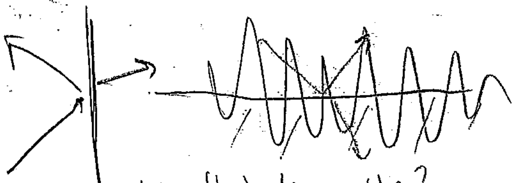
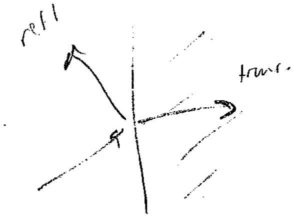
Almost
Ready to tackle the Big problem



how much reflected?
how much transmitted?



how much reflects/transmits?

in powerpoint

Reminder of boundary conditions.

(i) $\epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$ (from $\nabla \cdot \vec{D} = \rho_{free}$, if no surface charge; Maxwell Eq, pillbox)

(ii) $\vec{E}_{1\parallel} = \vec{E}_{2\parallel}$ from $\nabla \times E = -\partial B/\partial t$

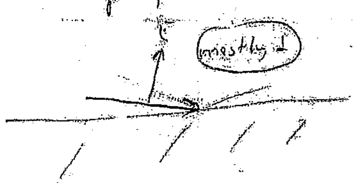
(iii) $B_{1\perp} = B_{2\perp}$ from $\nabla \cdot B = 0$

(iv) $\dfrac{\vec{B}_{1\parallel}}{\mu_1} = \dfrac{\vec{B}_{2\parallel}}{\mu_2}$ (from $\nabla \times H = J_{free} + \frac{\partial D}{\partial t}$, if no surface current; Maxwell Eq, like ...)

For us, $\mu_1 = \mu_2 = \mu_0$ so $\vec{B}_{1\parallel} = \vec{B}_{2\parallel}$

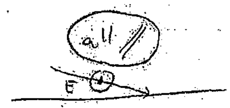
Complication: polarization will matter!

"p polarization" p = parallel to plane of incidence

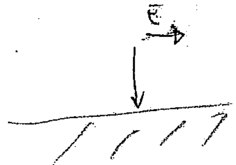


vs.

"s polarization" s = senkrecht, ⊥ to plane of incidence

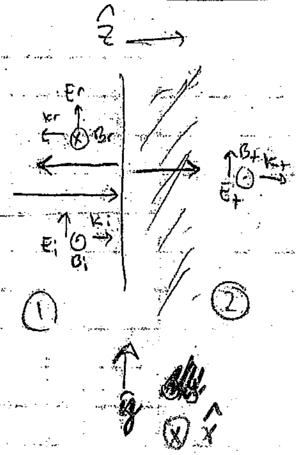


Getting our feet wet... lets tackle $\theta = 90°$ "Normal incidence"



then $\vec{E}$ is all $\parallel$, (s + p polarization indistinguishable; there is no "plane of incidence")

Reflection & Transmission at Normal Incidence



- Assume all $\vec{E}$'s in $\hat{y}$ direction
  - ↳ if not, they'll come out negative in the end
- Direction of B given by $\hat{E}\times\hat{B} = \hat{k}$
- each wave has form $\tilde{E}_0\, e^{i(kz-\omega t)}$
- Mag of B given by $\frac{E}{v}$

Write down waves:

incident $\quad \tilde{\vec{E}}_I = \tilde{E}_{0I}\, e^{i(k_1 z-\omega t)}\, \hat{y}$

$\tilde{\vec{B}}_I = \frac{1}{v_1}\tilde{E}_{0I}\, e^{i(k_1 z-\omega t)}\, (-\hat{x})$

reflected $\quad \tilde{\vec{E}}_R = \tilde{E}_{0R}\, e^{i(-k_1 z-\omega t)}\, \hat{y}$

$\tilde{\vec{B}}_R = \frac{1}{v_1}\tilde{E}_{0R}\, e^{i(-k_1 z-\omega t)}\, \hat{x}$

transmitted $\quad \tilde{E}_T = \tilde{E}_{T0}\, e^{i(k_2 z-\omega t)}\, \hat{y}$

$\tilde{B}_T = \frac{1}{v_2}\tilde{E}_{T0}\, e^{i(k_2 z-\omega t)}\, (-\hat{x})$

Plug into bndy eqns, set $z=0$, do some algebra then we're done!

