Review: Lorentz model

$$\tilde{\chi} = \frac{\omega_p^2}{\omega_0^2 - i\omega\gamma - \omega^2}$$

- $\omega_0 = \sqrt{k/m}$ from spring model
- $\omega_p = \sqrt{\frac{q^2 N}{\epsilon_0 m}}$ → #/volume
- summation if necessary for multiple resonances

Conductors: intuition says electrons free to move, no restoring force.
→ set $\omega_0 = 0$

$$\boxed{\tilde{\chi} = \frac{\omega_p^2}{-i\omega\gamma - \omega^2}}$$ ✓

More rigorously, can go back to wave eqn

$$\nabla^2 E - \mu_0\epsilon_0 \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial J_{free}}{\partial t} + \mu_0 \frac{\partial^2 P}{\partial t^2} - \frac{1}{\epsilon_0}\nabla(\nabla \cdot \vec{P})$$

↳ still set = 0

↳ J_{free} = current/area = $\frac{\text{charge}}{\text{area}} \cdot \frac{1}{\text{time}}$

$= \frac{\text{charge}}{\text{area}\cdot\text{distance}} \cdot \frac{\text{distance}}{\text{time}}$

$\vec{J}_f = Nq\vec{v}$

↳ drift velocity $= \frac{d\vec{r}}{dt}$ ↳ do Newton 2 again, w/o spring term

Gets same result! (done in book)

$$\tilde{N} = \sqrt{1+\tilde{\chi}}$$

$$\tilde{N} = \sqrt{1 - \frac{\omega_p^2}{i\omega\gamma + \omega^2}}$$ → If you remember how to take square root of complex #, you can separate into real + imag parts (or get Mathematica to do it)

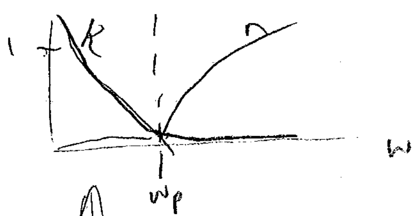


Fig 2.6 pg 54 (HW P2.8) on HW5

↑ K can be quite large

→ skin depth can be quite small

wave $\sim e^{-K\omega z/c}$

@ intensity skin depth $z = \frac{c}{2K\omega}$

Start with this as review?

Last time wave $\sim e^{-\kappa_{imag} z}$

$\tilde{k}^2 = \frac{\omega^2}{c^2}(1+\tilde{\chi})$

Now: $\tilde{k} = \frac{\omega}{c}(n+iK)$

$\kappa_{imag} = \frac{\omega K}{c}$

skin depth $d = \frac{1}{2\kappa_{imag}} = \frac{c}{2\omega K}$

Day 7 pg 2

Conductors let $\rho = 0$
$\nabla \cdot P = 0$

$$\nabla^2 E - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial J_{free}}{\partial t} + \mu_0 \frac{\partial^2 P}{\partial t^2} - \frac{1}{\epsilon_0} \nabla(\nabla \cdot P)$$

$$\vec{E} = \tilde{E}_0 e^{i(k \cdot r - \omega t)}$$

assume $J_{free} = \tilde{J}_0 e^{i(k \cdot r - \omega t)}$

$$\boxed{J_{free} = qN_v \quad \frac{\text{charge}}{\text{elec}} \cdot \frac{\text{electron}}{\text{volume}} \cdot \frac{\text{distance}}{\text{time}} = \frac{\text{charge/time}}{\text{area}}}$$

Ohm's Law $\tilde{J} = \sigma \tilde{E}$ Ohm's Law (same phase, same exponential)

$$\nabla^2 E - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = \mu_0 \sigma \frac{\partial E}{\partial t}$$

↳ damping! like Lorentz model
from single time derivative

Recall Lorentz model: $N^2 = 1 + \frac{\omega_p^2}{\omega_0^2 - i\gamma\omega - \omega^2}$ $\omega_p = \frac{Nq^2}{\epsilon_0 m_e}$

Method 1: solve diff eqn for E. (done in book) use model to relate J and E more carefully than $J = \sigma E$

