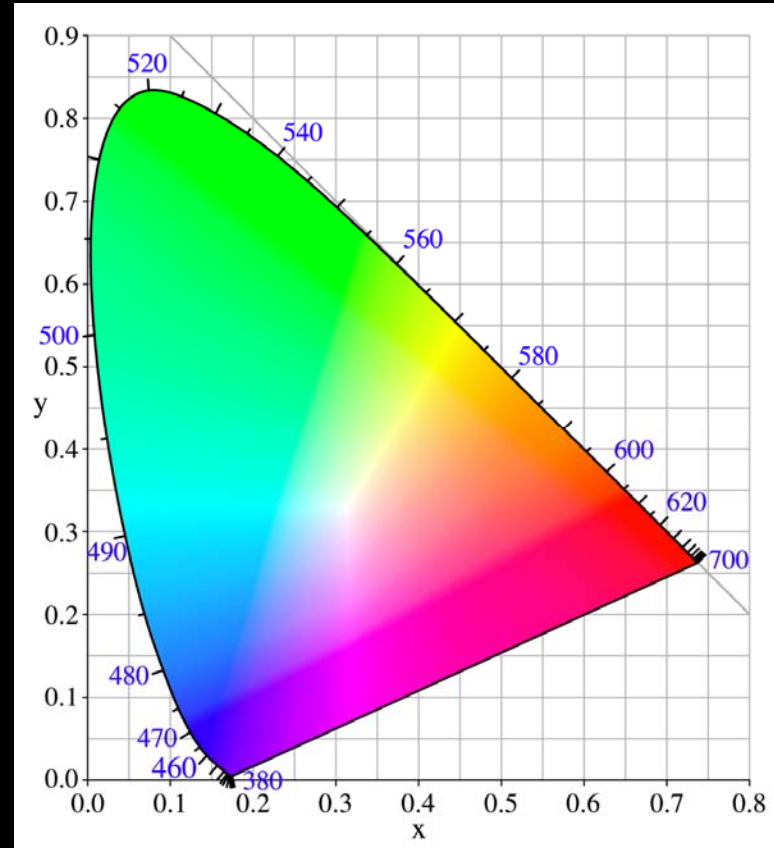


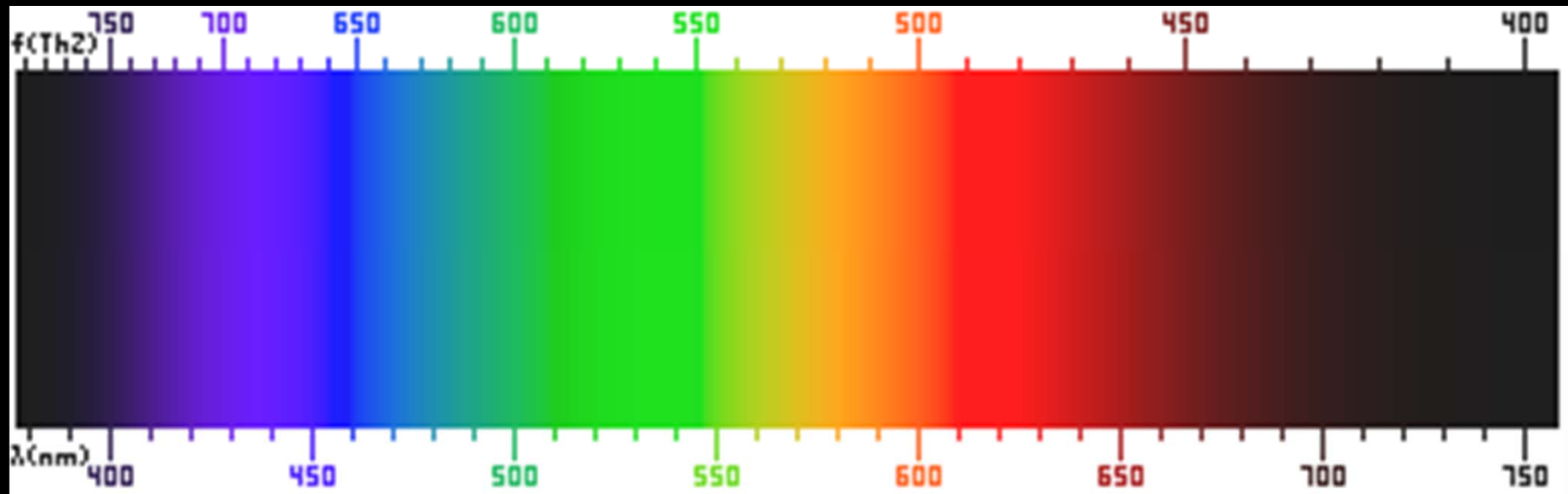
Color! Main Goals:

- Understand this thing:
"Chromaticity diagram"



- Given a spectrum, how to predict what color the spectrum will seem to you

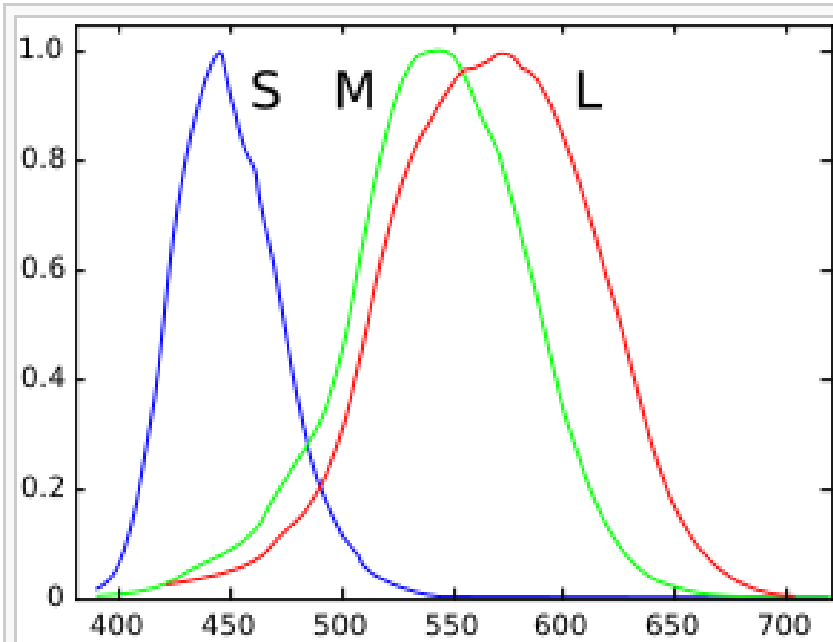
Visible Spectrum



From Wikipedia, “Visible Spectrum”

- “All the colors of the rainbow...”
→ Where is brown?? Where is pink?? Where is turquoise??

Cone cells

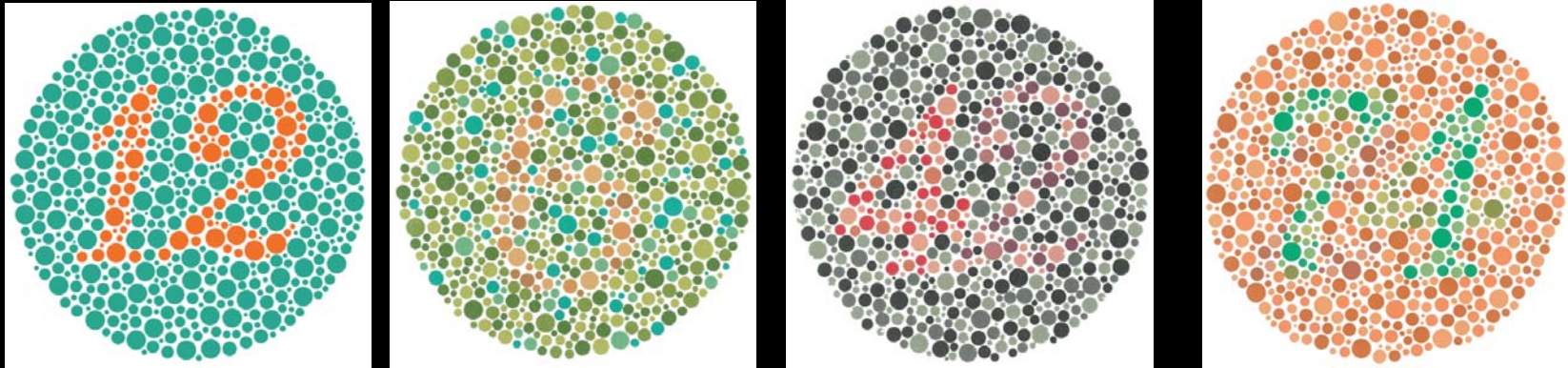


Normalized response spectra of human cones, S, M, and L types, to monochromatic spectral stimuli

From Wikipedia,
“Color Vision”

- “Short”
- “Medium”
- “Long”

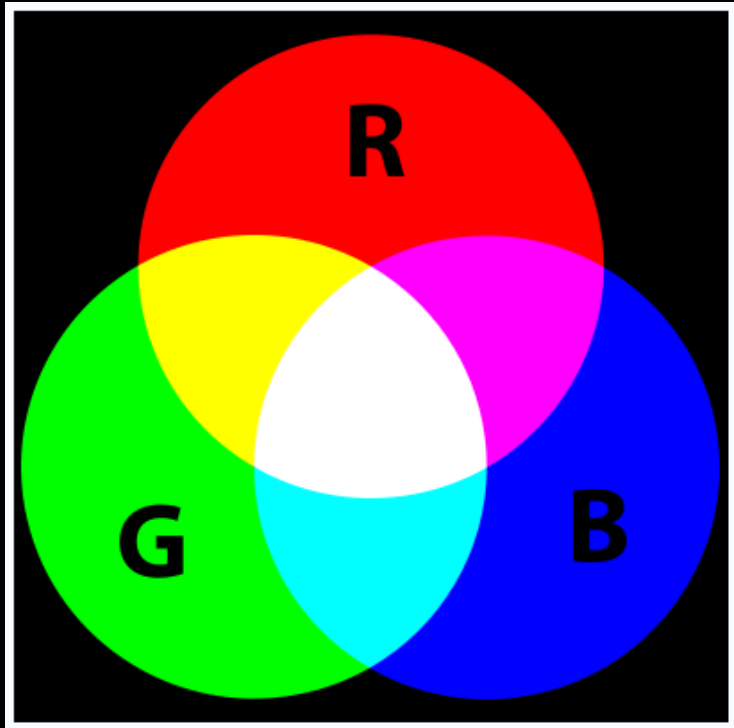
Color blindness



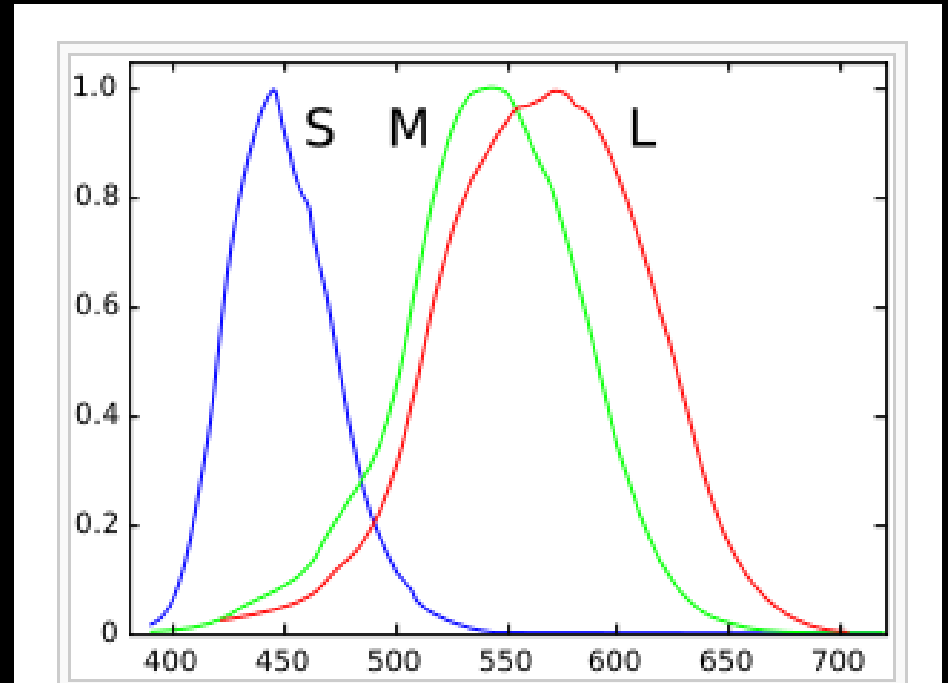
From Wikipedia, "Ishihara test"

- tritanopia – lacks S cones, <1% of males and females
- tritanomaly – S cones mutated, ~0.01% of males and females
- deuteranopia – lacks M cones, ~1% of males
- deuteranomaly – M cones mutated, standard "red-green color blindness", ~5% of European males; far fewer females
- protanopia – lacks L cones, ~1% of males
- protanomaly – L cones mutated, ~3% of European males, far fewer females
- achromatopsia – total color blindness, 0.003% of males and females