~~[illegible] always get B from [illegible]~~

region ① $\tilde{E} = \tilde{E}_{0I}\, e^{i(k_1 z-\omega t)}\, \hat{x} + \tilde{E}_{0R}\, e^{i(-k_1 z-\omega t)}\, \hat{x}$

② $\tilde{E} = \tilde{E}_{T0}\, e^{i(k_2 z-\omega t)}\, \hat{x}$

BC(iii) at $z=0$: $E_{1\parallel} = E_{2\parallel}$: $\quad \tilde{E}_{0I}\cancel{e^{-i\omega t}} + \tilde{E}_{0R}\cancel{e^{-i\omega t}} = \tilde{E}_{T0}\cancel{e^{-i\omega t}} \Rightarrow \boxed{\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{T0}}$

BC(iv) at $z=0$: $B_{1\parallel} = B_{2\parallel}$: $\quad \frac{\tilde{E}_{0I}}{v_1}\cancel{e^{-i\omega t}}(-\hat{x}) + \frac{\tilde{E}_{0R}}{v_1}\cancel{e^{-i\omega t}}(\hat{x}) = \frac{\tilde{E}_{0T}}{v_2}\cancel{e^{-i\omega t}}(-\hat{x})$

$$\boxed{\tilde{E}_{0I} - \tilde{E}_{0R} = \frac{v_1}{v_2}\tilde{E}_{0T}}$$

Solve for $\tilde{E}_{0T}$, plug in, → solve for $\tilde{E}_{0R}$ in terms of E_{0I}

let $\beta = \frac{v_1}{v_2} = \frac{n_2}{n_1}$

$$I - R = \beta (I + R)$$

$$I - R = \beta I + \beta R$$

$$I - \beta I = R + \beta R$$

$$I(1-\beta) = R(1+\beta)$$

$$\boxed{\tilde{E}_{0R} = \frac{1-\beta}{1+\beta}\tilde{E}_{0I}} = \frac{n_1 - n_2}{n_1 + n_2}\tilde{E}_{0I} = \frac{v_2 - v_1}{v_2 + v_1} E_{0I}$$ → 180° phase shift if $v_2 < v_1$, just like ropes!

Similarly $$\boxed{\tilde{E}_{0T} = \frac{2}{1+\beta}\tilde{E}_{0I}} = \frac{2n_1}{n_1 + n_2}\tilde{E}_{0I} = \frac{2v_2}{v_2 + v_1} \tilde{E}_{0I}$$

~~if $\mu_1 \approx \mu_2$ (since they both likely $\approx \mu_0$)~~

~~then $\tilde{E}_{0R} = \frac{1 - \frac{v_1}{v_2}}{1 + \frac{v_1}{v_2}} \tilde{E}_{0I}$~~

~~$\boxed{\tilde{E}_{0R} = \frac{v_2 - v_1}{v_2 + v_1}\tilde{E}_{0I}}$ identical to string waves!~~

~~and $\boxed{\tilde{E}_{0T} = \frac{2v_2}{v_2 + v_1} E_{0I}}$~~

~~in particular, get phase shift of 180° if $v_2 < v_1$ ($n_2 > n_1$)~~

~~↳ then $E_r = \downarrow$, not up!~~

~~Since $\frac{n_1}{n_2} = \frac{v_2}{v_1}$, it's also~~

~~$E_{0R} = \left|\frac{n_1 - n_2}{n_1 + n_2}\right| E_{0I}$ → the real amplitudes must be positive~~

~~$E_{0T} = \frac{2n_1}{n_1 + n_2} E_{0I}$~~

$\langle S \rangle = I = \frac{\text{Power}}{\text{area}} = \frac{1}{2} n \epsilon_0 c E^2$ as done before (lecture 7)

maybe save this for next lecture

energy/power coefficients:

$R = \frac{P_R}{P_I} \Rightarrow \boxed{R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2}$ reflectance

