quizzes

~~Quick Fourier series/transform summary~~

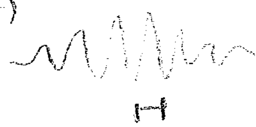
~~→ go over them fast~~

~~(that's in PPT)~~

$e^{-t^2/2T^2}$ ← ~~Explain~~ factor of 2 as in PHW so that T = std dev

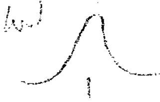
$f(t) = \cos\omega_0 t \times$ Gaussian

FT of Gaussian



$\omega_0 = \frac{2\pi}{\text{period}}$

$f(\omega)$



ω_0

symmetry note:

$f(\omega)$ is in general complex

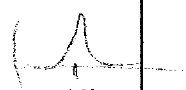
If $f(t)$ is real, then $f(\omega) = f^*(-\omega)$

If $f(t)$ is real + even, then $f(\omega)$ = real
real + odd   purely imag.

→ This case: $f(\omega)$ =

$-\omega_0$ ω_0

If $f(t) = e^{i\omega_0 t} \times e^{-t^2/T^2}$ then $f(\omega)$ =



ω_0

(only one peak)

Exact eqn on page after next

quiz

~~Parseval's thm~~

~~Since $I \sim |E|^2$~~

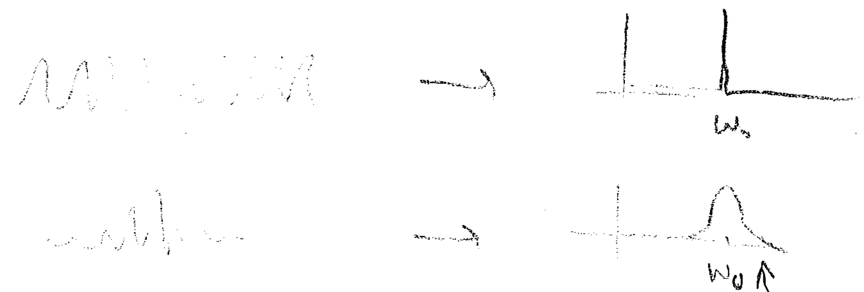
~~$\int |E(t)|^2 dt = \int |E(\omega)|^2 dt$~~

~~$I(\omega) = \frac{n\epsilon_0 c}{2} |E(\omega)|^2$~~ ~~(not FT of $I(t)$)~~

~~"Power spectrum" (often just $|E(\omega)|^2$ plotted)~~

long time →

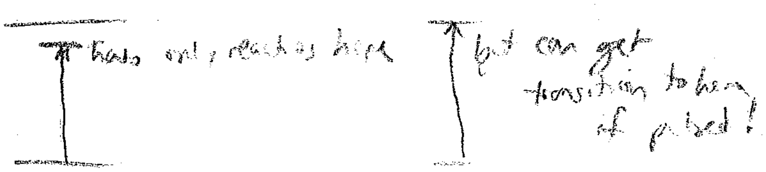
short time →



Contains some higher frequencies than ω_0!

These "extra" frequencies are real! Three evidence

(1) Excitation of forbidden energy levels



(2) Temporal dispersion through material
- diff frequency components travel at diff speeds
- when pulse exits material, it is <u>not as short</u>

(3) Audio: run through high pass filter which blocks fundamental (or even higher) You'll hear the higher harmonics!

Uncertainty Principle

For the Gaussian $\Delta t = \tau$

$$\Delta t \, \Delta\omega \gtrsim \frac{1}{2}$$

(Δ = std dev, <u>not</u> FWHM)

$$\Delta t \, \Delta E \gtrsim \frac{\hbar}{2}$$

since $E = \hbar\omega$

FT of Gaussian - exact equation

check

$$\boxed{f(t) = e^{-t^2/2T^2} e^{-i\omega_0 t}}$$

width = ? T, if you mean standard deviation

(FWHM is different)

($\frac{1}{e}$ width is different)

result of FT integral $\longrightarrow$

$$\boxed{f(\omega) = T e^{-(\omega-\omega_0)^2/2(\frac{1}{T})^2}}$$

width = $\frac{1}{T}$

as width of $f(t)$ increases,

width of $f(\omega)$ decreases! (and vice versa)

Uncertainty principle

$$\Delta t \, \Delta\omega \gtrsim \frac{1}{2} \qquad \text{with } \Delta = \text{"std dev"}$$

test: $(T)\left(\frac{1}{T}\right) \overset{?}{\geq} \frac{1}{2}$

$1 \overset{?}{\geq} \frac{1}{2}$ ✓

Quantum: $\Delta t \, \Delta E \gtrsim \frac{\hbar}{2}$ since $E = \hbar\omega$

Parseval's Thm: As it gets narrower, it also gets taller. Area conserved

$$\int |E(t)|^2 \, dt = \int |E(\omega)|^2 \, d\omega$$

units? ✓ each side is $\text{field}^2 \cdot \text{time}$ or $\frac{\text{field}^2}{\text{freq}}$

$\downarrow$

$I(\omega) = \frac{n\epsilon_0 c}{2} |E(\omega)|^2$ not the FT of $I(t)$

"power spectrum"

units of $I(\omega)$? $\left[\frac{n\epsilon_0 c}{2}\right] \cdot \frac{\text{field}^2}{\text{freq}^2} = \frac{\text{intensity}}{\text{freq}^2}$ interesting