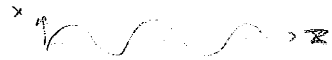
Calcite demo — Day 17 pg. 1 ← start w/ demo

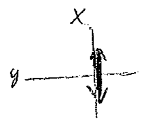
Polarization

We've been studying things like $\vec{E} = \vec{E}_0 \, e^{i(kz-\omega t)}$

$= (\text{say}) \; E_0 \hat{x} \cos(kz-\omega t)$



This is a "linearly polarized" wave, because the electric field goes back + forth in a line in the x-y plane

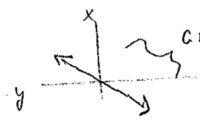


(wave coming out of page)

What happens when we add two waves together?

Example: $E_0 \hat{x} \cos(kz-\omega t) + E_0 \hat{y} \cos(kz-\omega t)$

$= E_0\sqrt{2} \left(\frac{\hat{x}+\hat{y}}{\sqrt{2}}\right) \cos(kz-\omega t)$



amplitude would be $E_0\sqrt{2}$

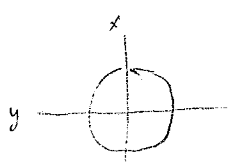
Still linear polarization!

Add a phase shift:

$E_0 \hat{x} \cos(kz-\omega t) + E_0 \hat{y} \underbrace{\sin(kz-\omega t)}_{\cos(kz-\omega t - 90^\circ)}$

Circular pol! See picture!

("amplitude" = E_0, not $E_0\sqrt{2}$)



(wave coming out of page)

(right circular, to be precise)

→ stop time, look at screw in field

same as regular screw → right handed

opposite → left handed

also: right handed: $\vec{E}$ rotates to right as seen from observer day 17 pg 2

writing circ. pol w/ complex numbers.

$$\vec{E}_0 = E_0\hat{x}\cos(kz-\omega t) + E_0\hat{y}\cos(kz-\omega t-90^\circ)$$
$$= E_0\hat{x}\,e^{i(kz-\omega t)} + E_0\hat{y}\,e^{i(kz-\omega t-90^\circ)}$$
$$= \left[E_0\hat{x} + E_0\hat{y}\,e^{i(-90^\circ)}\right] e^{i(kz-\omega t)}$$

$$E_0\begin{pmatrix}1\\ e^{i(-90^\circ)}\end{pmatrix} e^{i(kz-\omega t)}$$

$\to e^{i(-90^\circ)} = -i$

the "Jones vector"!

[Convention: vector has magnitude 1, so add $\sqrt{2}$]

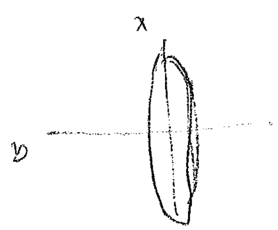
What if the y component not as strong as x-component?

$$E_0 = \left(E_0\hat{x} + .2E_0\hat{y}\,e^{i(-90^\circ)}\right) e^{i(kz-\omega t)}$$

$$\text{Jones} = \frac{1}{1.0198}\begin{pmatrix}1\\ .2e^{i(-90^\circ)}\end{pmatrix}$$

1, .2 → 1.0198...

Animation #3



Elliptically polarized

ellipticity = $\frac{E_{min}}{E_{max}}$

(like "eccentricity" of ellipse, but not quite the same mathematically)

Figure

Summary table 4.1

$\begin{pmatrix}1\\0\end{pmatrix}$ x

$\begin{pmatrix}0\\1\end{pmatrix}$ y

$\begin{pmatrix}\cos\alpha\\ \sin\alpha\end{pmatrix}$ $\angle\alpha$ from x

$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$ RCP

$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$ LCP

$\frac{1}{\sqrt{A^2+B^2}}\begin{pmatrix}A\\ Be^{i(-90^\circ)}\end{pmatrix}$ RCP-like elliptic; long axis oriented on x or y axis

$\frac{1}{\sqrt{A^2+B^2}}\begin{pmatrix}A\\ Be^{i\delta}\end{pmatrix}$ elliptic at whatever axis (angle α given in text) day 17 pg 3

[sidenote: Unpolarized light? Can't be described w/ Jones vectors.]

<u>How to polarize light</u> ~ figures in PPT

<u>Jones Matrices</u>

Certain optical components can affect the polarization state of light

linear polarizer → force into x, y, or other linear polarization

in y NIR polarizer:  reflects this E field

parallel wires

represent mathematically w/ matrix.

matrices to change value of a vector

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{for x-direction}$$

proof: $\text{new polar} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix}$ ✓

↑ current polar

leaves only x-polar

similarly $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ = forces into y-polar.

$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ = forces to 45°

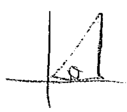
proof: $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \frac{1}{2}E_x + \frac{1}{2}E_y \\ \frac{1}{2}E_x + \frac{1}{2}E_y \end{pmatrix}$

↳ x + y components now equal → 45° ✓

$\begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$ = forces to linear @ angle θ from x-axis

could have overall phase shift

proof $\begin{pmatrix} \cos^2\theta & \sin\cos \\ \sin\cos & \sin^2\theta \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \cos^2\theta A + \sin\cos B \\ \sin\theta\cos\theta A + \sin^2\theta B \end{pmatrix}$

 $\tan\theta \stackrel{?}{=} \dfrac{\sin\theta\cos\theta A + \sin^2\theta B}{\cos^2\theta A + \sin\theta\cos\theta B} \stackrel{?}{=} \dfrac{\sin\theta\,\cancel{(\cos\theta A + \sin\theta B)}}{\cos\theta\,\cancel{(\cos\theta A + \sin\theta B)}}$ ✓

(finished ~10 mins early!)