$T = \frac{P_T}{P_I} \Rightarrow T = \frac{n_2}{n_1}\left(\frac{2n_1}{n_1 + n_2}\right)^2$

$$\boxed{T = \frac{4 n_1 n_2}{(n_1 + n_2)^2}}$$ transmittance

(T(x) = "transmission")

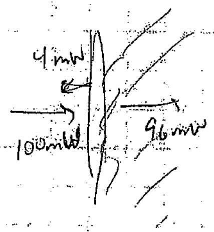
can show $R + T = 1$; conservation of energy!

easiest eqn for T: $\boxed{T = 1 - R}$

Example: air/glass $(n=1.5)$, 100mW laser
$(n=1)$

$$R = \left(\frac{1-1.5}{2.5}\right)^2 = 4\%$$

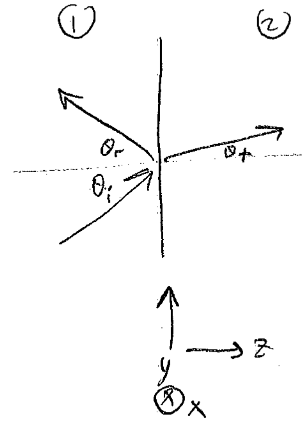
$$T = \frac{4 \cdot 1.5}{2.5^2} = 96\%$$



optics rule: lose 4% at every air/glass surface

→ might have to worry (safety) about reflections off of glass.

<u>At an angle</u> (but no polarization assumed yet)



{change axes to match handout}

$\vec{E}_{1\parallel} = \vec{E}_{2\parallel}$ B.C.

$$\tilde{E}_{0I}\, e^{i(\vec{k}_{1I}\cdot\vec{r}-\omega t)} + \tilde{E}_{0R}\, e^{i(\vec{k}_{1R}\cdot\vec{r}-\omega t)} = \tilde{E}_{0T}\, e^{i(\vec{k}_{2T}\cdot\vec{r}-\omega t)}$$

magnitude $= 2\pi/\lambda$

same mag as $k_{1I} = \frac{2\pi}{\lambda}$

- All $e^{-i\omega t}$ factors cancel (because freq is same)
- At interface $z=0$, Exponential factors must be all equal to each other (else can't work when $x, y \neq 0$)

Therefore $\vec{k}_{1I}\cdot\vec{r} = \vec{k}_{1R}\cdot\vec{r}$

$$k_{1Ix}X + k_{1Iy}Y = k_{1Rx}X + k_{1Ry}Y$$

k_{1I} has no x-component!

therefore k_{1Rx} must be 0

$\boxed{\text{Reflection is in plane of incidence}}$

Check this maybe because $z=0$ at bdy

(Same sort of thing with $\vec{k}_{1I}\cdot\vec{r} = \vec{k}_{2T}\cdot\vec{r}$

$\boxed{\text{Transmission is also in plane}}$)

$k_{1Iy} = k_{1Ry}$ magnitudes are same $2\pi/\lambda$ $\rightarrow \boxed{\theta_i = \theta_R}$

From $\vec{k}_{1I}\cdot\vec{r} = \vec{k}_{2T}\cdot\vec{r}$

$$k_{1Iy} = k_{2Ty} \rightarrow \frac{\omega n_1}{c}\sin\theta_1 = \frac{\omega n_2}{c}\sin\theta_2$$

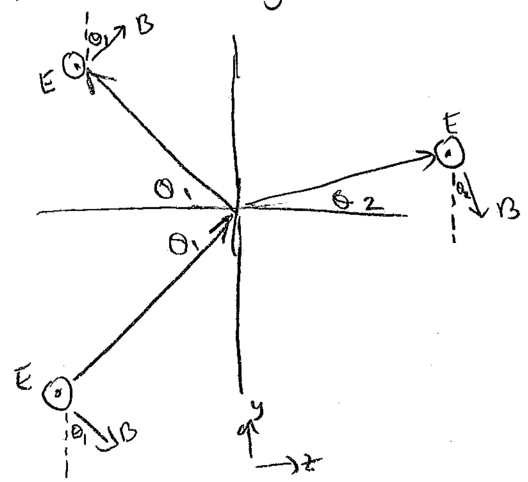
$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

Didn't even use any bndy conds yet!

