Converting T^{02} into potentially more useful form

start with

$$T^{02} = \frac{n_2}{n_0}\frac{\cos\theta_2}{\cos\theta_0}\frac{|t^{01}|^2|t^{12}|^2}{|e^{-ik_1 d\cos\theta_1} - r^{10}r^{12}e^{ik_1 d\cos\theta_1}|^2} \qquad \text{Eqn 4.14}$$

$$\text{denominator} = \left(e^{-ik_1 d\cos\theta_1} - r^{10}r^{12}e^{ik_1 d\cos\theta_1}\right) \times \text{complex conjugate}$$

$$= (\quad '' \quad\quad '' \quad)\left(e^{+ik_1 d\cos\theta_1} - r^{10*}r^{12*}e^{-ik_1 d\cos\theta_1}\right)$$

$$(\text{FOIL}) \quad = 1 \underbrace{- e^{2ik_1 d\cos\theta_1}r^{10}r^{12} - e^{-2ik_1 d\cos\theta_1}r^{10*}r^{12*}}_{\substack{= -(\text{something} + \text{complex conjugate}) \\ = -2 \times \text{Real part}(\text{something})}} + |r^{10}|^2|r^{12}|^2$$

$$\text{Write } r^{10} = |r^{10}|e^{i\delta_{10}} \qquad \delta = 2k_1 d\cos\theta_1$$

$$r^{12} = |r^{12}|e^{i\delta_{12}}$$

$$= 1 + |r^{10}|^2|r^{12}|^2 - 2\,\text{Real}\Big[\underbrace{|r^{10}|e^{i\delta_{10}}|r^{12}|e^{i\delta_{12}}e^{i\delta}}_{= |r^{10}||r^{12}|e^{i(\delta_{10}+\delta_{12}+\delta)}}\Big]$$

$$= 1 + |r^{10}|^2|r^{12}|^2 - 2|r^{10}||r^{12}|\underbrace{\cos(\delta_{10}+\delta_{12}+\delta)}_{\text{Trig trick: } \cos\phi = 1 - 2\sin^2\frac{\phi}{2}}$$

$$= \underbrace{1 - 2|r^{10}||r^{12}| + |r^{10}|^2|r^{12}|^2}_{(1-|r^{10}||r^{12}|)^2} + 4|r^{10}||r^{12}|\sin^2\frac{\phi}{2}$$

$$\boxed{\phi = \delta_{10} + \delta_{12} + 2k_1 d\cos\theta_1}$$

$$\text{denom.} = (1 - |r^{10}||r^{12}|)^2 + 4|r^{10}||r^{12}|\sin^2\frac{\phi}{2}$$

Put back in to T^{02} eqn

$$T^{02} = \frac{n_2}{n_0}\frac{\cos\theta_2}{\cos\theta_0}\frac{|t^{01}|^2|t^{12}|^2}{(1-|r^{10}||r^{12}|)^2 + 4|r^{10}||r^{12}|\sin^2\frac{\phi}{2}} \times \frac{\frac{1}{(1-|r^{10}||r^{12}|)^2}}{\frac{1}{(1-|r^{10}||r^{12}|)^2}}$$

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Converting T^{02}, continued

$$T^{02} = \frac{\frac{n_2}{n_0}\frac{\cos\theta_2}{\cos\theta_0}\frac{|t^{01}|^2|t^{12}|^2}{(1-|r^{10}||r^{12}|)^2}}{1 + \frac{4|r^{10}||r^{12}|}{(1-|r^{10}||r^{12}|)^2}\sin^2\frac{\phi}{2}}$$

or, using some other symbols to make it look simpler...

$$\boxed{T^{02} = \frac{T^{max}}{1 + F\sin^2\frac{\phi}{2}}}$$ Eqn 4.15

with $$\boxed{T^{max} = \frac{n_2}{n_0}\frac{\cos\theta_2}{\cos\theta_0}\frac{|t^{01}|^2|t^{12}|^2}{(1-|r^{10}||r^{12}|)^2}}$$ Eqn 4.16 equivalent

$$\boxed{F = \frac{4|r^{10}||r^{12}|}{(1-|r^{10}||r^{12}|)^2}}$$ Eqn 4.18 called "coefficient of finesse"

and ϕ as defined on previous page,

$$\boxed{\phi = \underbrace{\delta_{10}}_{\text{phase of } r^{10}} + \underbrace{\delta_{12}}_{\text{phase of } r^{12}} + 2k_1 d\cos\theta_1}$$ Eqn 4.17

Further Note: my T^{max} equation doesn't look exactly like eqn 4.16.

To complete things, write numerator like this:

$$T^{max}_{numer.} = \frac{n_2}{n_1}\frac{\cos\theta_2}{\cos\theta_1}|t^{12}|^2 \; \frac{n_1}{n_0}\frac{\cos\theta_1}{\cos\theta_0}|t^{01}|^2$$

(because n_1's and $\cos\theta_1$'s will cancel out)

$$= T_{12}T_{01}$$

Write $|r^{10}|$ as $\sqrt{R^{10}}$ and $|r^{12}|$ as $\sqrt{R^{12}}$, then

(alternate version) $$\boxed{T_{max} = \frac{T_{01}T_{12}}{(1-\sqrt{R^{10}R^{12}})^2}}$$ Eqn 4.16

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