Primary Colors



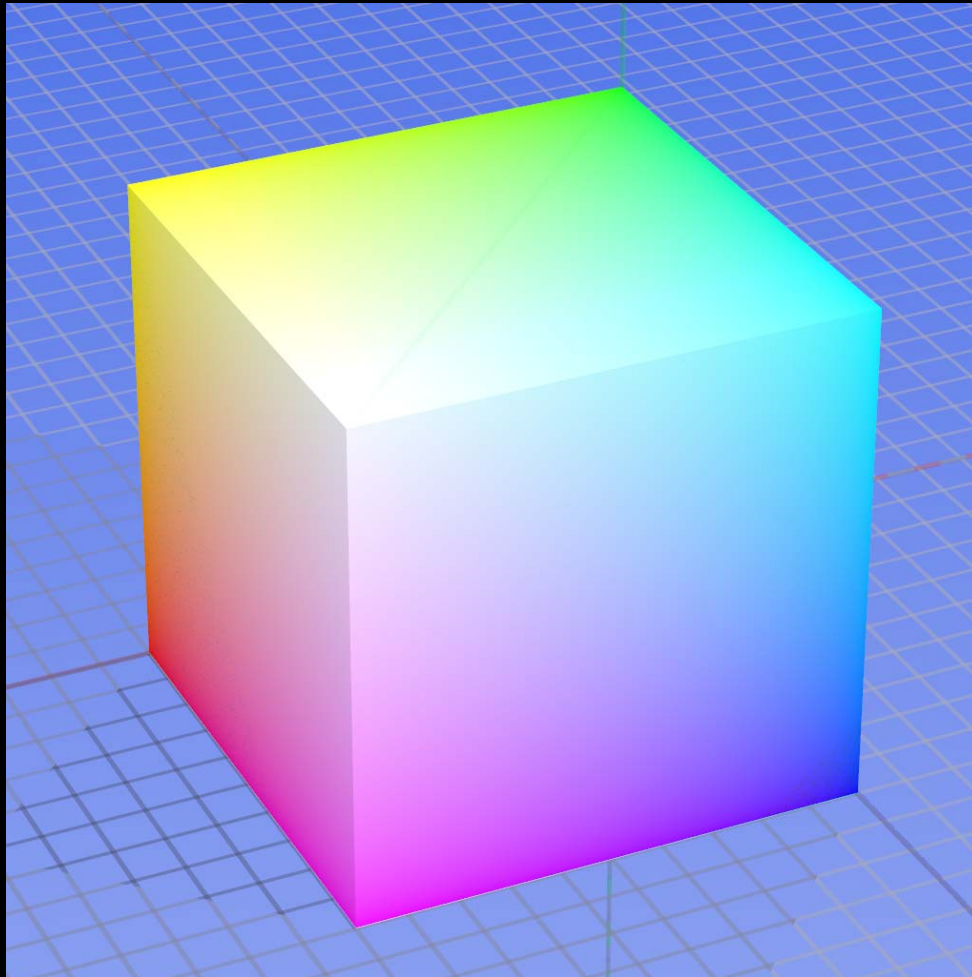
From Wikipedia,
“RGB Color Model”



Cone cell response, again

- How the primary song *should* go
- “Additive color mixing”
- (Pigments: “subtractive color mixing”)

Components of R, G, B: Plot in 3D “color space”

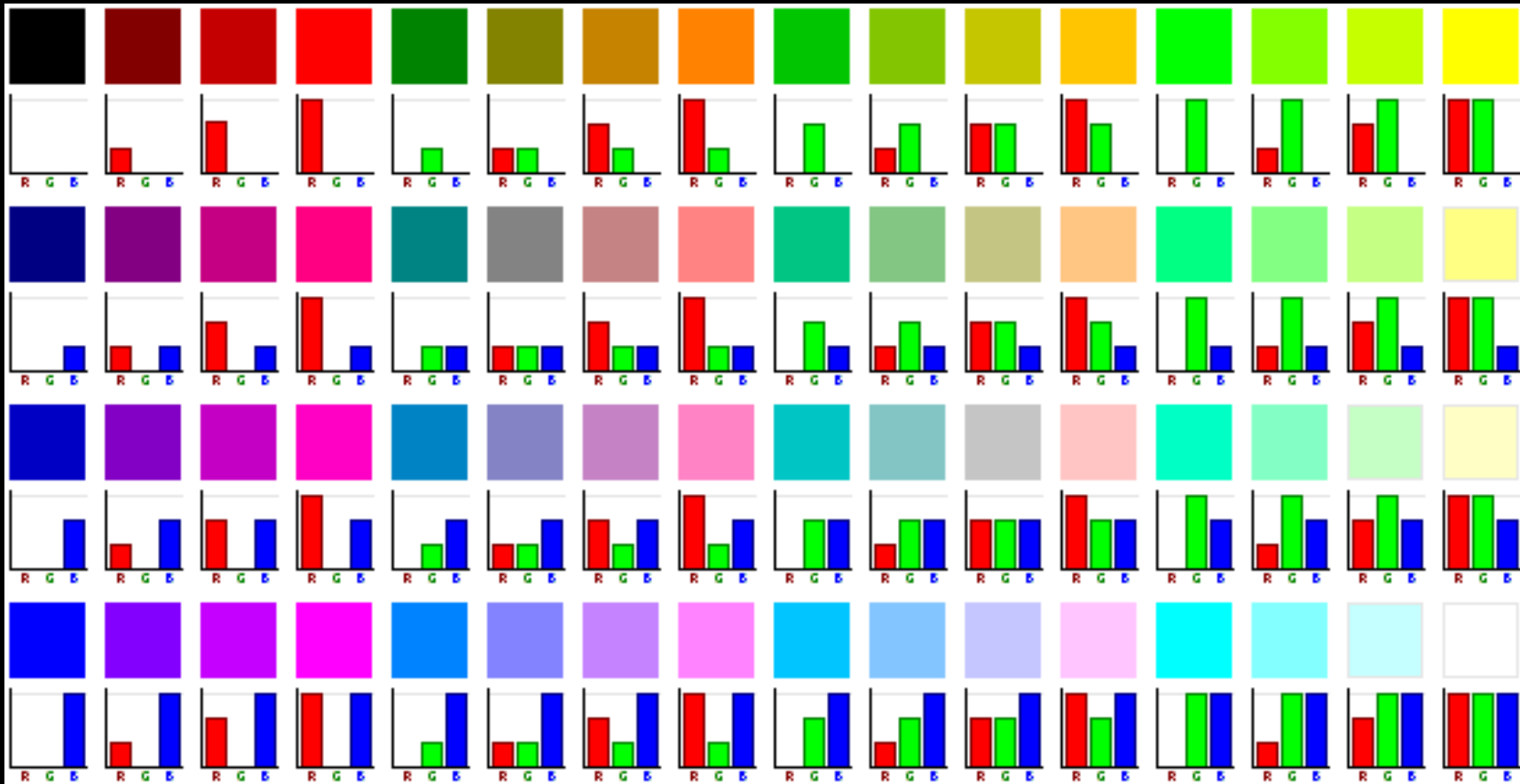


From Wikipedia,
“RGB Color Model”
(old version)

Viewing slices of the cube:

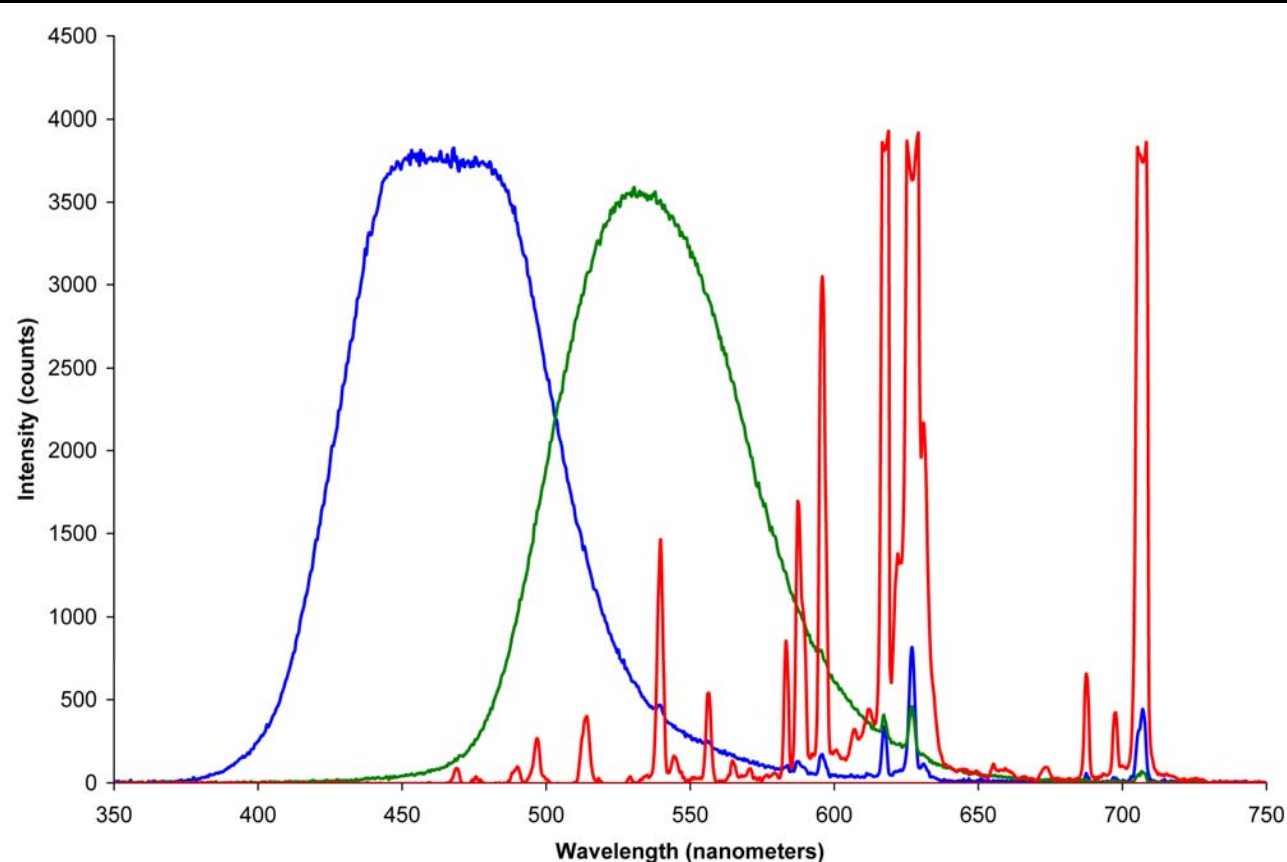
<https://programmingdesignsystems.com/color/color-models-and-color-spaces/index.html>

Components of R, G, B



From Wikipedia,
“RGB Color Model”
(old version)

How to Display Colors



The **emission spectra** of the three **phosphors** that define the **additive primary colors** of a **CRT** color video display. Other electronic color display technologies (**LCD**, **Plasma display**, **OLED**) have analogous sets of primaries with different emission spectra.

