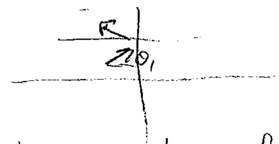
day 29 pg 1

Review Situation 1

Situation 2: Reflection from flat mirror



$y_2 = y_1$

What angle to use for θ_2?

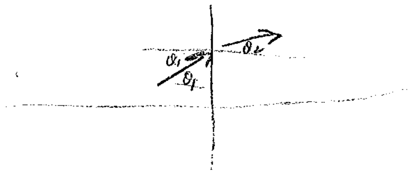
convention: θ_2 if going left

then $\theta_2 = \theta_1$

$$\begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}$$

$\hookrightarrow$ trivial M

Situation 3: Snell's Law
Refraction from flat surface



$y_2 = y_1$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$n_1 \theta_1 \approx n_2 \theta_2$

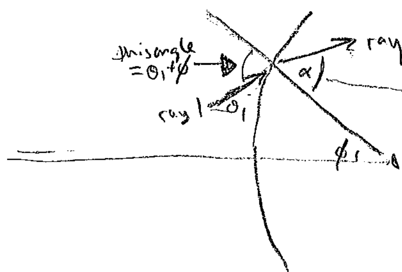
$\theta_2 \approx \frac{n_1}{n_2} \theta_1$

$$\begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix} \begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}$$

Situation 4: Snell's Law refraction from curved surface (like a lens)

R = radius of curvature

$y_2 = y_1$ again (obviously)

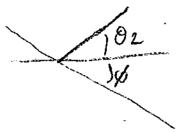


this angle via Snell's Law

$n_1 (\theta_1 + \phi) = n_2 \alpha$ (leaving out sine's)

$\alpha = \frac{n_1}{n_2} (\theta_1 + \phi)$

zoom in



$\alpha = \theta_2 + \phi \longrightarrow \theta_2 = \alpha - \phi$

$$\theta_2 = \frac{n_1}{n_2}\theta_1 + \frac{n_1}{n_2}\phi - \phi$$

$$= \frac{n_1}{n_2}\theta + \left(\frac{n_1}{n_2} - 1\right)\phi$$

What is ϕ in terms of y? $\phi = \frac{y}{R}$

$$\theta_2 = \frac{n_1}{n_2}\theta + \left(\frac{n_1}{n_2} - 1\right)\frac{1}{R} y$$

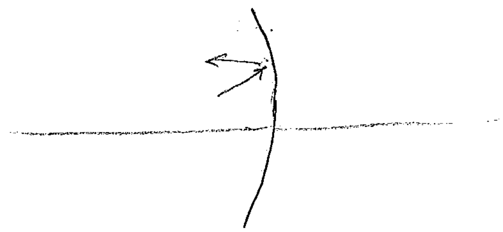
$$\begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{R}\left(\frac{n_1}{n_2} - 1\right) & \frac{n_1}{n_2} \end{pmatrix} \begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}$$

Notice this M reduces to our previous M when $R = \infty$

Note: opposite curvature handled via negative R

Day 29 pg 2

Situation 5 (almost done!) <u>Reflection</u> from curved surface



skip work

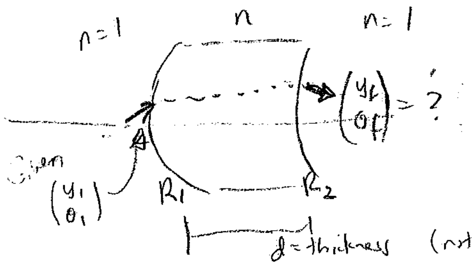
$$\begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \qquad \text{with } \boxed{f = \frac{R}{2}}$$

(opposite curvature → negative R)

We now have 3 very important matrices that we can piece together for more complicated situations

↳ because the two flat ones are subsets ($R \to \infty$)

Problem (thick lens)



R_1, R_2 = curvature of surfaces

d = thickness (not worrying about small ΔX from where ray hits curved surfaces)

$$\begin{pmatrix} y_f \\ \theta_f \end{pmatrix} = \begin{pmatrix} \text{surface 2} \end{pmatrix} \begin{pmatrix} \text{translation} \end{pmatrix} \begin{pmatrix} \text{surface 1} \end{pmatrix} \begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} 1 & 0 \\ \frac{1}{R_2}\left(\frac{n}{1}-1\right) & \frac{n}{1} \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{R_1}\left(\frac{1}{n}-1\right) & \frac{1}{n} \end{pmatrix}} \begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}$$

$$M_{tot} = \begin{pmatrix} 1 - \frac{d}{R_1}\left(1 - \frac{1}{n}\right) & \frac{t}{n} \\ \frac{n-1}{R_2} - \frac{\left(1-\frac{1}{n}\right)\left(n + \frac{d}{R_2}(n-1)\right)}{R_1} & 1 + \frac{d}{R_2}\left(1 - \frac{1}{n}\right) \end{pmatrix}$$

done w/ Mathematica

The answer!

