quiz

Last time:

We found that there are

two "magic" directions, for which the two n's are the same

(Formula in book, Eqn 5.25, I don't consider super important)

These directions called "optic axes"

quiz

Special cases: when $n_x = n_y$

then Eqn 5.25 predicts $\cos\theta = 1$ $(0,0,1)$
and $\cos\theta = -1$ $(0,0,-1)$

but those are both same direction!

This case, "optic axis" (only one) is in z-direction!

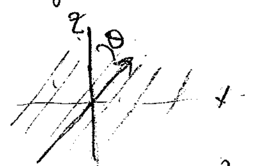
$$n = \begin{pmatrix} n_o & & \\ & n_o & \\ & & n_e \end{pmatrix}$$

called "extraordinary" direction

x, y called "ordinary" directions

Called uniaxial crystals.

Since x, y are indistinguishable let's orient x-y axes so that the light is traveling in x-z plane.

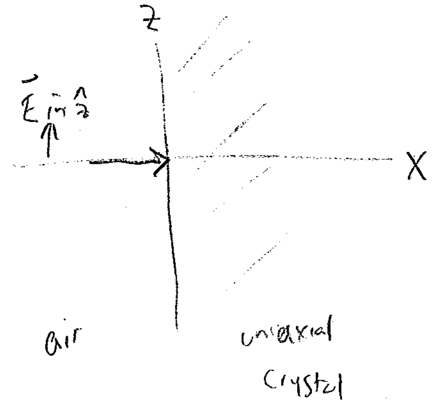


I.e., force $u_y = 0$
(as we said last time it must be for optic axis)

Fresnel Eqn $\quad \dfrac{1}{n^2} = \dfrac{\sin^2\theta}{n^2 - n_o^2} + 0 + \dfrac{\cos^2\theta}{n^2 - n_e^2}$

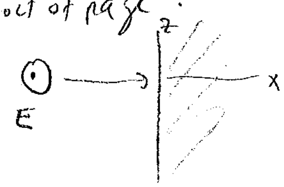
$n = ?$
(skip algebra)
$$\boxed{\begin{aligned} &= n_o \quad \text{(when polarized in } y\text{)} \\ &= \frac{n_o n_e}{\sqrt{n_o^2 \sin^2\theta + n_e^2 \cos^2\theta}} \quad \text{(when polarized in } x\text{-}z \text{ plane)} \end{aligned}}$$

Example: striking a surface, ⊥



What n will light see? answer: $n = n_e$ (field in z direction)
No eqn needed, but could set $\theta = 90°$ in previous Eqn

If E out of page? $\vec{E} = E\hat{y}$



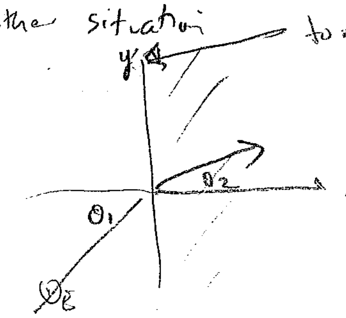
answer $n = n_o$ (field in y direction)
No eqn needed, but could set $\theta = 0°$ in Eqn

This direction used for <u>Waveplates</u>

if $\vec{E} = E_1\left(\frac{\hat{y}+\hat{z}}{\sqrt{2}}\right)$

y component + z component see different indices, have different wavelengths, get out of phase
if $\frac{1}{4}\lambda$ out of phase when leave crystal, produces "circular polarization."
More on that in next chapter.

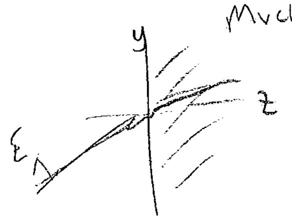
Another situation → to match book picture



Notice surface now $\perp$ to z

Thought question — what n does it see if $\vec{E} = E\hat{x}$ (s-polarized)

Much harder: if p-polarized



→ Angle depends on n

but n depends on angle!

$$n_2 = \frac{n_o n_e}{\sqrt{n_o^2 \sin^2\theta_2 + n_e^2 \cos^2\theta_2}}$$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_o n_e}{\sqrt{n_o^2 \sin^2\theta_2 + n_e^2 \cos^2\theta_2}}$$

Solve for $\sin^2\theta_1$, convert to $\tan^2\theta_1$

Skip!

$$\frac{\sin^2\theta_1}{\sin^2\theta_2} = \frac{n_o^2 n_e^2}{n_o^2 \sin^2\theta_2 + n_e^2(1-\sin^2\theta_2)}$$

denom $(n_o^2 - n_e^2)\sin^2\theta_2 + n_e^2$

$$\sin^2\theta_1 (n_o^2 - n_e^2)\sin^2\theta_2 + \sin^2\theta_1 n_e^2 = n_o^2 n_e^2 \sin^2\theta_2$$

$$\sin^2\theta_2 \left[n_o^2 n_e^2 - (n_o^2 - n_e^2)\sin^2\theta_1 \right] = \sin^2\theta_1 n_e^2$$

$$\sin\theta_2^2 = \frac{n_e^2 \sin^2\theta_1}{n_o^2 n_e^2 - (n_o^2 - n_e^2)\sin^2\theta_1}$$

$$\tan^2 = \frac{s^2}{1-s^2} = \frac{\text{num}}{\text{den}-\text{num}} = \frac{n_e^2 \sin^2\theta_1}{n_o^2 n_e^2 - (n_o^2 - \cancel{n_e^2})\sin^2\theta_1 - \cancel{n_e^2 \sin^2\theta_1}}$$

$$\boxed{\tan\theta_2 = \frac{n_e \sin\theta_1}{n_o \sqrt{n_e^2 - \sin^2\theta_1}}}$$ ✓