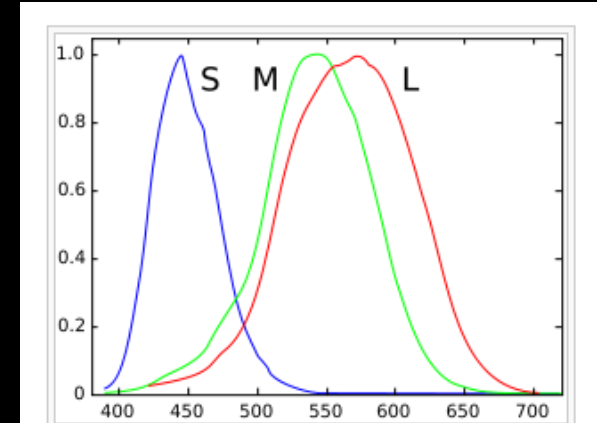
From Wikipedia, "Primary Color"

1920's Color Matching Experiments

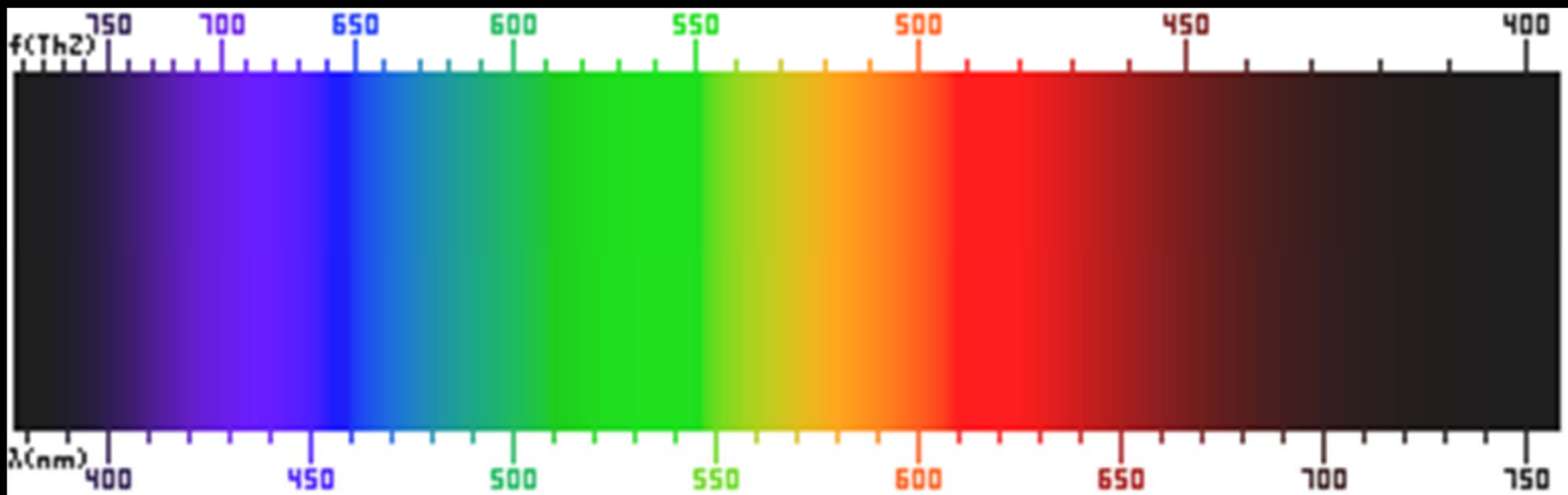
Using combinations of these three:

- Narrow red source at 700 nm
- Narrow green source at 546.1 nm
- Narrow blue source at 435.8 nm

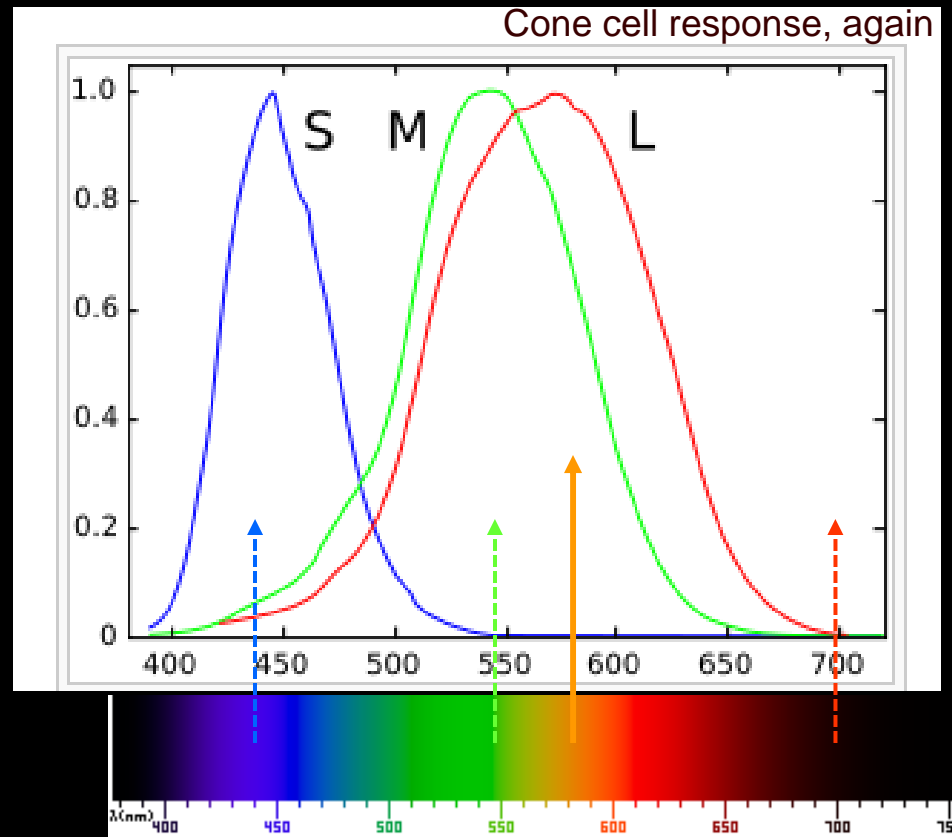
How much of each is required to match the wavelengths in the visible spectrum (“pure colors”)?



Cone cell response, again



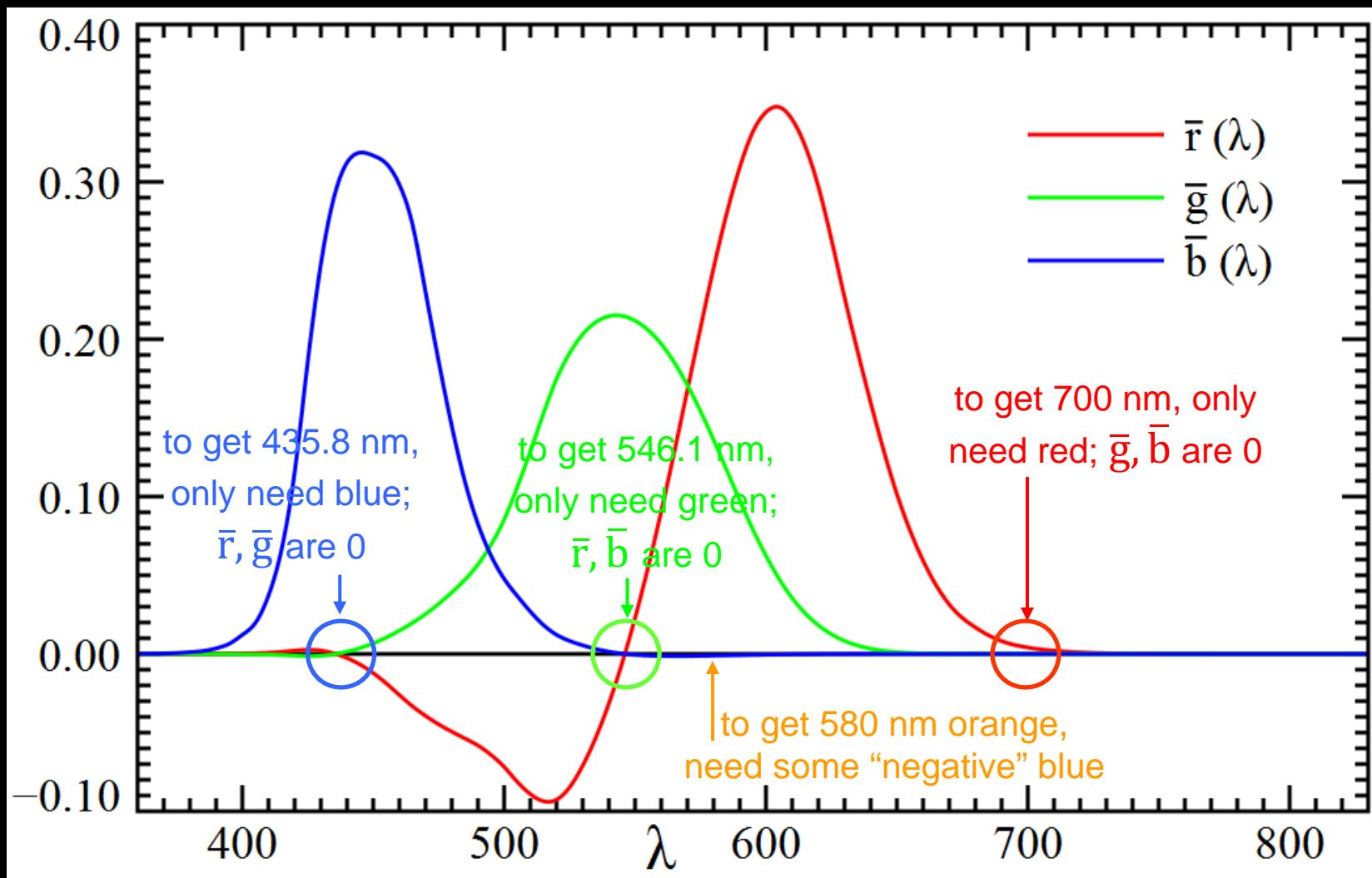
A Concern



- Say you want to mix the dashed lines to look the same as 580 nm orange line.
- You may start by turning up the red light. But soon you also need to turn up the green light. However...
 - Green light will excite some S! (a small amount, but nonzero)
 - 580 nm alone will never excite S! Therefore 580 nm cannot be matched.
 - You need some "negative blue" to counteract

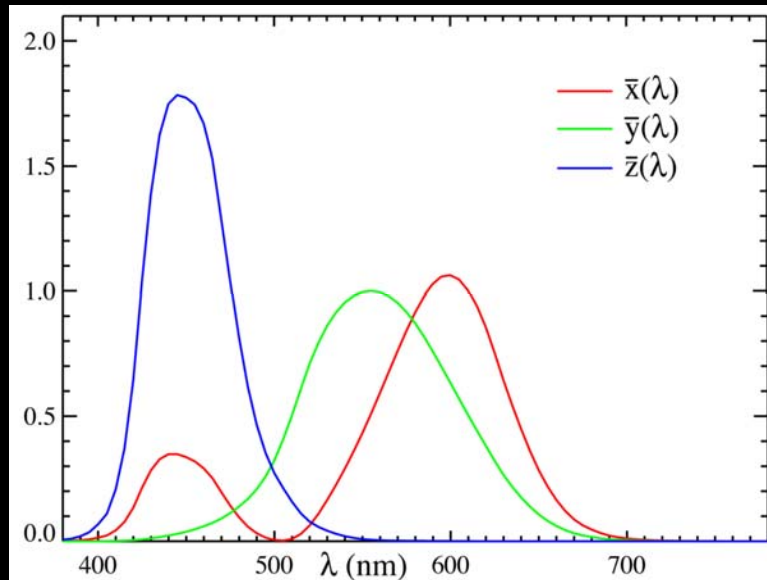
Results: \bar{r} , \bar{g} , \bar{b} functions

red source = 700 nm green source = 546.1 nm blue source at 435.8 nm

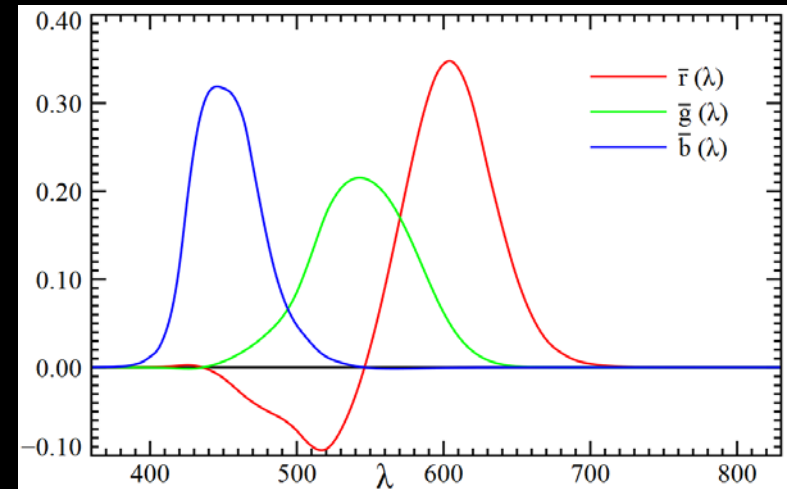


From Wikipedia, "1931 Color Space" (also in P&W)

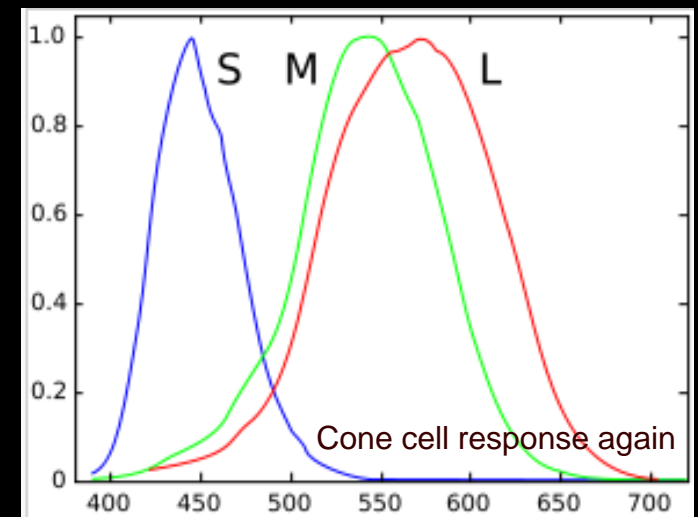
$\bar{x}, \bar{y}, \bar{z}$ functions vs. $\bar{r}, \bar{g}, \bar{b}$ functions



