Constants (integrals may also be given as needed)

$c = 2.998 \times 10^8$ m/s

$h = 6.626 \times 10^{-34}$ J·s

$k_B = 1.381 \times 10^{-23}$ J/K

$N_A = 6.022 \times 10^{23}$

$m_{electron} = 9.109 \times 10^{-31}$ kg

$e = 1.602 \times 10^{-19}$ C

$\epsilon_0 = 8.854 \times 10^{-12}$ C²/N·m²

$\mu_0 = 4\pi \times 10^{-7}$ T·m/A

$\sigma = 5.670 \times 10^{-8}$ W/m²·K⁴

General E&M

$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon_0 \epsilon_r \mathbf{E}$

$\nabla \cdot \mathbf{D} = \rho_{free}$

$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} = \frac{\mathbf{B}}{\mu_0 \mu_r} \approx \frac{\mathbf{B}}{\mu_0}$

$\nabla \times \mathbf{H} = \mathbf{J}_{free} + \frac{\partial \mathbf{D}}{\partial t}$

$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$

$\nabla^2 E - \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial \mathbf{J}_{free}}{\partial t} + \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} - \frac{1}{\epsilon_0} \nabla(\nabla \cdot \mathbf{P})$

Lorentz Model

$\omega_p^2 = \frac{Nq^2}{m\epsilon_0}$

$\chi = \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\omega\gamma}$ (dielectrics)

$\chi = \frac{\omega_p^2}{-\omega^2 - i\omega\gamma}$ (metals)

Poynting

$\nabla \cdot \mathbf{S} + \frac{\partial u_{field}}{\partial t} = -\frac{\partial u_{medium}}{\partial t}$

$\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$

$u_{field} = \frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2$

$\frac{\partial u_{medium}}{\partial t} = \mathbf{E} \cdot \mathbf{J}$

$I = \langle S \rangle = \frac{1}{2} n \epsilon_0 c E^2$

Fresnel Eqs

$\alpha = \frac{\cos\theta_2}{\cos\theta_1}, \ \beta = \frac{n_2}{n_1}$

$r = \frac{\alpha - \beta}{\alpha + \beta}, \quad t = \frac{2}{\alpha + \beta}$ (p-polar)

$r = \frac{1 - \alpha\beta}{1 + \alpha\beta}, \quad t = \frac{2}{1 + \alpha\beta}$ (s-polar)

Two Interfaces

$t_{02} = \frac{t_{01} t_{12}}{\exp(-ik_1 d \cos\theta_1) - r_{10} r_{12} \exp(ik_1 d \cos\theta_1)}$

$T_{02} = \frac{n_2 \cos\theta_2}{n_0 \cos\theta_1} \frac{|t_{01}|^2 |t_{12}|^2}{|1 - r_{10} r_{12} \exp(i2k_1 d \cos\theta_1)|^2}$

$T_{02} = \alpha_{02} \beta_{02} |t_{02}|^2 = \frac{T_{max}}{1 + F \sin^2 \Phi/2}$

$T_{max} = \frac{T_{01} T_{12}}{(1 - \sqrt{R_{10}} \sqrt{R_{12}})^2}$

$F = \frac{4|r_{10}||r_{12}|}{(1 - |r_{10}||r_{12}|)^2}$

$\Phi = \delta_{10} + \delta_{12} + 2k_1 d \cos\theta_1$

$\Delta\Phi_{\text{FWHM}} = 4/\sqrt{F}$

$\Delta\lambda_{FWHM} = \frac{\lambda^2}{\pi n_1 d \cos\theta_1 \sqrt{F}}$

$\Delta\lambda_{FSR} = \frac{\lambda^2}{2 n_1 d \cos\theta_1}$

Multilayers

$t_{02} = 1/a_{11}$

$r = a_{21}/a_{11}$

$\beta_j = k_j \ell_j \cos\theta_j$

p-polar:

$$M_j = \begin{pmatrix} \cos\beta_j & -\frac{i \sin\beta_j \cos\theta_j}{n_j} \\ -\frac{i n_j \sin\beta_j}{\cos\theta_j} & \cos\beta_j \end{pmatrix}$$

$$A = \frac{1}{2 n_0 \cos\theta_0} \begin{pmatrix} n_0 & \cos\theta_0 \\ n_0 & -\cos\theta_0 \end{pmatrix} \times \left(\prod_{j=1}^N M_j\right) \begin{pmatrix} \cos\theta_{N+1} & 0 \\ n_{N+1} & 0 \end{pmatrix}$$

s-polar:

$$M_j = \begin{pmatrix} \cos\beta_j & -\frac{i \sin\beta_j}{n_j \cos\theta_j} \\ -i n_j \sin\beta_j \cos\theta_j & \cos\beta_j \end{pmatrix}$$

$$A = \frac{1}{2 n_0 \cos\theta_0} \begin{pmatrix} n_0 \cos\theta_0 & 1 \\ n_0 \cos\theta_0 & -1 \end{pmatrix} \times \left(\prod_{j=1}^N M_j\right) \begin{pmatrix} 1 & 0 \\ n_{N+1} \cos\theta_{N+1} & 0 \end{pmatrix}$$

