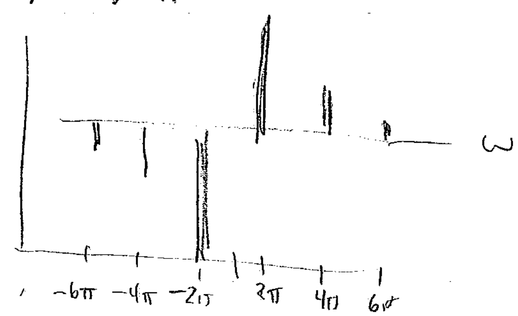
Quizzes

Fourier handout, continued

Fourier series

- exponential notation $f(t) = \sum_{n} c_n e^{-in\omega_0 t}$ ← sometimes +
- coefficients $c_n = \frac{1}{T}\int_0^T f(t)\, e^{in\omega_0 t}\, dt$
- Example

do math $\rightarrow f(t) = \ldots + \left(-\frac{2i}{5\pi}\right) e^{i10\pi t} + \left(-\frac{2i}{3\pi}\right) e^{i6\pi t} + \ldots$ (see handout)



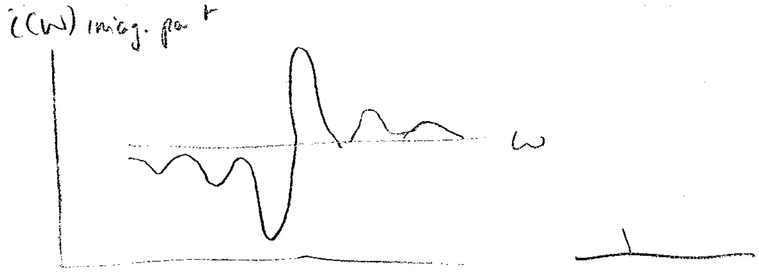
c_n is function of ω, since $\omega = n\omega_0$

Fourier transform

$$f(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} c(\omega)\, e^{-i\omega t}$$

$$c(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\, e^{+i\omega t}$$

- Symmetry Notes
- Notation Notes
- Example



width of this ⟷ time of pulse duration

These frequency components are real!

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Back to book. Example in section 7.2

Two interfering plane waves $E_1 = E_0 e^{i(k_1 r - \omega_1 t)}$

~~[illegible]~~ $E_2 = E_0 e^{i(k_2 r - \omega_2 t)}$ same amp

Like beats video

$$I = \frac{n\epsilon_0 c}{2} E_{tot} E_{tot}^*$$

$$= \frac{n\epsilon_0 c}{2} E_0 \cdot E_0^* \left[e^{i(k_1 r - \omega_1 t)} + e^{i(k_2 r - \omega_2 t)}\right]\left[e^{-i(k_1 r - \omega_1 t)} + e^{-i(k_2 r - \omega_2 t)}\right]$$

$$2 + \underbrace{e^{i((k_2 - k_1) r - (\omega_2 - \omega_1) t)} + e^{-i(\quad)}}_{2\cos(\quad)}$$

$$= n\epsilon_0 c \vec{E}_0 \cdot \vec{E}_0^* \left(1 + \cos(\Delta k - \Delta\omega t)\right)$$

travels at $v_g = \frac{\Delta\omega}{\Delta k}$

(spread in ω's often called "bandwidth")

in general $v_g = \frac{d\omega}{dk}$ is just expression if $\Delta\omega$ not too large

Could be given $\omega(k)$ or $k(\omega)$

or could be given $v(\omega)$ or $n(\omega)$

etc. $\frac{\omega}{k} = \frac{c}{n}$ true for each phase

So. $k = \frac{\omega n(\omega)}{c}$

Then take $\frac{dk}{d\omega}$

still true: $\frac{\omega}{k} = \frac{c}{n}$

$\omega = \frac{c}{n} k$

Only way ω = nonlinear function of k is if n is a function of ω (dispersion!)

if no dispersion then $v_p = v_g$

often (usually?) written as $k = \frac{1}{c}\omega n(\omega)$

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(~~cont from last lecture~~)

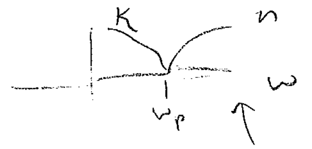
Example:

review: metal/plasma $\chi = \frac{\omega_p^2}{\cancel{\omega_0^2} - i\omega\gamma - \omega^2}$

$= -\frac{\omega_p^2}{\omega^2}$ if no damping

$n = \sqrt{1+\chi}$

$\tilde{n} = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$



$n<1$ when $\omega > \omega_p$

so $V_{phase} = \frac{\omega}{k} = \frac{c}{n}$ is larger than c

let $\omega_p = 10^8/s$

We have 2 plane waves superimposed

$\omega_1 = 5 \cdot 10^8/s$

$\omega_2 = 6 \cdot 10^8/s$

What is velocity of group?

$V_g = \frac{\Delta\omega}{\Delta k}$

Method 1 $\frac{\omega}{k} = \frac{c}{n} \rightarrow \omega = \frac{c}{n} k$

$k = \omega \cdot \frac{n}{c}$

$k_1 = \frac{5 \cdot 10^8}{3 \cdot 10^8} \sqrt{1 - \left(\frac{10^8}{5 \cdot 10^8}\right)^2} = 1.6330$

$k_2 = \frac{6 \cdot 10^8}{3 \cdot 10^8} \sqrt{1 - \left(\frac{1}{6}\right)^2} = 1.9720$

$\frac{\Delta\omega}{\Delta k} = \frac{1 \cdot 10^8}{1.972 - 1.633} = 2.950 \cdot 10^8 m/s$

Method 2 $\frac{\Delta\omega}{\Delta k} \approx \frac{d\omega}{dk}$ since ω_1 close to ω_2

$\frac{dk}{d\omega}$ easier: $k = \frac{\omega n}{c} = \frac{1}{c} \omega \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$

$\frac{dk}{d\omega} =$ (product rule)

$\frac{1}{c}\left[\sqrt{1 - \frac{\omega_p^2}{\omega^2}} + \frac{\omega \cdot \frac{1}{2}}{\sqrt{}} \cdot (-2\omega_p^2 \omega^{-3})\right]$

$= \frac{1}{c}\left(n + \frac{\omega_p^2}{n\omega^2}\right)$

[compare $V_{phase} = \frac{c}{n}$]

$\frac{d\omega}{dk} = \frac{c}{n + \frac{\omega_p^2}{n\omega^2}} \cdot \frac{n}{n} = \frac{nc}{n^2 + \frac{\omega_p^2}{\omega^2}} = nc$

since denom $= 1$ from $n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$, $n^2 = 1 - \frac{\omega_p^2}{\omega^2}$

$\frac{d\omega}{dk} = \left[\sqrt{1 - \left(\frac{1 \cdot 10^8}{5.5 \cdot 10^8}\right)^2}\right] c = 2.950 \cdot 10^8 m/s$