From Wikipedia, "CIE 1931 Color Space"
(Very Important, but not in Peatross & Ware)



- all are positive
- \bar{z} = close to S cones, close to \bar{b}
- \bar{y} = matches intensity response of eye, close to M cones
- \bar{x} = chosen so that white is equal parts of all three



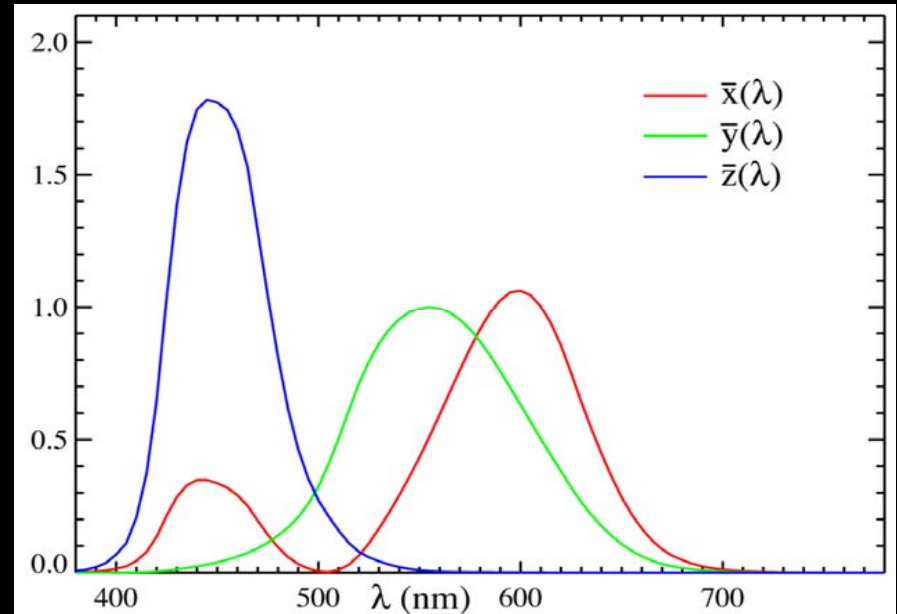
Projections

- Given a spectrum $I(\lambda)$, how much \bar{x} , \bar{y} , and \bar{z} does it have?

$$X = \int I(\lambda) \bar{x}(\lambda) d\lambda$$

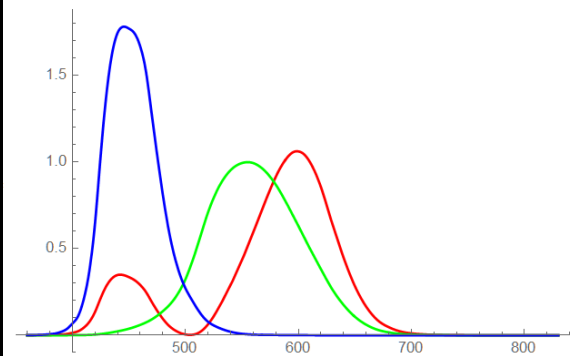
$$Y = \int I(\lambda) \bar{y}(\lambda) d\lambda$$

$$Z = \int I(\lambda) \bar{z}(\lambda) d\lambda$$

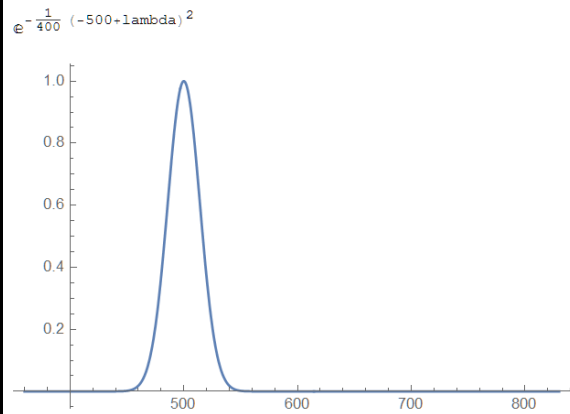


Example from homework (P2.13 part b)

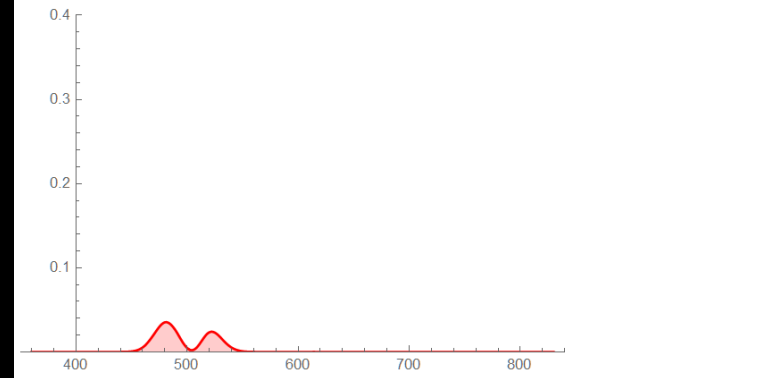
Plot of the color matching functions:



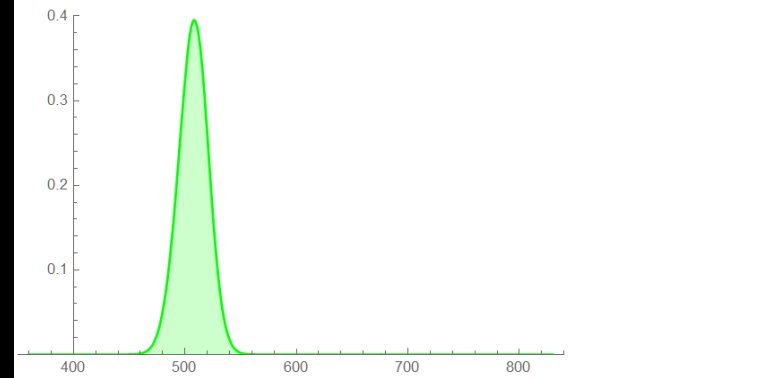
Plot of the given intensity function:



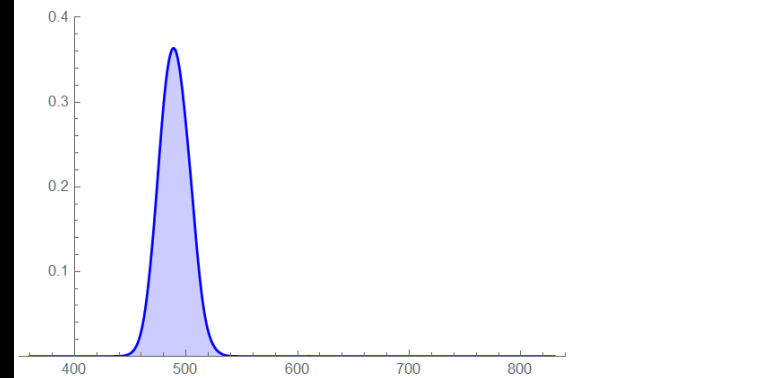
Projection of the intensity function onto the X color matching function



Projection of the intensity function onto the Y color matching function



Projection of the intensity function onto the Z color matching function



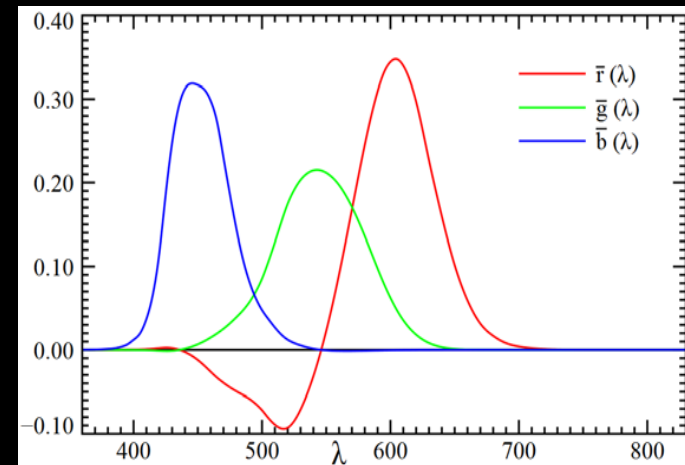
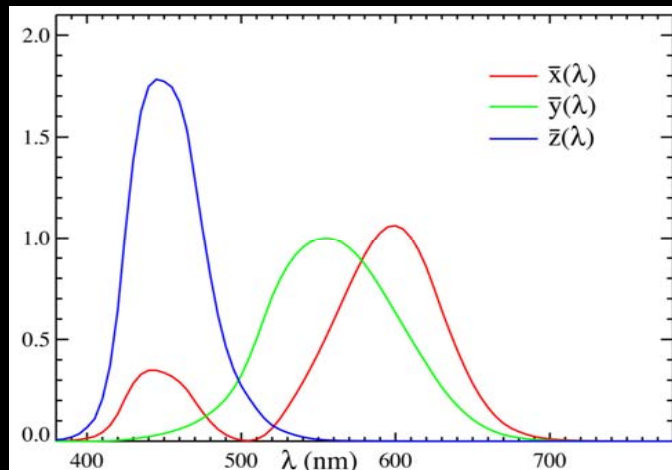
Calculate the areas: X, Y, Z
→ Then normalize so they add up to 1 (call them x, y, z)

Side Note: Linear Transformations of Projection coordinates (X,Y,Z) and (R,G,B)

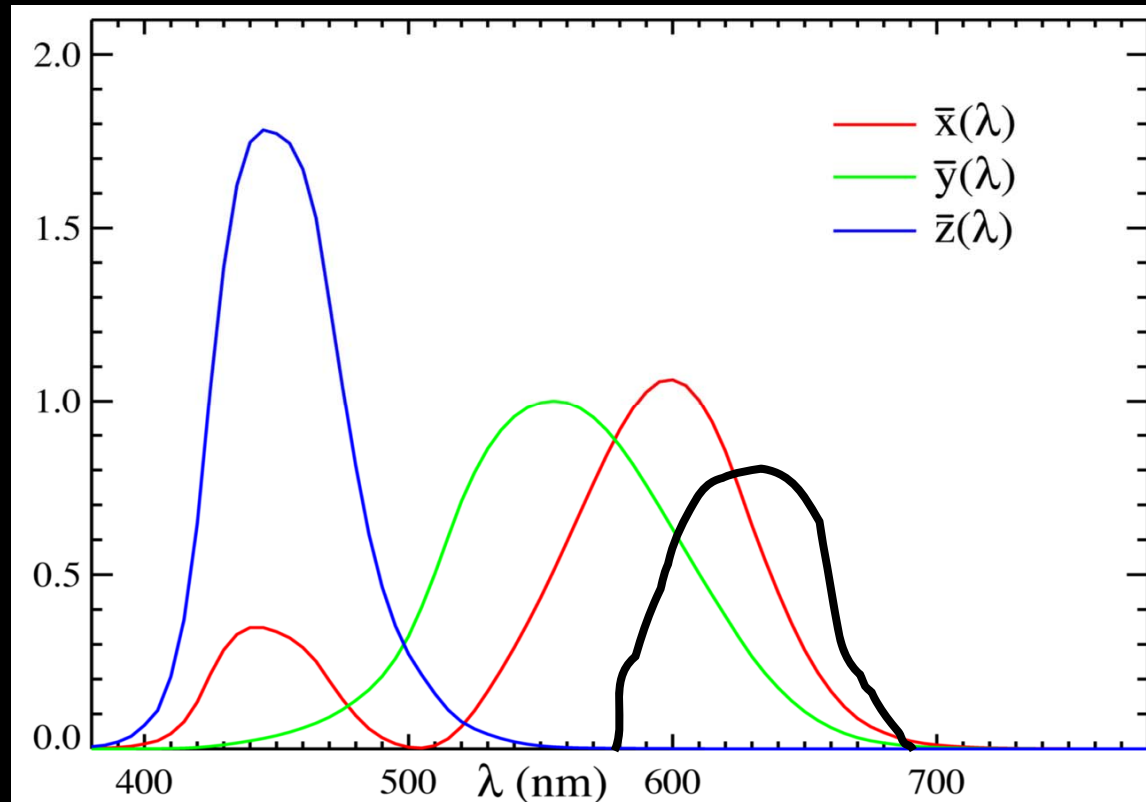
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{0.17697} \begin{bmatrix} 0.49 & 0.31 & 0.20 \\ 0.17697 & 0.81240 & 0.01063 \\ 0.00 & 0.01 & 0.99 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

From P&W Example 2.4

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 0.4185 & -0.1587 & -0.08283 \\ -0.09117 & 0.2524 & 0.01571 \\ 0.0009209 & -0.002550 & 0.1786 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



