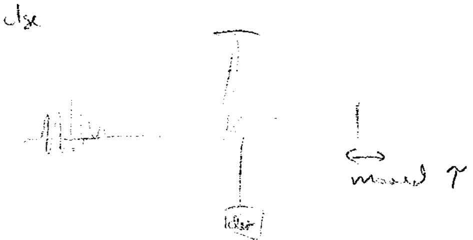
Dec 25

→ Review everything Justin taught

Worked example (like P 8.4 due Tuesday)
(Example 8.1 pg 207 and Example 8-2 pg 209)

Gaussian pulse



$$E(t) = E_0 e^{-t^2/2T^2} e^{-i\omega_0 t}$$ what does detector measure as mirror is moved?

Step 1: FT: $E(\omega) = T E_0 e^{-(\omega-\omega_0)^2/2\left(\frac{1}{T}\right)^2}$ done last week

Step 2: calculate $\mathcal{E}$

$$\mathcal{E} = \int_{-\infty}^{\infty} I(\omega)\, d\omega$$ remember $I(\omega) = \frac{1}{2} n \epsilon_0 c\, |E(\omega)|^2$

$$= \int_{-\infty}^{\infty} \frac{1}{2} n \epsilon_0 c\, T^2 E_0^2 e^{-T^2(\omega-\omega_0)^2} d\omega$$ (lose factor of 2 due to squared)

$$= \underbrace{\frac{1}{2} n \epsilon_0 c E_0^2}_{I_0} T^2 \underbrace{\int_{-\infty}^{\infty} e^{-T^2(\omega-\omega_0)^2} d\omega}_{\frac{1}{T}\sqrt{\pi}}$$

$$\mathcal{E} = I_0 \sqrt{\pi} \cdot T$$ = total energy of pulse (if you multiply by area; $I = \frac{\text{Joules}}{m^2}$)

Step 3: calculate γ

$$\gamma = \frac{1}{\mathcal{E}} \cdot \underbrace{\int_{-\infty}^{\infty} I(\omega) e^{-i\omega\tau} d\omega}_{\text{focus on this}}$$

$$= \underbrace{\frac{1}{2} n \epsilon_0 c E_0^2}_{I_0} T^2 \underbrace{\int_{-\infty}^{\infty} e^{-T^2(\omega-\omega_0)^2} e^{-i\omega\tau} d\omega}_{\frac{\sqrt{\pi}}{T} e^{-\tau^2/4T^2} e^{-i\omega_0\tau}}$$

$$= I_0 T \sqrt{\pi}\, e^{-\tau^2/4T^2} e^{-i\omega_0\tau}$$

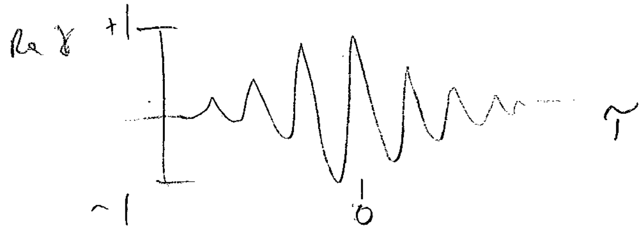
combine
$$\gamma = \frac{\cancel{I_0}\,\cancel{T}\sqrt{\pi}\; e^{-\tau^2/4T^2}\, e^{-i\omega_0\tau}}{\cancel{I_0}\sqrt{\pi}\,\cancel{T}}$$

$$\underline{\gamma = e^{-\tau^2/4T^2}\, e^{-i\omega_0\tau}}$$

$-\frac{\tau^2}{2(2T^2)}$
γ width $= \sqrt{2}\,T$

recall what symbols stand for
T = width of Gaussian you send in
ω_0 = ~~freq~~ central freq. of pulse
τ = position of mirror

Step 4: consider (a) $\mathrm{Re}\,\gamma$, (b) $|\gamma| = V$, (c) $\int |\gamma|^2 d\tau = \tau_c$
sig ~ $1 + \mathrm{Re}\,\gamma$



width = $T\sqrt{2}$

(a) – sig looks same, but goes from 0 to 2

(b) – $\underline{|\gamma| = e^{-\tau^2/4T^2}}$ = envelope of the cosine oscillation
fringes not visible when τ far from 0

(c) – $\tau_c = \int_{-\infty}^{\infty} |\gamma|^2 d\tau$
$= \int_{-\infty}^{\infty} e^{-\tau^2/4T^2} d\tau$
$= \underline{(\sqrt{2\pi})\,T}$

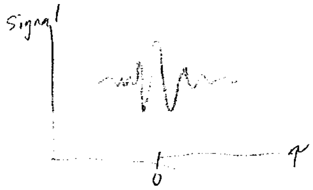
coherence time ≈ width of Gaussian. makes sense

Problem P8.4: just need part (a), but (b) and (c) would be good problems too

Day 25 pg 3

Section 8.4 Fourier Spectroscopy

$$\cancel{\text{Signal}(\tau) \sim \left\{2\int_{-\infty}^{\infty} I(t)\,dt\right\}\left\{1 + Re(\gamma(\tau))\right\}} = 2\mathcal{E}_0\left(1 + \frac{Re\int I(\omega)e^{-i\omega\tau}d\omega}{\mathcal{E}_0}\right)$$

If we measure  can we deduce the frequency spectrum?

Yes!

$$Sig(\tau) = 2\mathcal{E}_0 + 2Re\int I(\omega)e^{-i\omega\tau}d\omega$$

(really sig is prop. to that; could have units of volts, etc. Ignore)

Take FT of $sig(\tau)$

$$FT(Sig(\tau)) = FT(2\mathcal{E}_0) + FT\Big(2\,\underbrace{Re\int I(\omega)e^{-i\omega\tau}d\omega}_{Re\, x = \frac{1}{2}(x + x^*)}\Big)$$

$FT(Sig(\tau))$ → experimental, can do with computer

$FT(2\mathcal{E}_0)$ → $2\mathcal{E}_0\,\underbrace{FT(1)}_{\sqrt{2\pi}\delta(\omega)}$

~~assume~~ $I(\omega)$ must be real, since $\sim |E|^2$

$$\sqrt{2\pi}\;FT\left(\frac{1}{\sqrt{2\pi}}\int I(\omega)e^{-i\omega\tau}d\omega + \frac{1}{\sqrt{2\pi}}\int I(\omega)e^{+i\omega\tau}d\omega\right)$$

$\omega' = -\omega$

$$FT\left[IFT(I(\omega)) + IFT(I(-\omega))\right]$$

$$= 2\sqrt{2\pi}\,\mathcal{E}_0\,\delta(\omega) + \sqrt{2\pi}\,I(\omega) + \sqrt{2\pi}\,I(-\omega)$$

$$\frac{FT(NS(\tau))}{\sqrt{2\pi}} = 2\mathcal{E}_0\,\delta(\omega) + I(\omega) + I(-\omega)$$

experimental graph

Typically  don't overlap

$I(\omega)$ identified!

Side Note: if CW source? Then $\int_{-\infty}^{\infty} I(t)\,dt$ = infinity

Use ave value $\langle I_{det}(\tau)\rangle = 2\langle I\rangle(1 + Re\,\gamma(\tau))$

(Final Eqn on Eqn sheet)