Crystals

$\frac{1}{n^2} = \frac{u_x^2}{n^2 - n_x^2} + \frac{u_y^2}{n^2 - n_y^2} + \frac{u_z^2}{n^2 - n_z^2}$

Uniaxial:

$n = n_o, \ n = \frac{n_o n_e}{\sqrt{n_o^2 \sin^2\theta_2 + n_e^2 \cos^2\theta_2}}$

p-polar, optic axis $\perp$ to surface:

$\tan\theta_2 = \frac{n_e}{n_o} \frac{\sin\theta_1}{\sqrt{n_e^2 - \sin^2\theta_1}}$

$\tan\theta_s = \frac{n_o}{n_e} \frac{\sin\theta_1}{\sqrt{n_e^2 - \sin^2\theta_1}}$

Polarization

General Jones vector, standard form: $\begin{pmatrix} A \\ B e^{i\delta} \end{pmatrix}$

RCP: $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i(-90°)} \end{pmatrix}$

LCP: $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i(+90°)} \end{pmatrix}$

Angle of elliptical: $\alpha = \frac{1}{2} \tan^{-1}\left(\frac{2AB\cos\delta}{A^2 - B^2}\right)$

$E_\alpha = |E_{eff}| \times \sqrt{A^2 \cos^2\alpha + B^2 \sin^2\alpha + AB\cos\delta \sin 2\alpha}$

$E_{\alpha \pm 90°} = |E_{eff}| \times \sqrt{A^2 \sin^2\alpha + B^2 \cos^2\alpha - AB\cos\delta \sin 2\alpha}$

Linear polarizer: $\begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$

$\frac{\lambda}{4}$: $\begin{pmatrix} \cos^2\theta + i\sin^2\theta & \sin\theta\cos\theta - i\sin\theta\cos\theta \\ \sin\theta\cos\theta - i\sin\theta\cos\theta & \sin^2\theta + i\cos^2\theta \end{pmatrix}$

$\frac{\lambda}{2}$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

Fourier, Delta, Convolution

$f(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt$

$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(\omega) e^{-i\omega t} d\omega$

$E_0 e^{-t^2/2T^2} e^{-i\omega_0 t} \Leftrightarrow E_0 T e^{-T^2(\omega - \omega_0)^2/2}$

$\delta(t - t_0) \Leftrightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega(t - t_0)} d\omega$

$a(t) \otimes b(t) = \int_{-\infty}^{\infty} a(t') b(t - t') dt'$

Linear dispersion

$v_g \approx \left(\frac{d}{d\omega} k_{real}\Big|_{\omega = \omega_0}\right)^{-1}$

$t' \approx \frac{d}{d\omega} \mathbf{k}_{\mathbf{real}}\Big|_{\omega = \omega_0} \cdot \Delta\mathbf{r}$

$I \sim e^{-2\mathbf{k}_{\mathbf{imag}}(\omega_0) \cdot \Delta\mathbf{r}} |E(t - t', \mathbf{r_0})|^2$

Quadratic dispersion

$k = k_0 + \frac{1}{v_g}(\omega - \omega_0) + \alpha(\omega - \omega_0)^2$

$\frac{1}{v_g} = \frac{1}{c}(n'\omega + n)|_{\omega = \omega_0}$

$\alpha = \frac{1}{2c}(n''\omega + 2n')|_{\omega = \omega_0}$

Gaussian wavepacket, through thickness z:

$\Phi = 2\alpha z/T^2$

$\tilde{T} = T\sqrt{1 + \Phi^2}$

$E(t, z) = \frac{E_0 e^{i(kz - \omega_0 t)}}{(1 + \Phi^2)^{1/4}} \exp\left(\frac{i}{2} \tan^{-1}\Phi - \frac{i}{2} \frac{\Phi}{\tilde{T}^2} \left(t - \frac{z}{v_g}\right)^2\right) \exp\left(-\frac{1}{2\tilde{T}^2} \left(t - \frac{z}{v_g}\right)^2\right)$

Michelson, Temporal Coherence

Single ω: $I_{det}(\tau) = 2I_0(1 + \cos\omega\tau)$

Band of ω's:

$\varepsilon = \int_{-\infty}^{\infty} I(\omega) d\omega$

$\gamma(\tau) = \frac{1}{\varepsilon} \int_{-\infty}^{\infty} I(\omega) e^{-i\omega\tau} d\omega$