Worked Example



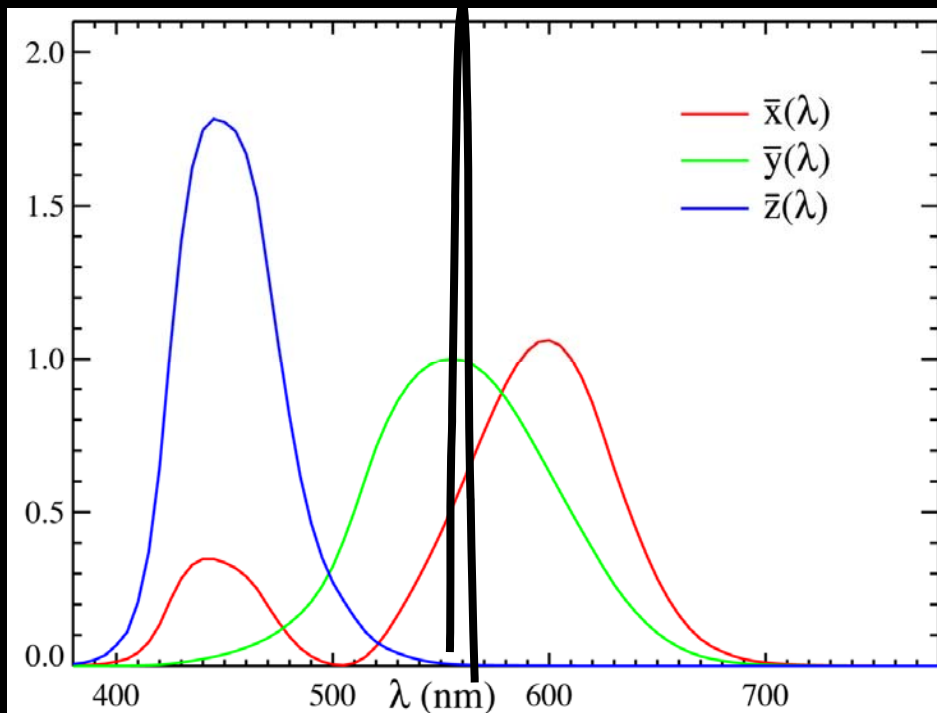
- $X =$
- $Y =$
- $Z =$

Normalize so they add up to 1 ("color" should not depend on overall intensity)

- $x =$
- $y =$
- $z = 1 - x - y =$

Another Worked Example

- What is (x,y) for a delta function at 560 nm?



My estimates:

$$X = 0.59$$

$$Y = 0.98$$

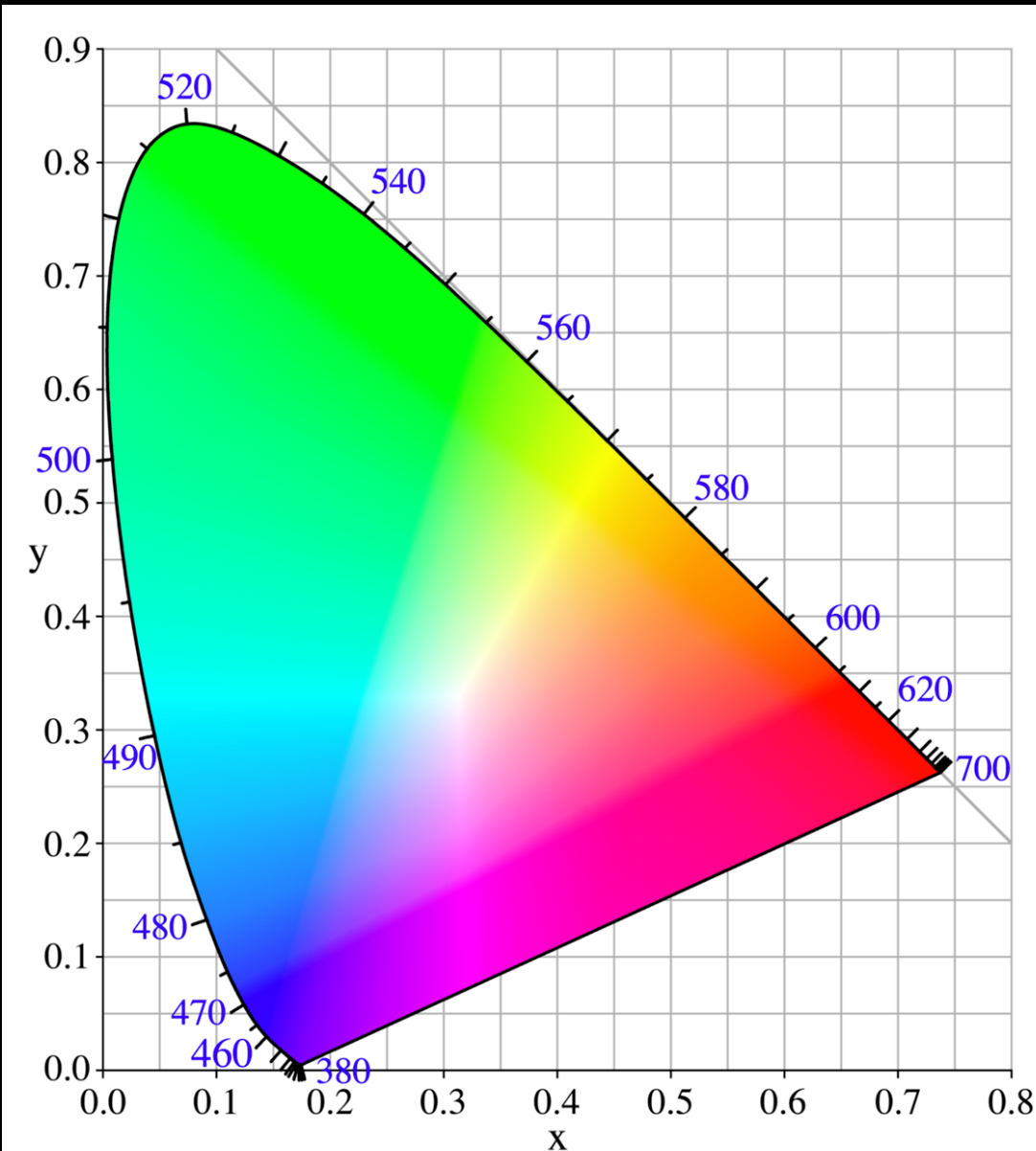
$$Z = 0$$

$$x = 0.38$$

$$y = 0.62$$

Do that for every wavelength \rightarrow the “locus” curve

Chromaticity Diagram



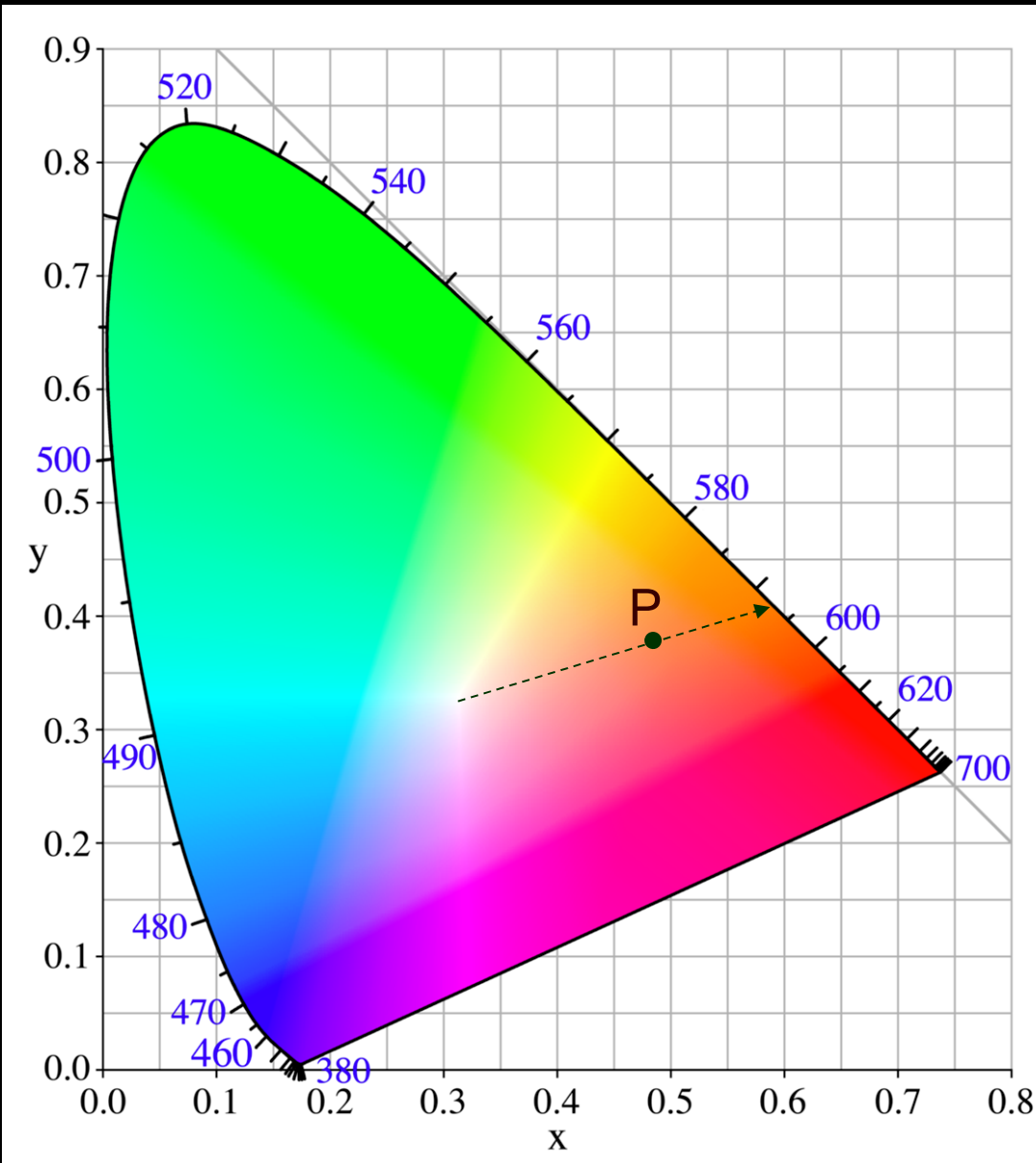
Things to observe

- The locus curve
Example: 560 nm = (0.38, 0.62)
- The white point = (0.33, 0.33)
- "Line of purples"

Linear effects

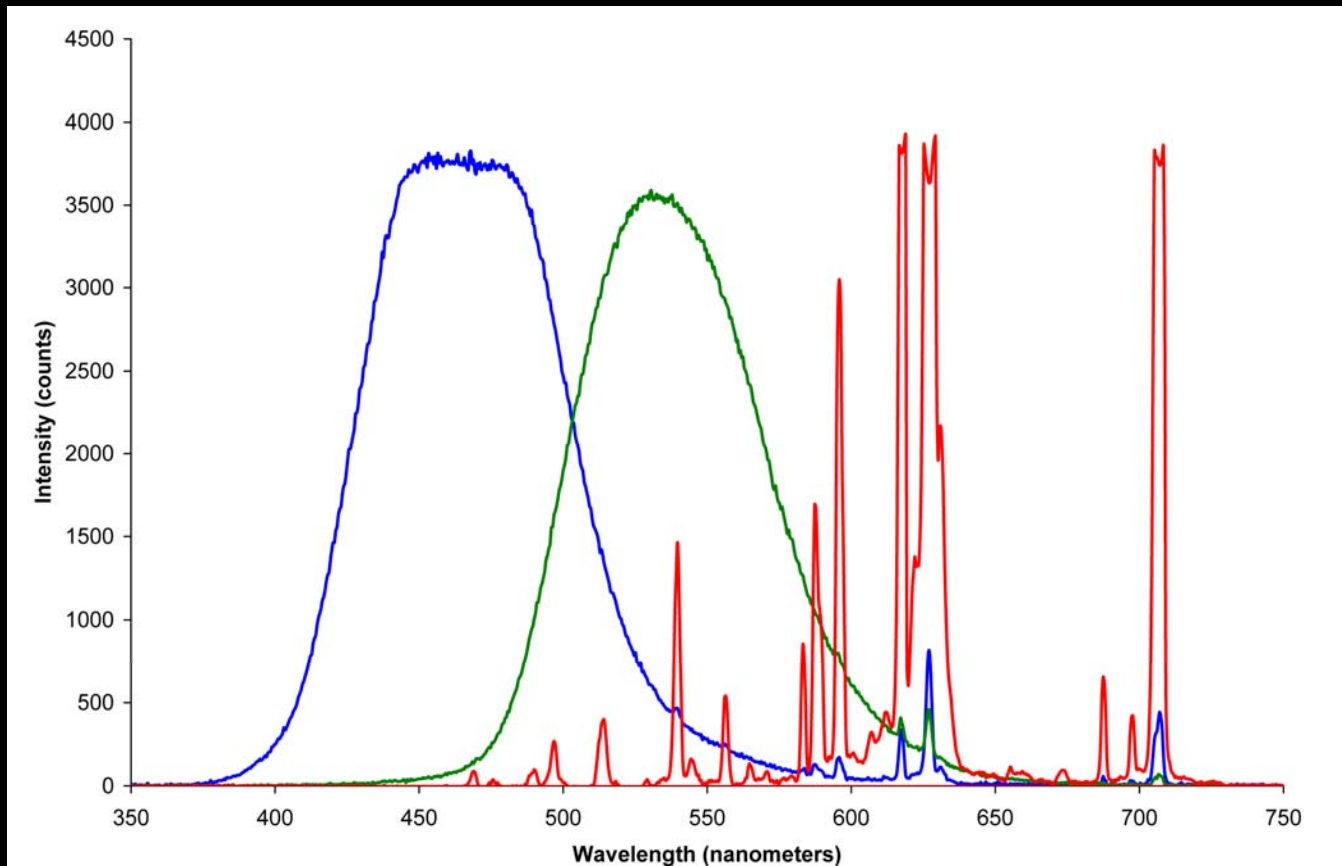
- Color mixing along line connecting two points
- "Complementary colors": can mix to get white

Chromaticity Diagram



- Hue
 - a. Draw the line connecting the white point to P, what λ does it hit? (Example: 593 nm)
- Saturation
 - a. How far along that line is the point? (Example: 65%)
- Brightness
 - a. Overall intensity, Not on this diagram

Remember this?