{Handout: S-polarization}

Optics - Deriving Fresnel Eqns for s-polarization by Dr Colton

(using method of Griffiths)



$(\hat{x} = \text{into paper})$

s-polarization → E = perp. to interface

Directions of $\vec{B}$ chosen to make $\vec{E} \times \vec{B}$ be in direction of propogation

incident
$$\tilde{E}_I = \tilde{E}_{0I}\, e^{i(\vec{k}_I\cdot\vec{r} - \omega t)}\,(-\hat{x})$$
$$\tilde{B}_I = \frac{1}{v_1}\tilde{E}_{0I}\, e^{i(\vec{k}_I\cdot\vec{r} - \omega t)}\,(-\cos\theta_1\,\hat{y} + \sin\theta_1\,\hat{z})$$

reflected
$$\tilde{E}_R = \tilde{E}_{0R}\, e^{i(\vec{k}_R\cdot\vec{r} - \omega t)}\,(-\hat{x})$$
$$\tilde{B}_R = \frac{1}{v_1}\tilde{E}_{0R}\, e^{i(\vec{k}_R\cdot\vec{r} - \omega t)}\,(+\cos\theta_1\,\hat{y} + \sin\theta_1\,\hat{z})$$

transmitted
$$\tilde{E}_T = \tilde{E}_{0T}\, e^{i(\vec{k}_T\cdot\vec{r} - \omega t)}\,(-\hat{x})$$
$$\tilde{B}_T = \frac{1}{v_2}\tilde{E}_{0T}\, e^{i(\vec{k}_T\cdot\vec{r} - \omega t)}\,(-\cos\theta_2\,\hat{y} + \sin\theta_2\,\hat{z})$$

BC 1: $(E_{\parallel})_1 = (E_{\parallel})_2$ this is x-component

$$\tilde{E}_{0I}\, e^{i(\ )} + \tilde{E}_{0R}\, e^{i(\ )} = \tilde{E}_{0T}\, e^{i(\ )}$$

exponentials must all be equal, so cancel them out

Eqn (1) $$\boxed{\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}}$$

BC 2: $\frac{1}{\mu_1}(B_{\parallel})_1 = \frac{1}{\mu_2}(B_{\parallel})_2$ this is y-component

Non magnetic, so $\mu_1 = \mu_2 = \mu_0$. Cancel them.

Cancel exponentials again.

Eqn (2) $$\boxed{\frac{1}{v_1}\tilde{E}_{0I}(-\cos\theta_1) + \frac{1}{v_1}\tilde{E}_{0R}(+\cos\theta_1) = \frac{1}{v_2}\tilde{E}_{0T}(-\cos\theta_2)}$$

Let $\alpha = \frac{\cos\theta_2}{\cos\theta_1}$, $\beta = \frac{v_1}{v_2}$ $\left(= \frac{n_2}{n_1}\right)$

multiply Eqn 2 by $v_1/\cos\theta_1$ on both sides

call fields I, R, T for simplicity

$$\begin{aligned} &\text{Eqn (1)} \quad I + R = T \\ &\text{Eqn (2)} \quad -I + R = -\alpha\beta T \end{aligned}$$

some algebra... subtract eqns

$$2I = (1+\alpha\beta)T$$

★ $$\frac{T}{I} = \boxed{t = \frac{2}{1+\alpha\beta}}$$

"transmission coefficient" for s

plug back in for α and β, and this is obviously far right hand side of Eqn 3.19

more algebra... multiply top eqn by $\alpha\beta$, then add

$$\begin{aligned} \alpha\beta I + \alpha\beta R &= \alpha\beta T \\ \text{add} \quad -I + R &= -\alpha\beta T \\ \hline (\alpha\beta - 1)I + (1+\alpha\beta)R &= 0 \end{aligned}$$

★ $$\frac{R}{I} = \boxed{r = \frac{1-\alpha\beta}{1+\alpha\beta}}$$

"reflection coefficient" for s

plug back in for α and β, and this is obviously far right hand side of Eqn 3.18

TOPS. 35500