☆ another useful M!

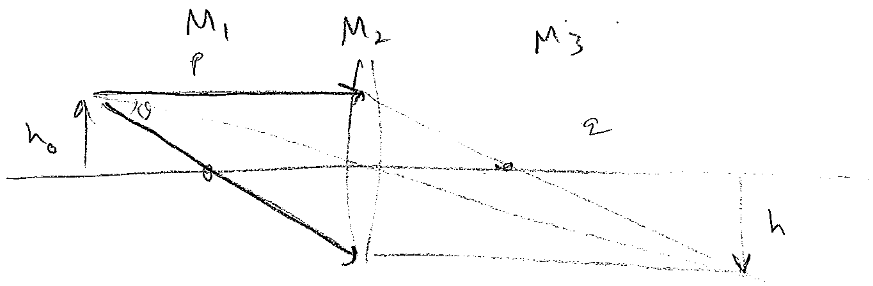
Interesting: $d \to 0$ (thin lens)

$$M_{tot} = \begin{pmatrix} 1 & 0 \\ (n-1)\left(\frac{1}{R_2} - \frac{1}{R_1}\right) & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

with $\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$ "lensmakers eqn"

if outside: n_1, inside: n_2 then $n \to \frac{n_2}{n_1}$

day 29 pg 3



Where do these two rays meet up?

Should be at $q = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1}$ and $h = -\left(\frac{q}{p}\right) \times h_0$ } let's prove that w/ ABCD stuff

ray 1: $\begin{pmatrix} y \\ \theta \end{pmatrix} = \begin{pmatrix} h_0 \\ 0^\circ \end{pmatrix}$

$M_1 = \begin{pmatrix} 1 & p \\ 0 & 1 \end{pmatrix}$

$M_2 = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$

$M_3 = \begin{pmatrix} 1 & q \\ 0 & 1 \end{pmatrix}$

$$M_3 M_2 M_1 \begin{pmatrix} y \\ \theta \end{pmatrix} = \begin{pmatrix} 1 & q \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 1 & p \\ 0 & 1 \end{pmatrix} \begin{pmatrix} h_0 \\ 0 \end{pmatrix}}_{\begin{pmatrix} h_0 \\ 0 \end{pmatrix}}$$

$$\begin{pmatrix} h_0 \\ -h_0/f \end{pmatrix}$$

$$\begin{pmatrix} h_0 - q h_0/f \\ -\frac{h_0}{f} \end{pmatrix} \begin{matrix} \leftarrow \text{final height} \\ \leftarrow \text{does this make sense?} \checkmark \end{matrix} \quad (q = \text{unknown})$$

ray 2: $\begin{pmatrix} y \\ \theta \end{pmatrix} = \begin{pmatrix} h_0 \\ -\frac{h_0}{p-f} \end{pmatrix}$ ← since $\theta \simeq \tan\theta$ (p-f, θ, h_0)

$$M_3 M_2 M_1 \begin{pmatrix} y \\ \theta \end{pmatrix} = \begin{pmatrix} 1 & q \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & p \\ 0 & 1 \end{pmatrix} \begin{pmatrix} h_0 \\ -\frac{h_0}{p-f} \end{pmatrix}$$

(Mathematica)

$$= \begin{pmatrix} \frac{f h_0}{f-p} \\ 0 \end{pmatrix} \begin{matrix} \leftarrow \text{final height (no q dependence... makes sense} \checkmark) \\ \leftarrow \text{does this make sense?} \checkmark \end{matrix}$$

Rays intersect when final heights are same

$$h_0 - \frac{q h_0}{f} = \frac{f h_0}{f-p} \longrightarrow 1 - \frac{q}{f} = \frac{f}{f-p} \longrightarrow \frac{q}{f} = 1 - \frac{f}{f-p} = \frac{\cancel{f} - p \cancel{- f}}{f-p}$$

and final height $\boxed{h = -(q/p) h_0}$ ⟵ $\boxed{q = \frac{+pf}{p-f}}$ ✓ same eqn.