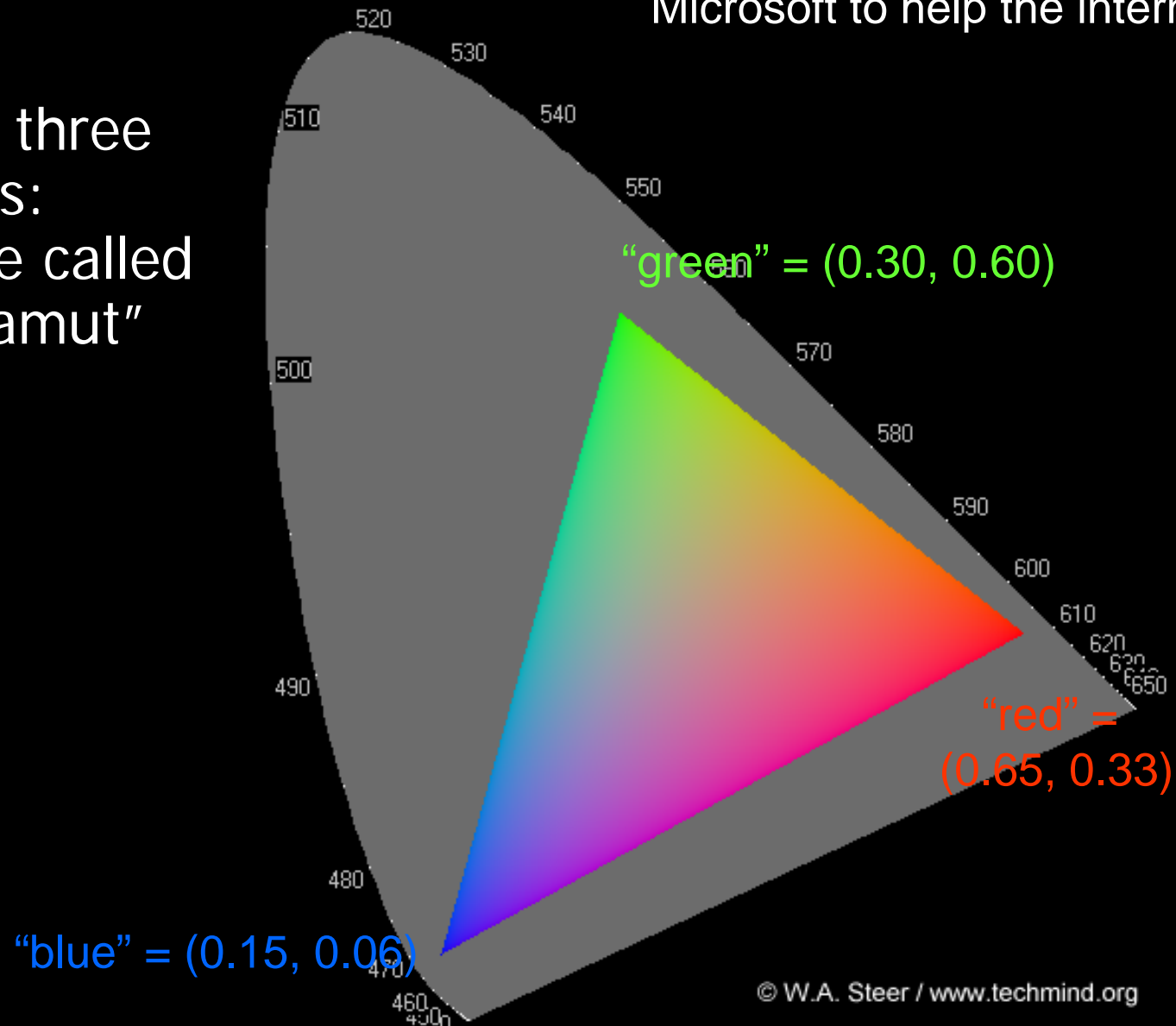


The **emission spectra** of the three **phosphors** that define the **additive primary colors** of a **CRT** color video display. Other electronic color display technologies (**LCD**, **Plasma display**, **OLED**) have analogous sets of primaries with different emission spectra.

sRGB: three specific color sources

Standard created in 1996 by HP and Microsoft to help the internet

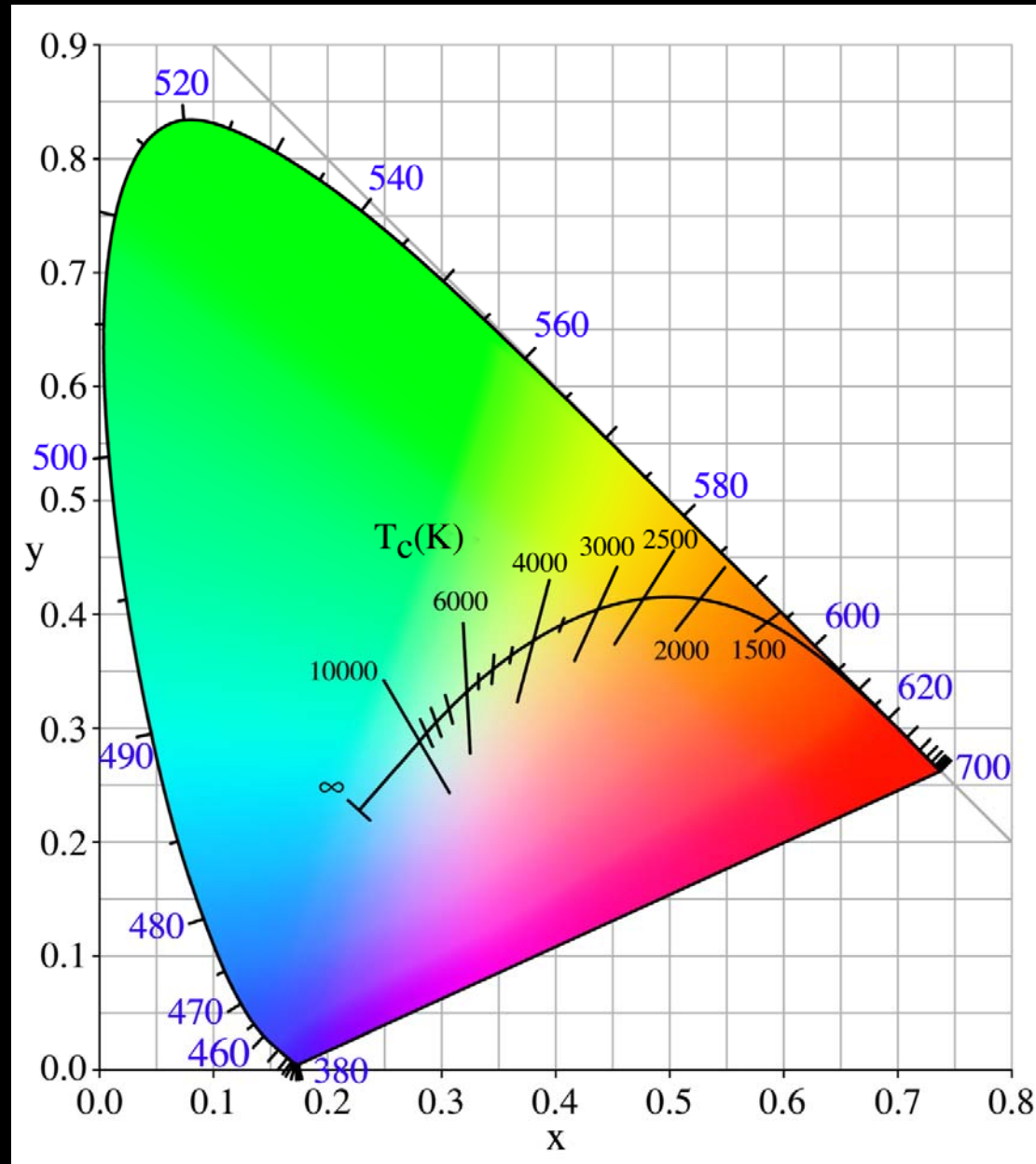
- Mixing three sources: triangle called the "gamut"



Review (end of Monday's class?)

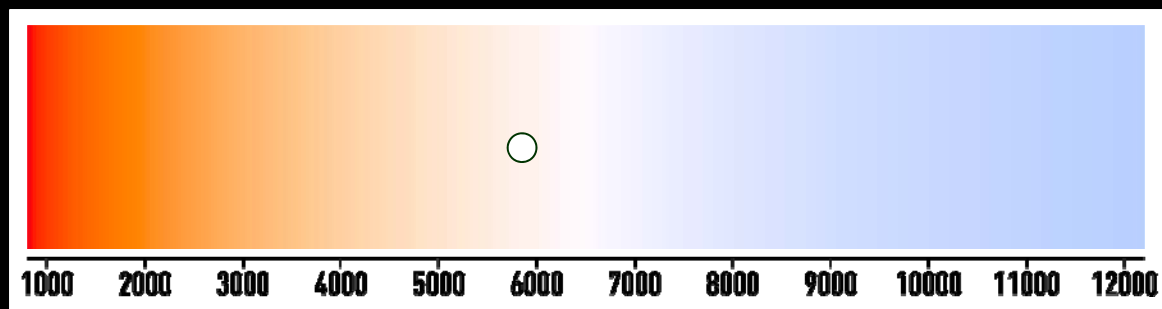
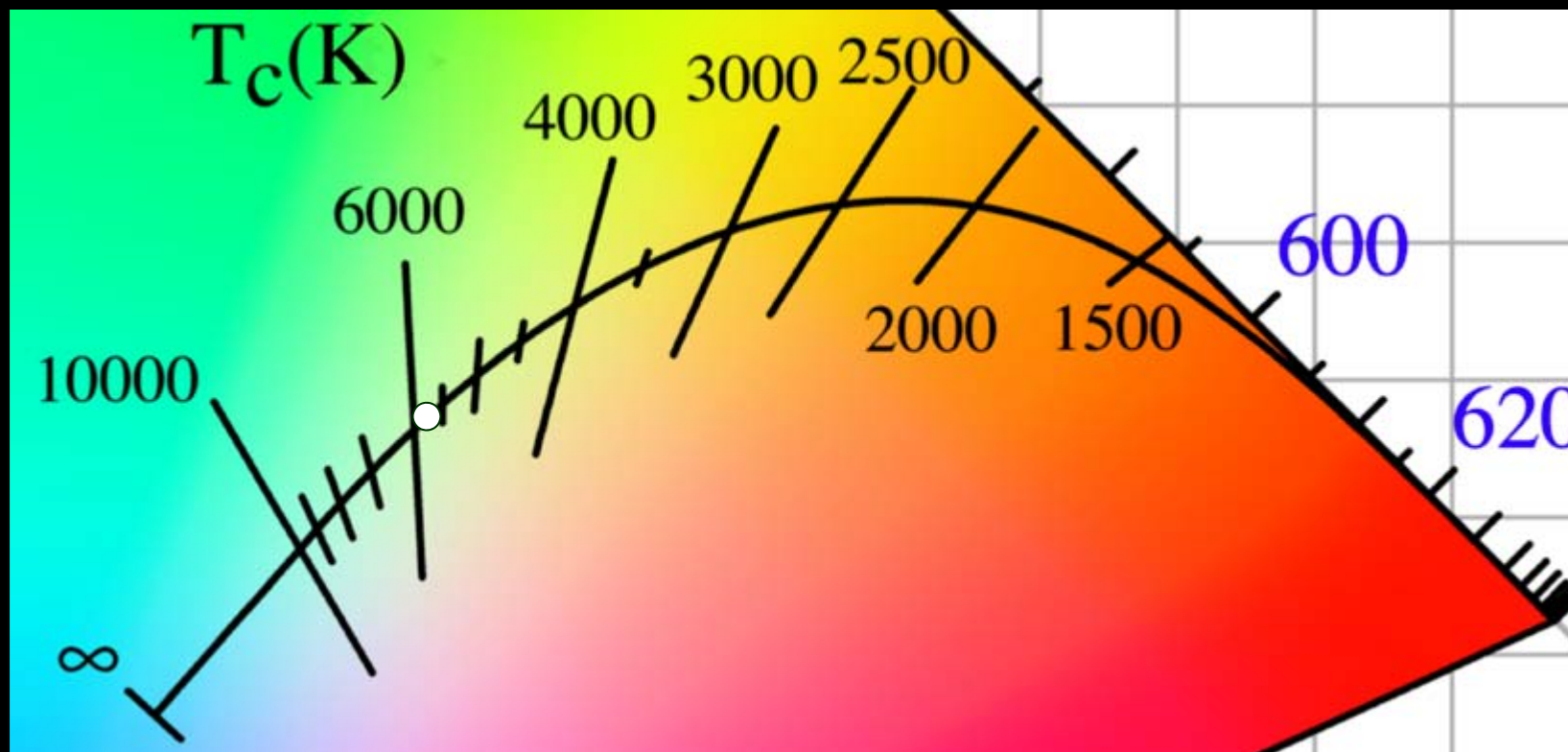
- Three types of cones responsive to three different wavelength ranges (short, medium, long, aka SML)
- Three dimensional color space (RGB)
- Color matching experiments produced \bar{r} , \bar{g} , \bar{b} "color matching functions"
- Alternate set developed with better properties, \bar{x} , \bar{y} , \bar{z}
- X, Y, Z are the projections of a given spectrum on to the $\bar{x}, \bar{y}, \bar{z}$ functions
- x, y, z are the normalized X, Y, Z values
- x, y are the chromaticity coordinates (z is superfluous), can be plotted on the chromaticity diagram
- Colors on the chromaticity diagram combine linearly
- Actual RGB sources fall on the chromaticity diagram, their enclosed triangle is the gamut of possible colors they can display

