day 4 pg 1

Maxwell Eqns in matter

what about regions inside matter ~~[illegible]~~

must modify M.E. (at least it's useful to do so!)

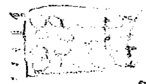
$\nabla \cdot E = \rho/\epsilon_0$

↳ ρ from two sources

(1) "free charge", aka existing charges

(2) "bound charge" aka "separated charge"

E ⟶



even if no free charge, ρ is not zero everywhere

ρ negative, ρ positive

define $\vec{P}$ = dipole moment / volume "polarization"

dipole moment = $q d$ for an individual molecule

then a changing polarization creates a "sloshing" current

$\vec{J}_p = \frac{\partial \vec{P}}{\partial t}$ Eqn of cont $\rightarrow \nabla \cdot J = -\frac{\partial \rho}{\partial t} \rightarrow \nabla \cdot \left(\frac{\partial \vec{P}}{\partial t}\right) = -\frac{\partial \rho}{\partial t} \rightarrow \rho_b = -\nabla \cdot \vec{P}$

charge density associated w/ that current

$\nabla \cdot \vec{E} = \frac{\rho_{free} + \rho_{bound}}{\epsilon_0} = \frac{\rho_{free}}{\epsilon_0} + \frac{-1}{\epsilon_0} \vec{\nabla} \cdot \vec{P}$

$\nabla \cdot \left(\vec{E} + \frac{1}{\epsilon_0} \vec{P}\right) = \frac{\rho_{free}}{\epsilon_0}$

$\nabla \cdot \underbrace{\left(\epsilon_0 \vec{E} + \vec{P}\right)}_{} = \rho_{free}$

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ = a mathematically useful field "displacement field"

$\boxed{\vec{\nabla} \cdot \vec{D} = \rho_{free}}$ ME #1 for materials "Macroscopic ME"

↳ don't need to worry about microscopic details of ρ, just the ρ that's easy to observe

(test: $\nabla \cdot \vec{E} = -\frac{\nabla \cdot \vec{P}}{\epsilon_0}$ when $\rho_{free} = 0$)

$\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$

↳ J from 3 sources

(1) "free current" aka macroscopic current

(2) "polarization current" from "sloshing" of material as it's becoming polarized, as just discussed

$\vec{J}_p = \frac{\partial \vec{P}}{\partial t}$

(3) bound current (next page)

↳ Not mentioned in P+W!

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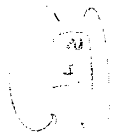
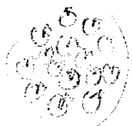
bound currents from intrinsically magnetic objects

 same B's as 

must be "currents" inside!

QM: spin of electrons = intrinsic angular momentum

also partial orbital ang mom



Griffiths fig 6.15

turns out $J_b = \nabla \times \vec{M}$ define $M = \frac{\text{mag. dipole moment}}{\text{volume}}$

mag dip. mom $= I \times \pi r^2$ for wire loop

then $\nabla \times B = \mu_0 J_{free} + \mu_0 J_p + \mu_0 J_b + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$

$$\boxed{\nabla \times \vec{B} = \mu_0 J_{free} + \mu_0 \left(\frac{\partial P}{\partial t}\right) + \mu_0 (\nabla \times M) + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}}$$

Ampere's law form in P etc, if $\nabla \times M = 0$

Working a bit more... $\nabla \times B - \nabla \times \mu_0 M = \mu_0 J_f \; \mu_0 \frac{\partial P}{\partial t} + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$

divide by μ_0,

$$\nabla \times \underbrace{\left(\frac{B}{\mu_0} - M\right)}_{} = J_f + \frac{\partial}{\partial t}\underbrace{(P + \epsilon_0 E)}_{D \text{ already}}$$

define $H = \frac{B}{\mu_0} - M$

$$\boxed{\nabla \times H = J_{free} + \frac{\partial D}{\partial t}}$$ the usual Maxwell #4 in matter

↳ "displacement current"

ME in matter:

$$\boxed{\begin{aligned} \nabla \cdot D &= \rho_f \\ \nabla \times E &= -\frac{\partial B}{\partial t} \\ \nabla \cdot B &= 0 \\ \nabla \times H &= J_f + \frac{\partial D}{\partial t} \end{aligned}}$$

$\nabla \cdot E = -\frac{\nabla \cdot \vec{P}}{\epsilon_0}$ if no ρ_{free}

$\nabla \times B = \mu_0 J_{free} + \mu_0 \frac{\partial P}{\partial t} + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$ if no M

constitutive eqns

$D = \epsilon_0 E + P$ $= \epsilon \vec{E}$ often ($\vec{P} \parallel \vec{E}$) — permittivity
$= \epsilon_0 \epsilon_r E$

$H = \frac{B}{\mu_0} - M$ $= \frac{1}{\mu} \vec{B}$ often ($M \parallel B$) — permeability
$= \frac{1}{\mu_0 \mu_r} B$ — relative permeability

ϵ_r = relative permittivity = dielectric constant (K?)