Method 2: use Lorentz model stuff, w/ no restoring force $\rightarrow \omega_0 = 0$

$$\boxed{N^2 = 1 + \frac{\omega_p^2}{-i\omega\gamma - \omega^2}}$$

(same result, with $\sigma = \frac{Nq^2}{m\gamma}$

→ can get $n + i\kappa$ from that

---

if conductor has bound electrons also, which can absorb also,

$$N^2 = 1 + \omega_p^2 \left[\frac{f_e}{-\omega^2 + i\gamma_e \omega} + \sum_j \frac{f_j}{\omega_{0j}^2 - \omega^2 + i\gamma_j \omega} \right]$$

$\omega_p^2 = \frac{Nq^2}{\epsilon_0 m}$

$f_e = \frac{\#\text{ cond. electrons}}{\text{atom}}$

$f_j = \frac{\#\text{ bound electrons at that res. freq.}}{\text{atom}}$

$N = \frac{\#\text{ atoms}}{\text{vol}}$

instead of $\frac{\#\text{ electrons}}{\text{volume}}$

What does ω_p represent? $E \rightarrow$ then kill $\vec{E} \rightarrow$ what happens?

"plasma oscillations"

"Good conductors" γ = small, $\ll \omega$. Then $\tilde{\chi} = \frac{\omega_p^2}{-\omega^2}$

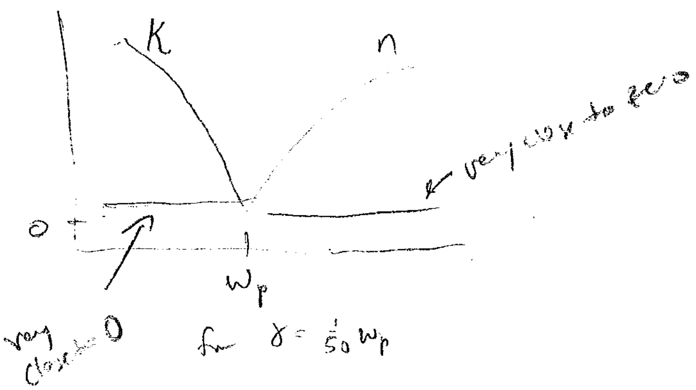
then $N^2 = 1 - \frac{\omega_p^2}{\omega^2}$

Hecht: "ω_p serves as a critical value below which the index is complex and the penetrating wave drops off exponentially."

$\omega > \omega_p$: n = real, absorption = small ("lossless"), conductor = transparent

$\omega < \omega_p$, n = complex, absorption important, conductor reflective (I think)

Book figure ~~[illegible]~~



for $\gamma = \frac{1}{50}\omega_p$

Reflectance Figure from Hecht (in PowerPoint)

Energy + Power in E+M fields

Appendix (also in Griffiths) : (I won't derive)

energy stored in electric field $U = \frac{\epsilon_0}{2} \int E^2 \, dv$ ← compare $U = \frac{1}{2} C V^2$

" " " magnetic " $U = \frac{1}{2\mu_0} \int B^2 \, dv$ ← $U = \frac{1}{2} L I^2$

= how much energy it took to assemble the charges / set up the currents producing the fields

energy density $\frac{U}{\text{volume}}$ $u_{field} = \frac{\epsilon_0}{2} E^2 + \frac{B^2}{2\mu_0}$

lowercase

Poynting Derivation on Power Point

energy flux = power (will actually do $\frac{\text{power}}{\text{volume}}$)

Start with Maxell Eqns 2 + 4

$\nabla \times E = -\frac{\partial B}{\partial t}$ $\qquad \nabla \times B = \mu_0 J + \epsilon_0 \mu_0 \frac{\partial E}{\partial t}$