What's the Color of Blackbody Radiation?

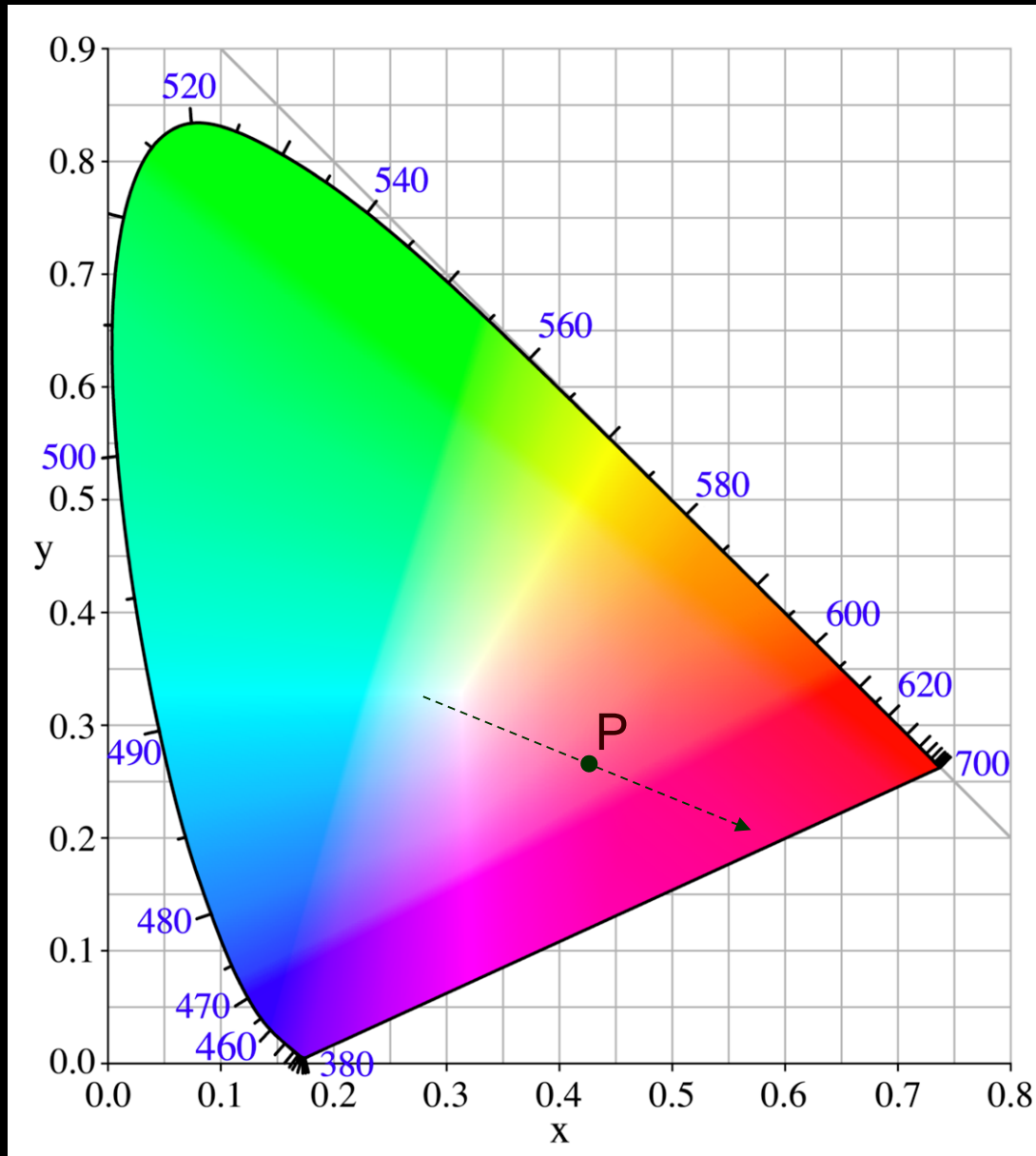


From Wikipedia
"Color Temperature"

Color of the Sun? $T = 5778\text{K}$



Complementary colors



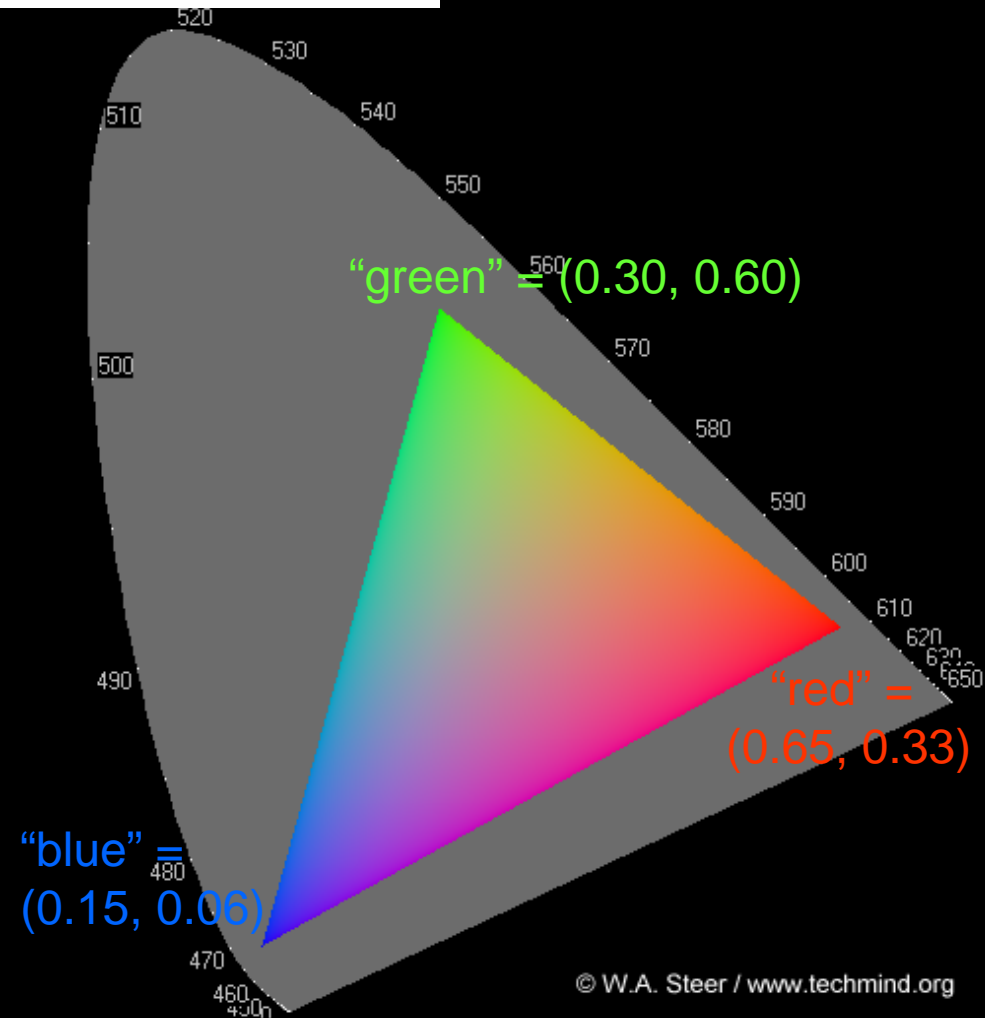
- What is the hue of this point P?
- Complementary color, λ_c

XYZ to sRGB

$$\begin{bmatrix} \tilde{R} \\ \tilde{G} \\ \tilde{B} \end{bmatrix} = \begin{bmatrix} 3.2406 & -1.5372 & -0.4986 \\ -0.9689 & 1.8758 & 0.0415 \\ 0.0557 & -0.2040 & 1.0570 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

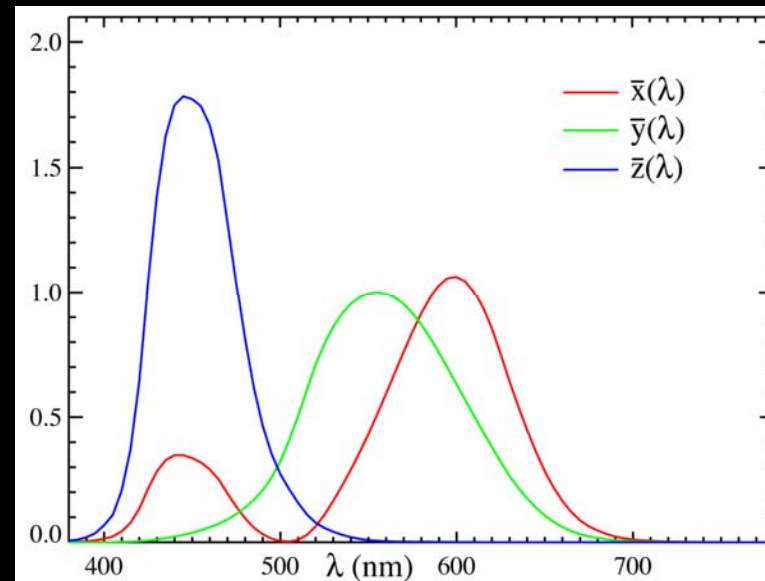
From P&W P2.14

- Step 1: linear transformation
- Step 2: nonlinear (function given in P2.14)
- Normalize R,G,B values to be integers from 0 to 255
 - 256×256×256 = 16,777,216 possible colors



xyY

- Reminder: what was Y?



