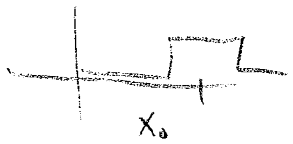
day 34 pg 1

FT of shifted function



FT(shifted rect) = FT(rect) ⊗ FT(δ at x_0)

$\delta(x-x_0)$

$= a \,\text{sinc}\, \frac{k_x a}{2} \cdot \underbrace{\int_{-\infty}^{\infty} \delta(x-x_0)\, e^{ikx}\, dx}_{e^{ikx_0}}$ (sifting property)

$= a \,\text{sinc}\, \frac{k_x a}{2}\, e^{ikx_0}$

2D: $\boxed{\text{FT (shifted aperture)} = e^{ik_x x_0} e^{ik_y y_0} \cdot \text{FT (regular aperture)}}$

"Array Thm"

$\boxed{\text{FT (array of identical apertures)} = \text{FT (aperture)} \cdot \sum_{\text{all apertures}} e^{i(k_x x_{0j} + k_y y_{0j})}}$

Again not worrying about factors of $1/\sqrt{2\pi}$!

(x_{0j}, y_{0j}) : center of j^{th} aperture

= FT of (↑ ↑ ↑ ↑ ↑ ↑ etc.) located at centers !

Don't even have to be in an array!

Random vs Non-random array

Random: Σ phase factors cancels

$I = I(\text{one aperture})$

Non-random: Σ phase factors gives you fine detail

$I = I(\text{array})$ modulated by $I(\text{one aperture})$

like cosines × sinc²

for double slit example

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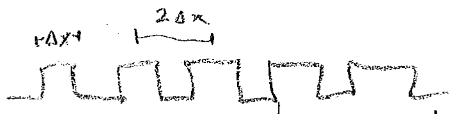
quiz 11.7

Diffraction Grating

aperture function



(ignore y-direction)

"comb function", remember?
2N+1 total delta functions

Fourier Transform $\frac{1}{\sqrt{2\pi}}\, a\, \mathrm{sinc}\frac{ka}{2}$

$\frac{1}{\sqrt{2\pi}}\, \Delta x\, \mathrm{sinc}\left(k_x\frac{\Delta x}{2}\right)$

$$FT = \frac{1}{\sqrt{2\pi}}\, \frac{\sin(2N+1)\frac{\omega t_0}{2}}{\sin\frac{\omega t_0}{2}}$$

from handout
$t_0 \to 2\Delta x$
$\omega \to k_x$

let N now = total number

$$= \frac{1}{\sqrt{2\pi}}\, \frac{\sin N k_x \Delta x}{\sin k_x \Delta x}$$

$$= \frac{1}{2\pi}\, \Delta x\, \mathrm{sinc}\frac{k_x \Delta x}{2d}\; \frac{\sin N k_x \frac{\Delta x}{d}}{\sin k_x \frac{\Delta x}{d}}$$

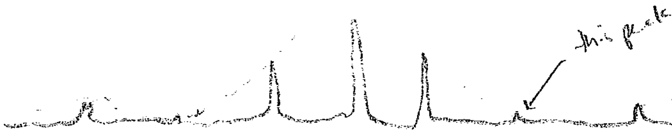
$$\boxed{I = I_0\, \mathrm{sinc}^2\frac{k_x \Delta x}{2d}\; \frac{\sin^2 N k_x \frac{\Delta x}{d}}{N^2 \sin^2 k_x \frac{\Delta x}{d}}}$$

added N^2 in denominator (and removed Δx from numer.)
so $I(x=0) = I_0$

recall $\frac{\sin Nx}{\sin x} \to N$ as $x \to 0$
also from delta function handout

Recall $\frac{\sin Nx}{x}$ plotted in delta function handout for $N=10$

this now squared
& multiplied by sinc^2



this peak happens to fall on minima of sinc^2 (because delta function spacing = $2\Delta x$ exactly)

position of this "first order peak" (& other peaks) depends on wavelength (via k)

"Fraunhofer n-slit"
Multiple Slit
Animation

if narrower aperture
wider sinc^2

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section 11.3

Spectrometer

use (first order) diffraction spot to give you wavelength separation

Let grating spacing $= h$ now instead of $2\Delta x$

1) position of peaks set by comb function

2) overall width of peaks set by sinc function

1) peaks occur when denominator goes to zero $\sin x = 0 \rightarrow x = m\pi$

$$k \frac{x}{d}(h/2) = m\pi$$

compare old method

$$\boxed{n\lambda = d\sin\theta}$$

$$\frac{2\pi}{\lambda}\frac{x}{d}(h/2) = m\cancel{\pi}$$

$$\lambda = \frac{xh}{md}$$

x = position on screen

d = distance from grating to screen

h = grating spacing (meters/line)

Again from delta function handout

distance (in k space) to first zero $= \frac{2\pi}{N_{tot} t_0}$

so

$$\Delta k_x = \frac{2\pi}{Nh} \qquad k_x = \frac{kx}{d}$$

$$\frac{x}{d}\Delta k = \frac{2\pi}{Nh} \qquad k = \frac{2\pi}{\lambda}$$

$$\frac{x}{d}\frac{2\pi\Delta\lambda}{\lambda_0^2} = \frac{2\pi}{Nh} \qquad \Delta k = \frac{2\pi\Delta\lambda}{\lambda^2}$$

$$\Delta\lambda = \frac{d}{x}\frac{\lambda_0^2}{Nh}$$

$$\frac{xh}{d} = m\lambda$$

$$= \frac{1}{m\lambda}\frac{\lambda^2}{N}$$

$$\boxed{\Delta\lambda = \frac{\lambda}{mN}}$$ close to $\Delta\lambda_{FWHM}$

$$RP = \frac{\lambda}{\Delta\lambda}$$

$$\boxed{RP = mN}$$

increase RP by increasing N (# lines illuminated by light) and/or m (but this decreases intensity)