Wave Eqn inside dielectric/magnetic material

→ some ρ_b, $\boxed{\text{no } \rho_f}$

some J_b $\boxed{\text{no } J_f}$

1. $\nabla \cdot D = \rho_f$
2. $\nabla \times E = -\partial B/\partial t$
3. $\nabla \cdot B = 0$
4. $\nabla \times H = J_f + \partial D/\partial t$

⇒

$\nabla \cdot D = 0$

$\nabla \times E = -\partial B/\partial t$

$\nabla \cdot B = 0$

$\nabla \times H = \partial D/\partial t$

if linear, $D = \epsilon_0 \epsilon_r E$

$H = \frac{1}{\mu_r \mu_0} B$

if $\vec{D} \sim \vec{E}$ ⇒

$\nabla \cdot E = 0$

$\nabla \times E = -\partial B/\partial t$

$\nabla \cdot B = 0$

$\nabla \times \left(\frac{1}{\mu_r \mu_0} B\right) = \epsilon_0 \epsilon_r \frac{\partial E}{\partial t}$

$\nabla \times B = \mu_0 \epsilon_0 \underbrace{\mu_r \epsilon_r}_{\text{only change}} \frac{\partial E}{\partial t}$

get wave eqn again

$$\nabla^2 E = \mu_0 \epsilon_0 \mu_r \epsilon_r \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 B = \mu_0 \epsilon_0 \mu_r \epsilon_r \frac{\partial^2 B}{\partial t^2}$$

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \frac{1}{\sqrt{\mu_r \epsilon_r}}$$

$$\boxed{v = \frac{c}{\sqrt{\epsilon_r \mu_r}}}$$

Compare $v = \frac{c}{n}$ learned somewhere

if $\mu_r \simeq 1$, as it often is, then $n = \sqrt{\epsilon_r}$

always assumed in P+W

index of refraction = $\sqrt{\text{dielectric constant}}$

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To get P+W's wave eqn; Eqn 1.41

start with $\nabla \cdot D = \rho_{free} \rightarrow \nabla \cdot \vec{E} = -\frac{1}{\epsilon_0} \nabla \cdot \vec{P}$ if no ρ_{free}

$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$ keep

$\boxed{\nabla \cdot B = 0}$ keep

$\nabla \times H = J_{free} + \frac{\partial D}{\partial t} \rightarrow \nabla \times B = \mu_0 J_{free} + \mu_0 \frac{\partial P}{\partial t} + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$ if no M

~~Restrict to materials where~~ $\rho_{free} = 0$

$M = 0$

Then $\nabla \cdot (\epsilon_0 E + P) = 0 \rightarrow \boxed{\nabla \cdot E = -\frac{1}{\epsilon_0} \nabla \cdot P}$

$\nabla \times \left(\frac{B}{\mu_0}\right) = J_{free} + \frac{\partial}{\partial t}(\epsilon_0 E + P)$

$\rightarrow \boxed{\nabla \times B = \mu_0 J_{free} + \mu_0 \epsilon_0 \frac{\partial E}{\partial t} + \mu_0 \frac{\partial P}{\partial t}}$

Then do the usual $\nabla \times (\nabla \times E) = \nabla \times \left(-\frac{\partial B}{\partial t}\right)$ trick

$\nabla(\nabla \cdot E) - \nabla^2 E = -\frac{\partial}{\partial t}(\nabla \times B)$

$\nabla \cdot E \rightarrow -\frac{1}{\epsilon_0} \nabla \cdot P$

$\nabla \times B \rightarrow \mu_0 J_{free} + \mu_0 \epsilon_0 \frac{\partial E}{\partial t} + \mu_0 \frac{\partial P}{\partial t}$

P+W Wave Eqn

$$\boxed{\nabla^2 E - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial J_{free}}{\partial t} + \mu_0 \frac{\partial^2 P}{\partial t^2} - \frac{1}{\epsilon_0} \nabla(\nabla \cdot P)}$$

usual wave eqn in vacuum

electric currents, important when free charges (plasma, metals)

dipole oscillations very important when no J_{free}

happens when eg $\vec{P}$ not in same direction as E

Give out Complex # handout!