day 37 pg 1

Back to main Eqn for Gaussian beams $E = E_0 \frac{w_0}{w} e^{-r^2/w^2} e^{i(\dots)}$

$I \propto |E|^2$

$$I = I_0 \left(\frac{w_0}{w}\right)^2 e^{-2r^2/w^2}$$

$$= \frac{I_0}{1 + z^2/z_0^2} e^{-2r^2/w^2}$$

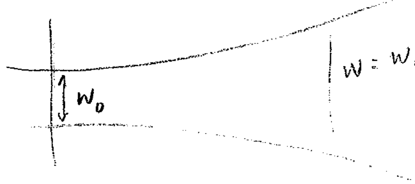
z_0 = "Rayleigh range" = aka "confocal parameter"

$w \approx$ width of beam, expands with z

Also note it's a measure of intensity decrease

maybe widths should be $2w_0$ or $2w$

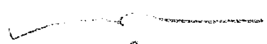
w_0 - same as used for Gaussian wells on last page



$w = w_0 \sqrt{1 + \frac{z^2}{z_0^2}}$

w_0

$z = 0$



z_0 = measure of how fast beam expands

At $z = z_0$ $\quad I = \frac{I_0}{2} e^{-2r^2/(w_0\sqrt{2})^2}$ $\quad$ width up by $\sqrt{2}$, center intensity down by 2

$w = w_0\sqrt{2}$

PPT plots

At $z = 2z_0$ $\quad I = \frac{I_0}{5} e^{-2r^2/(w_0\sqrt{5})^2}$ $\quad$ width up by $\sqrt{5}$, center intensity down by 5

etc

Large z: $w = \frac{w_0 z}{z_0}$ $\quad$ $\theta/2$ goes with a straight line $y = mx$

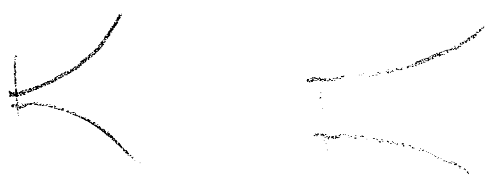
$\tan\theta/2 = \frac{w/2}{z} \rightarrow \frac{\theta}{2} = \frac{w}{2z} \rightarrow \theta = \frac{w}{z} = \frac{w_0}{z_0}$

$\theta = \frac{2}{k w_0}$

$\boxed{\theta = \frac{\lambda}{\pi w_0}}$

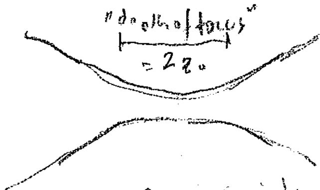
$\left(\lambda = \frac{\lambda_{vac}}{n}\right)$

Small w_0 (tight focus) $\rightarrow$ small z_0 (fast diverging)



Note: Run light backwards in time, get focusing (and then diverging beam after it focuses to w_0)

"depth of focus" $= 2z_0$



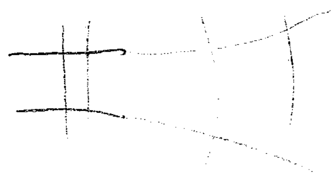
coming into and out of focus

day 37 pg 2

(this should go earlier, when  is first explained.)

Note: the Fresnel integral tacitly assumed that all points on Gaussian (at $z=0$) had <u>same phase</u> $\rightarrow$ plane wave wave fronts at $z=0$

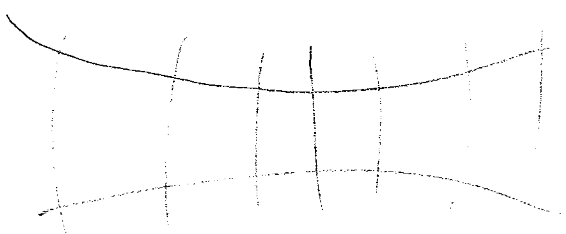
i.e. from plane wave, only possibility is for beam to expand.



If we had done integral with varying phase across the integral, as in a partially circular w, we would have gotten

(($\rightarrow$ shrinks in diameter

)) $\rightarrow$ increases in diameter



day 37 pg 3

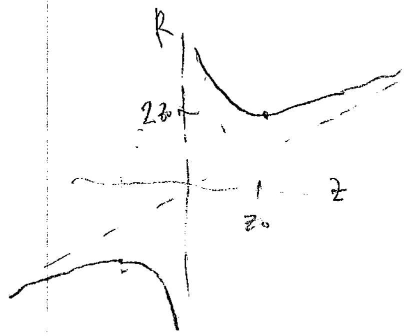
phase factors of E

A) e^{ikz} : obvious

B) $e^{i\frac{kr^2}{2R}}$ curves wave front

limits: 1) $z \to 0$
$R = \infty$ this term goes away

2) $z \to$ large
$R \approx z$



then $e^{ik(z + \frac{r^2}{2R})} = e^{ik(z + \frac{r^2}{2z})}$

$= e^{ik\sqrt{z^2+r^2}}$ $(z \gg r)$

$r^2 \to x^2+y^2$

$= e^{ikr}$ (real r!)

spherical wave front!