Snell's Law replacement for p-polar when plane $\perp$ z

$\frac{\sin^2}{s^2} + \frac{\cos^2}{s^2} = \frac{1}{s^2}$

$\cancel{\frac{1}{\tan}}$ $\frac{1}{s^2} - 1 = \frac{1-s^2}{s^2}$

$\tan^2 = \frac{s^2}{1-s^2}$ $\sin^2 = \frac{\text{num}}{\text{den}}$

$= \frac{\frac{\text{num}}{\text{den}}}{1 - \frac{\text{num}}{\text{den}}} \times \frac{\text{de}}{\text{de}}$

$= \frac{\text{num}}{\text{den}-\text{num}}$

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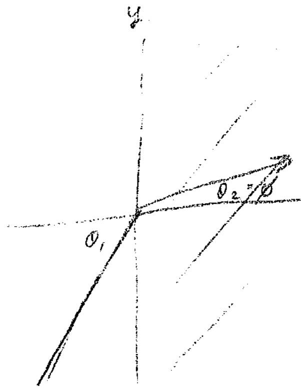
day 15 pg 4

<u>Poynting vector</u>

s polar: same as normal
p polar: weird!



k_{trans} direction given by $\tan\theta_2 = \dfrac{n_e \sin\theta_1}{n_o\sqrt{n_e^2 - \sin^2\theta_1}}$

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$$

what direction is E? $\nabla\cdot D = 0$ / phase = 0

$$\nabla\cdot(\epsilon_0 E + P) = 0$$

plane waves: $\vec{k}\cdot(\epsilon_0 E + P) = 0$ $P = \epsilon_0(\chi)E$

$$\underbrace{\vec{k}}_{\begin{pmatrix}0\\ k\sin\phi\\ k\cos\phi\end{pmatrix}}\cdot\epsilon_0\left[\begin{pmatrix}E_x\\E_y\\E_z\end{pmatrix} + \begin{pmatrix}\chi_{xx}E_x\\ \chi_{yy}E_y\\ \chi_{zz}E_z\end{pmatrix}\right] = 0$$

$$k\sin\phi\,(1+\underbrace{\chi_{yy}}_{n_y^2 = n_o^2})E_y + k\cos\phi\,(1+\underbrace{\chi_{zz}}_{n_z^2 = n_e^2})E_z = 0$$

$$E_z = -\tan\phi\,\frac{n_o^2}{n_e^2}E_y$$

$$E = \begin{pmatrix}0\\ E_y\\ -\tan\phi\,\frac{n_o^2}{n_e^2}E_y\end{pmatrix}$$ field inside crystal

$$\vec{B} = \frac{1}{\omega}\vec{k}\times\vec{E}$$

$$= \frac{1}{\omega}\begin{pmatrix}0\\ k\sin\phi\\ k\cos\phi\end{pmatrix}\times\begin{pmatrix}0\\ E_y\\ -\tan\phi\frac{n_o^2}{n_e^2}E_y\end{pmatrix}$$

$$= \frac{1}{\omega}\left[-k\sin\phi\tan\phi\frac{n_o^2}{n_e^2}E_y - k\cos\phi\,E_y\right]\hat{x}$$

$$= -\frac{kE_y}{\omega}\hat{x}\left(\frac{n_o^2}{n_e^2}\sin\phi\tan\phi - \cos\phi\right)$$

can skip details

day 15 pg 5

$$\vec{S} = \frac{-k E_y^2}{\mu_0 \omega} \begin{pmatrix} 0 \\ 1 \\ -\tan\phi \frac{n_o^2}{n_e^2} \end{pmatrix} \times \begin{pmatrix} \frac{n_o^2}{n_e^2} \sin\phi \tan\phi \cdot \cos\phi \\ 0 \\ 0 \end{pmatrix}$$

$$= \frac{+k}{\mu_0 \omega} E_y^2 \left(\frac{n_o^2}{n_e^2} \sin\phi \tan\phi \tan\phi \right) \underbrace{\left(\hat{z} + \hat{y} \frac{n_o^2}{n_e^2} \tan\phi \right)}_{\text{not same direction as } \vec{k}!}$$

$\hat{k}$ ($= \sin\phi \, \hat{y} + \cos\phi \, \hat{z}$) ← important part

dir of $\vec{S}$ "θ_s": $\tan\theta_s = \dfrac{\frac{n_o^2}{n_e^2} \tan\phi}{1}$

or $\tan\theta_s = \dfrac{n_o^2}{n_e^2} \dfrac{n_e \sin\theta_1}{n_o \sqrt{n_e^2 - \sin^2\theta_1}}$

$$\boxed{\tan\theta_s = \frac{n_o}{n_e} \frac{\sin\theta_1}{\sqrt{n_e^2 - \sin^2\theta_1}}}$$ ← important

this is what is seen by an observer!

"extraordinary" beam doesn't appear to follow Snell's law, even after angular dependence of n_2 is taken into account.

That's all for non isotropic crystals