$\text{Sig}(\tau) \sim 2\varepsilon(1 + \text{Re}\,\gamma(\tau))$

$V(\tau) = \text{visibility} = |\gamma(\tau)|$

$\tau_c = \int_{-\infty}^{\infty} |\gamma(\tau)|^2 d\tau$

$FT(\text{Sig}(\tau)) \sim 2\varepsilon_0 \delta(\omega) + I(\omega) + I(-\omega)$

Young, Spatial Coherence

Pt source: $I_{screen} = 2I_0\left(1 + \cos\left(\frac{kyh}{D}\right)\right)$

Extended source:

$\varepsilon = \int_{-\infty}^{\infty} I(y') dy'$

$\gamma(h) = \frac{1}{\varepsilon} e^{-ikyh/D} \int_{-\infty}^{\infty} I(y') e^{-ikhy'/R} dy'$

$I_{screen} = 2\langle I_{oneslit} \rangle (1 + \text{Re}\,\gamma(h))$

$V(h) = \text{visibility} = |\gamma(h)|$

$h_c = \int_{-\infty}^{\infty} |\gamma(h)|^2 dh$

Rays: $\nabla R(\mathbf{r}) = n(\mathbf{r}) \hat{s}(\mathbf{r})$

ABCD Matrices

Translation: $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$

Flat surface refraction: $\begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix}$

Curved surface refraction:

$\begin{pmatrix} 1 & 0 \\ \frac{1}{R}(n_1/n_2 - 1) & n_1/n_2 \end{pmatrix}$

R = + for convex, – for concave

Spherical mirror/thin lens: $\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$

$f_{lens} = ((n_2/n_1 - 1)(1/R_1 - 1/R_2))^{-1}$;

R = + for convex surface, – for concave

$f_{mirror} = R/2$; R = + for concave

$p_1 = (1 - D)/C, \ p_2 = (1 - A)/C, \ f = -1/C$

Cavity stability: $-1 < \frac{A + D}{2} < 1$

Diffraction formulas

Helmholtz Eq.: $\nabla^2 \mathbf{E}(\mathbf{r}) = -k^2 \mathbf{E}(\mathbf{r})$

Fresnel: $E(x, y, z) = -\frac{i e^{ikz} e^{ik(x^2 + y^2)/2z}}{\lambda z} \times \iint_{aper} E(x', y', 0) e^{ik(x'^2 + y'^2)/2z} e^{-ik(xx' + yy')/z} dx' dy'$

Fraunhofer: $E(x, y, z) = -\frac{i e^{ikz} e^{ik(x^2 + y^2)/2z}}{\lambda z} \times \iint_{aper} E(x', y', 0) e^{-ik(xx' + yy')/z} dx' dy'$

More Fourier Transforms

Comb function (N deltas): $FT = \frac{1}{\sqrt{2\pi}} \frac{\sin N\omega t_0/2}{\sin \omega t_0/2}$

Single slit: $FT = \frac{1}{\sqrt{2\pi}} a \,\text{sinc}(k_x a/2)$

Rectangle: $FT = \frac{1}{2\pi} ab \,\text{sinc}(k_x a/2) \,\text{sinc}(k_y b/2)$

Top hat: $FT = \frac{a^2}{2} \frac{2\,\text{J}_1(k_\rho a)}{k_\rho a} = \frac{a^2}{2} \,\text{jinc}(k_\rho a)$

Spectrometer: $\lambda = \frac{xh}{mz}, \ \Delta\lambda = \frac{\lambda}{mN}$

Rayleigh: $\theta_{min} \approx \frac{1.22\lambda}{D}$

Gaussian Beams

$E(x, y, z) = E_0 \frac{w_0}{w} \exp\left(-\frac{\rho^2}{w^2}\right) \times \exp\left(ikz + \frac{ik\rho^2}{2R} - i\tan^{-1}\left(\frac{z}{z_0}\right)\right)$

$z_0 = \frac{k w_0^2}{2}, \ R = z + \frac{z_0^2}{z}, \ w = w_0 \sqrt{1 + \frac{z^2}{z_0^2}}$

$q_2 = \frac{A q_1 + B}{C q_1 + D}, \ q = z + i z_0$

Plank Blackbody Formulas:

$E = \frac{hf}{\exp(hf/k_B T) - 1}$

$u = \frac{8}{15} \frac{\pi^5 k_B^4}{h^3 c^3} T^4$

$I = \frac{2}{15} \frac{\pi^5 k_B^4}{h^3 c^2} T^4 = \sigma T^4$

$\rho_f = \frac{8\pi h f^3/c^3}{\exp(hf/k_B T) - 1}$

$\rho_\lambda = \frac{8\pi hc/\lambda^5}{\exp(hc/\lambda k_B T) - 1}$

$\lambda_{max} = \frac{0.00290}{T}$ m · K

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