$\nabla \times \left(\frac{B}{\mu_0}\right) = \epsilon_0 \frac{\partial E}{\partial t} + \underbrace{\frac{\partial P}{\partial t}}_{J_p} + J_f$ (book's version) when $\rho_{free} = 0$, $M = 0$

dot $\vec{B}$ $\qquad$ dot $\vec{E}$

$B \cdot (\nabla \times E) = -\underbrace{B \cdot \frac{\partial B}{\partial t}}_{\frac{1}{2}\frac{\partial}{\partial t}(B^2)}$ $\qquad \frac{1}{\mu_0} \vec{E} \cdot (\nabla \times B) = \epsilon_0 \underbrace{E \cdot \frac{\partial E}{\partial t}}_{\frac{1}{2}\frac{\partial}{\partial t}(E^2)} + \vec{E} \cdot (J_p + J_f)$

vector math

$\nabla \cdot (E \times B) + E \cdot (\nabla \times B)$

$E \cdot (\nabla \times B) = -\frac{1}{2}\frac{\partial}{\partial t}(B^2) - \nabla \cdot (E \times B)$

substituting up here

$\Downarrow$

$\frac{1}{\mu_0}\left[-\frac{1}{2}\frac{\partial}{\partial t}(B^2) - \nabla \cdot (E \times B)\right] = \epsilon_0 \frac{1}{2}\frac{\partial}{\partial t}(E^2) + \vec{E} \cdot (\vec{J}_p + \vec{J}_f)$

Poynting Thm $\qquad \nabla \cdot \underbrace{\left(\vec{E} \times \frac{\vec{B}}{\mu_0}\right)}_{\vec{S}} + \frac{\partial}{\partial t} \underbrace{\left(\frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}\right)}_{u_{field}} = \underbrace{-\vec{E} \cdot (\vec{J}_p + \vec{J}_f)}_{-\frac{\partial u_{medium}}{\partial t}}$

each term = $\frac{\text{power}}{\text{volume}}$

$$\boxed{\nabla \cdot S + \frac{\partial u_{field}}{\partial t} = -\frac{\partial u_{medium}}{\partial t}}$$

Power vec !

energy flowing out of region

change in energy of E+M fields

work done on charges, per time

$E \times H$ in general, if need to worry about magnetic material

$$\nabla \cdot S + \frac{dU_{free}}{dt} = -\frac{dU_{medium}}{dt}$$

integrate

$$\int (\nabla \cdot S)\, dv + \frac{dU_{free}}{dt} = -\frac{dU_{medium}}{dt}$$

$\frac{dU_{free}}{dt}$ = − work done − energy transported away

→ Give Thought Question on $\vec{E}$ vs $\vec{B}$ always in phase? Or give as review question since I mentioned that last lecture.

Final notes on $\vec{S}$:

$$\vec{S} = \frac{1}{\mu_0} \underbrace{\vec{E} \times \vec{B}}$$

$\vec{E} \times \vec{B}$ = direction of energy flow

fits with $\hat{E} \times \hat{B} = \hat{k}$ picture

Remember plane waves $\vec{B}_0 = \frac{1}{v} E_0 \; \hat{u}$ ← $\hat{u}$ is k direction

from lecture 5

Aside: complex n → complex k → E vs B not in phase

free space plane wave: $B \perp E$

no absorption: E and B in phase

$$S(t) = \frac{1}{\mu_0} (E \cos \omega t) \left(\frac{E}{c} \cos \omega t \right)$$

$$S(t) = \frac{E^2}{\mu_0 c} \cos^2(\omega t)$$

time average of $\cos^2 = \frac{1}{2}$

$$\boxed{\langle S \rangle = \frac{1}{2} \frac{E^2}{\mu_0 c}}$$

Notation: time average

$\frac{\text{power}}{\text{area}}$ = intensity of light ☆

$\epsilon v E^2$

Note: $c^2 = \frac{1}{\epsilon_0 \mu_0} \rightarrow \mu_0 c = \frac{1}{\epsilon_0 c}$

$$\boxed{\langle S \rangle = \frac{1}{2} \epsilon_0 c E^2}$$

in matter: $B_0 = \frac{1}{c/n} E_0$

E vs B → factor of n in some ...

$$\boxed{I = \langle S \rangle = \frac{1}{2} n \epsilon_0 c E^2}$$

End of Ch 2!