{12}|^2 - 2|r^{10}||r^{12}|\underbrace{\cos(\delta_{10}+\delta_{12}+\delta)}_{\text{Trig trick: } \cos\phi = 1 - 2\sin^2\frac{\phi}{2}}$$

$$= \underbrace{1 - 2|r^{10}||r^{12}| + |r^{10}|^2|r^{12}|^2}_{(1-|r^{10}||r^{12}|)^2} + 4|r^{10}||r^{12}|\sin^2\frac{\phi}{2}$$

$$\boxed{\phi = \delta_{10}+\delta_{12}+2k_1 d\cos\theta_1}$$

$$\text{denom.} = (1-|r^{10}||r^{12}|)^2 + 4|r^{10}||r^{12}|\sin^2\frac{\phi}{2}$$

Put back in to T^{02} eqn

$$T^{02} = \frac{n_2}{n_0}\frac{\cos\theta_2}{\cos\theta_0}\frac{|t^{01}|^2|t^{12}|^2}{(1-|r^{10}||r^{12}|)^2 + 4|r^{10}||r^{12}|\sin^2\frac{\phi}{2}} \times \frac{\frac{1}{(1-|r^{10}||r^{12}|)^2}}{\frac{1}{(1-|r^{10}||r^{12}|)^2}}$$

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$$T^{02} = \frac{\frac{n_2}{n_0}\frac{\cos\theta_2}{\cos\theta_0}\frac{|t^{01}|^2 |t^{12}|^2}{(1-|r^{10}||r^{12}|)^2}}{1 + \frac{4|r^{10}||r^{12}|}{(1-|r^{10}||r^{12}|)^2}\sin^2\frac{\phi}{2}}$$

or, using some other symbols to make it look simpler...

☆ $$\boxed{T^{02} = \frac{T^{max}}{1 + F\sin^2\frac{\phi}{2}}}$$ Eqn 4.15

with $$\boxed{T^{max} = \frac{n_2}{n_0}\frac{\cos\theta_2}{\cos\theta_0}\frac{|t^{01}|^2|t^{12}|^2}{(1-|r^{10}||r^{12}|)^2}}$$ Eqn 4.16

$$\boxed{F = \frac{4|r^{10}||r^{12}|}{(1-|r^{10}||r^{12}|)^2}}$$ Eqn 4.18

called "coefficient of finesse"

and ϕ as defined on previous page,

$$\boxed{\phi = \underbrace{\delta_{10}}_{\text{phase of } r^{10}} + \underbrace{\delta_{12}}_{\text{phase of } r^{12}} + 2k_1 d\cos\theta_1}$$ Eqn 4.17

Oh ... my T^{max} equation doesn't look exactly like eqn 4.16

to complete things, write numerator like this:

$$T^{max}_{numer.} = \frac{n_2}{n_1}\frac{\cos\theta_2}{\cos\theta_1}|t^{12}|^2 \; \frac{n_1}{n_0}\frac{\cos\theta_1}{\cos\theta_0}|t^{01}|^2$$

(because n_1's and $\cos\theta_1$'s will cancel out)

$$= T^{12} \cdot T^{01} \checkmark$$

Also, obviously all of my $|r^{10}|$ type terms can be written as $\sqrt{R^{10}}$ or equivalent