C) $e^{-i\tan^{-1}(z/z_0)}$ Jackson's phase shift between actual wave + pure plane wave

$= -\frac{\pi}{2}$ when $z = -\infty$

$= +\frac{\pi}{2}$ when $z = +\infty$

also $\to$ $= \pm\frac{\pi}{4}$ when $z = \pm z_0$

$\to$ when light goes through a focus,
it has an overall phase shift of π

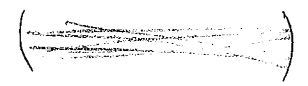
"Gouy shift" (book: seen for other profiles, not just Gaussian)

Aside on Mus.. pleated lossy modes $\to$ PPT

Jan 27 09

Gaussian beams + stable resonators

before

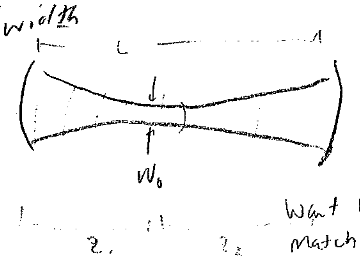


infinitely thin rays bouncing back + forth

y not infinite $\rightarrow$ $0 < \left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right) < 1$ criterion

(HW P9.15)

Now, beam w/ width



Want mirror curvature to match wavefront curvature at that point. $\rightarrow$ $R = z + \frac{z_0^2}{z}$

$zR + z^2 = z_0^2$

$z^2 - zR + z_0^2 = 0$

$z = \frac{R}{2} \pm \frac{\sqrt{R^2 - 4z_0^2}}{2}$

take $+\sqrt{\ }$

① $z_1 + \frac{z_0^2}{z_1} = R_1$ $\rightarrow$ $z_1 = \frac{R_1}{2} + \frac{\sqrt{R_1^2 - 4z_0^2}}{2}$

② $z_2 + \frac{z_0^2}{z_2} = R_2$ $\rightarrow$ $z_2 =$ similar

③ $z_1 + z_2 = L$

$$\left(\frac{R_1}{2} + \frac{\sqrt{R_1^2 - 4z_0^2}}{2}\right) + \left(\frac{R_2}{2} + \frac{\sqrt{R_2^2 - 4z_0^2}}{2}\right) = L$$

$$z_0^2 = \frac{-L(L-R_1)(L-R_2)(L-R_1-R_2)}{(2L-R_1-R_2)^2}$$ (done w/ Mathematica)

$$= \frac{+L(R_1-L)(R_2-L)(R_1+R_2-L)}{(R_1-L+R_2-L)^2}$$

$$\text{Num} = L R_1\left(1-\frac{L}{R_1}\right)R_2\left(1-\frac{L}{R_2}\right)\left(\underbrace{\frac{R_1R_2}{L}\left[1-\left(1-\frac{L}{R_1}\right)\left(1-\frac{L}{R_2}\right)\right]}_{\text{since this } = R_1+R_2-L}\right)$$

$$= R_1^2R_2^2(x)(1-x)$$

with $x = \left(1-\frac{L}{R_1}\right)\left(1-\frac{L}{R_2}\right)$

z_0^2 must be positive

$\rightarrow x$ must be between 0 and 1

$$\boxed{0 < \left(1-\frac{L}{R_1}\right)\left(1-\frac{L}{R_2}\right) < 1}$$

same condition as before!

day 37 pg 5

Can also work out z_1 and z_2 in terms of z_0^2

turns out $\rightarrow$ if $\left(1-\frac{L}{R_1}\right)\left(1-\frac{L}{R_2}\right)$ too close to 0 or 1,

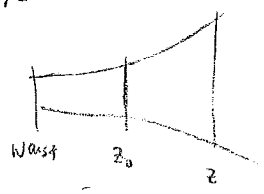
then one or both of $z_1, z_2 \rightarrow \infty$

and $w_1, w_2 \rightarrow \infty$

(would require infinitely large mirrors)

Oct 31 p6

q parameter



All important info contained in z_0 and z

z_0 → sets overall curvature

z → sets where you are

(could use W_0 and W, since given W_0 and W you can determine z_0 and z, but it's not done)

combine this info into one "q-parameter"

$$q = z + i z_0$$

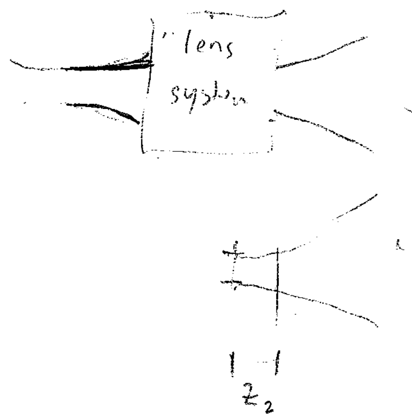
How does q change as you pass through optical element?

Thm: $$\boxed{q_2 = \frac{Aq_1 + B}{Cq_1 + D}}$$ when $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ = normal (ray) matrix for the element

$\boxed{\text{"ABCD Law for Gaussian beams"}}$

z_{02} = new Rayleigh range

z_2 = how far from "virtual waist"



} new beam looks like it's coming from a new waist, with a new curvature ← not necessarily out part of lens system

Proof in book (App II.A)
(1) free space
(2) thin lenses } arbitrary lens
(3) thick window

} I won't go over

eq 37 eg 7

incidentally for a thin lens $M = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$

Can probably skip this. Could be ok exam problem.

This means

$$q_2 = \frac{1 \cdot q_1 + 0}{(-1/f) q_1 + 1}$$

$$q_2 = \frac{q_1}{-\frac{q_1}{f} + 1} \times \frac{1/q_1}{1/q_1}$$

$$q_2 = \frac{1}{-\frac{1}{f} + \frac{1}{q_1}}$$

$$\frac{1}{q_2} = -\frac{1}{f} + \frac{1}{q_1}$$

$$\boxed{\frac{1}{f} = \frac{1}{q_1} - \frac{1}{q_2}}$$

almost like $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$

but they aren't distances, they are beam shape parameters