- all are positive
- \bar{z} = close to S cones, close to \bar{b}
- \bar{y} = matches intensity response of eye, close to M cones
- \bar{x} = chosen so that white is equal parts of all three

XYZ \leftrightarrow xyY Transformations

(Nonlinear)

$$\begin{bmatrix} x \\ y \\ Y \end{bmatrix} = \begin{bmatrix} \frac{X}{X+Y+Z} \\ \frac{Y}{X+Y+Z} \\ Y \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \frac{x}{y} Y \\ Y \\ \frac{1-x-y}{y} Y \end{bmatrix}$$

"Hue, brightness, saturation"

- Hue – use RGB values to turn locus into a hexagon, then written as 0 to 360°
- Saturation called "chroma", as before
- Brightness from Y, scaled as 0 to 1

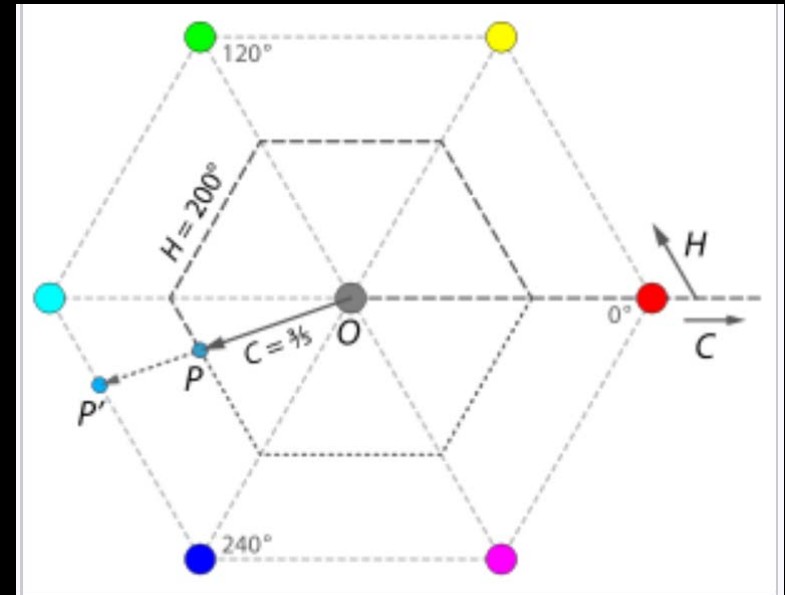
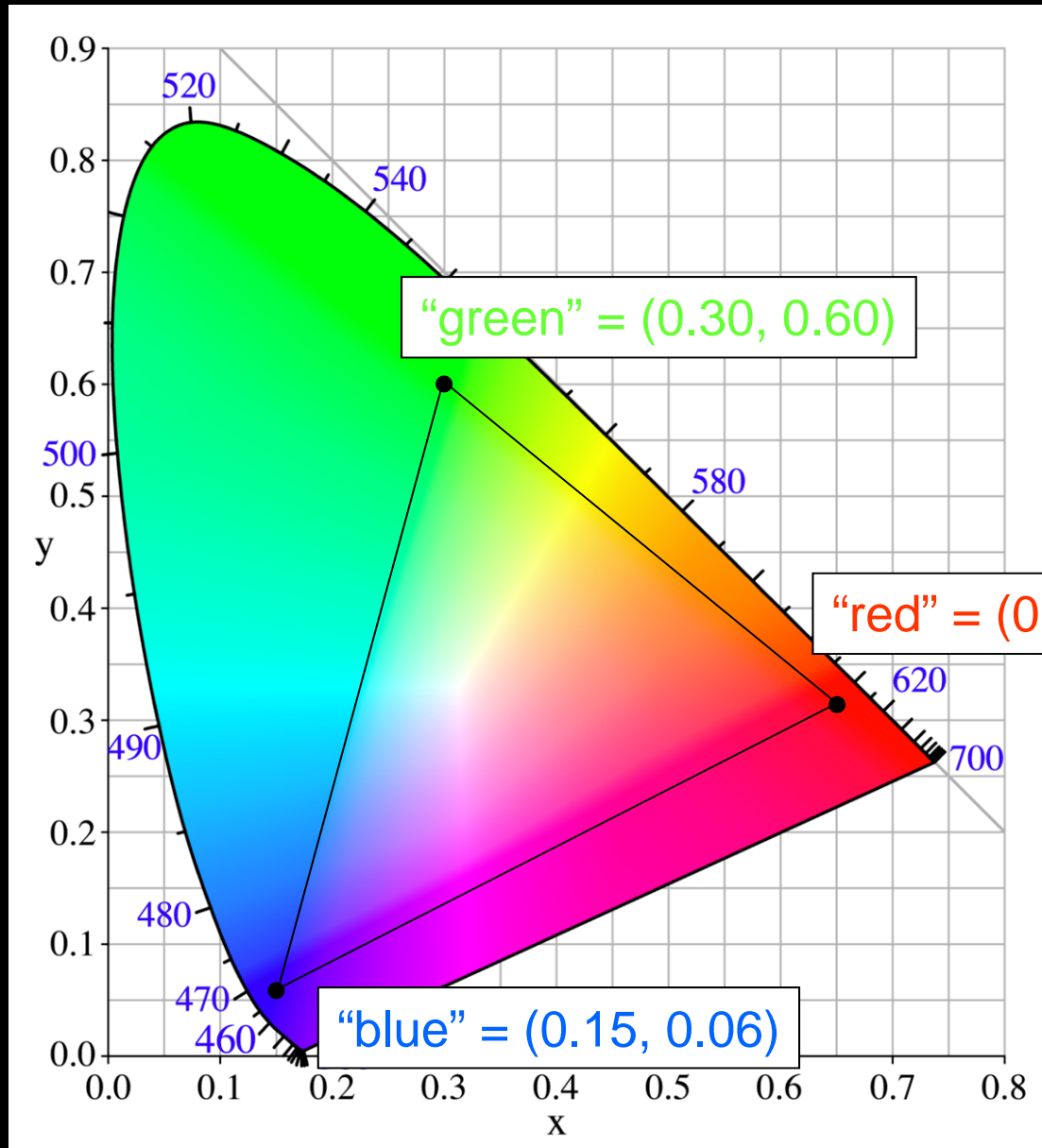


Fig. 9. Both hue and chroma are defined based on the projection of the RGB cube onto a hexagon in the "chromaticity plane". Chroma is the relative size of the hexagon passing through a point, and hue is how far around that hexagon's edge the point lies.

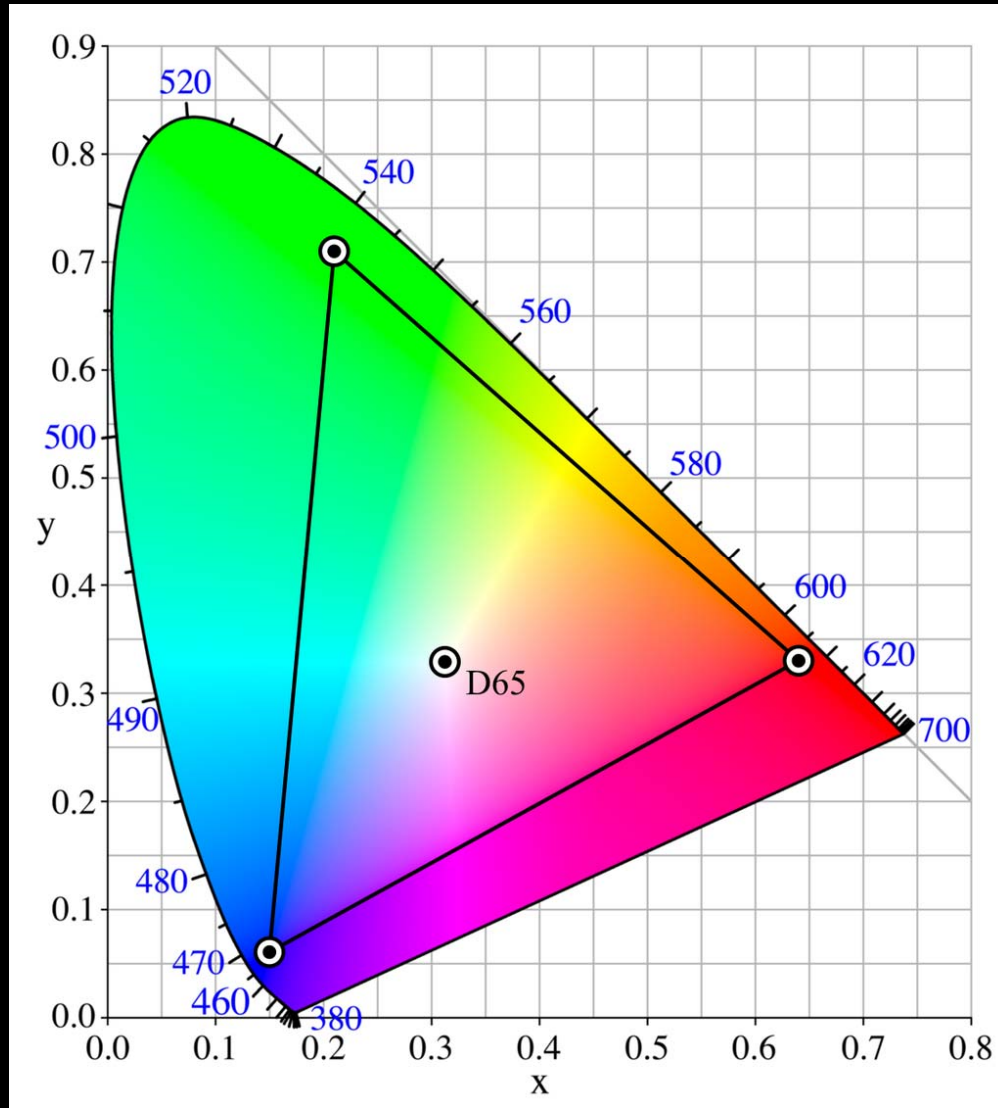
From Wikipedia
"HSL and HSV"

sRGB gamut, again

- What would be the ideal set of three light sources for your monitor?



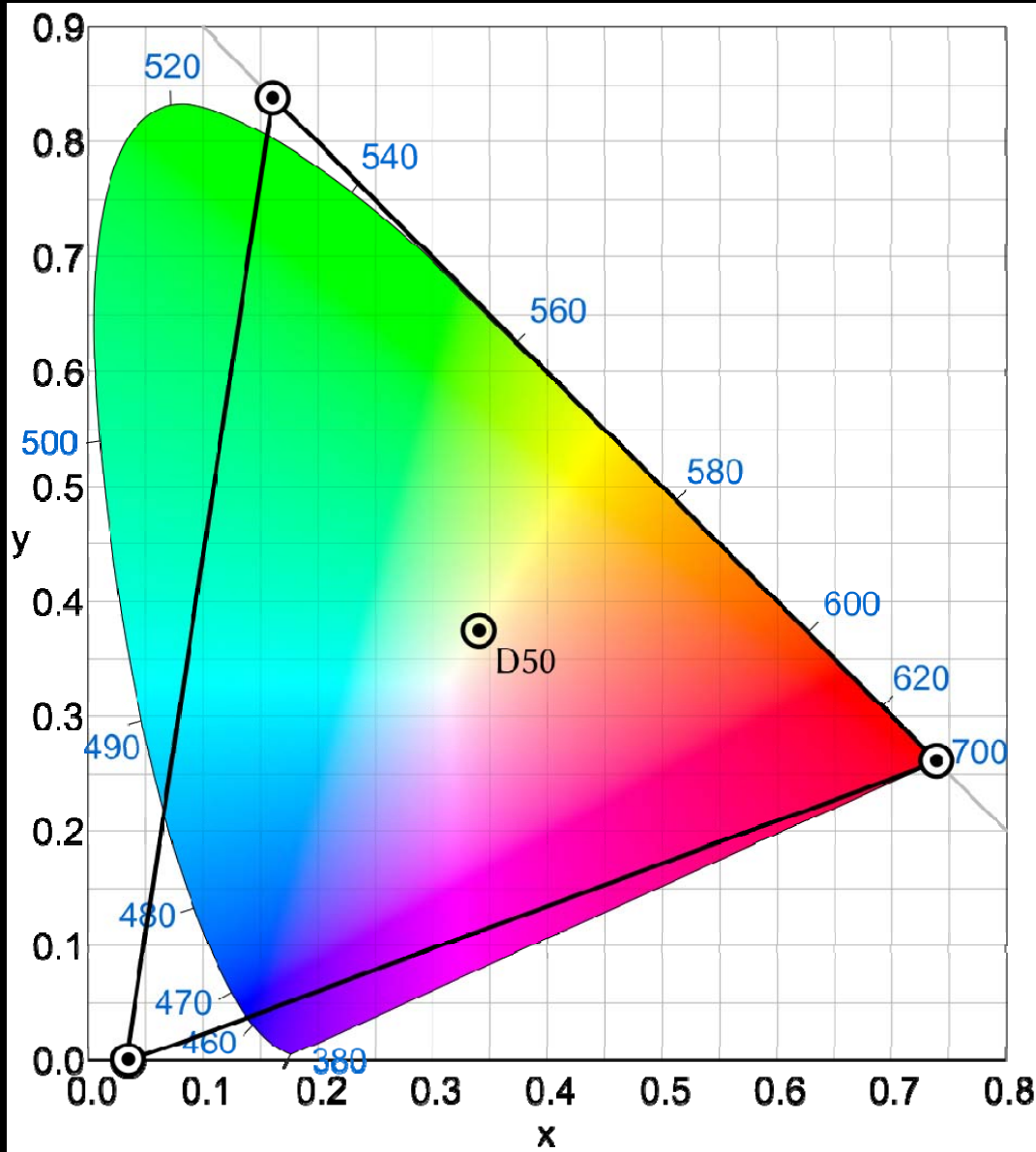
Adobe RGB



- Better than sRGB?
- Only if your camera/display/ printer are all calibrated for it
- Not for use on internet
- Also: there's a wider range of possible colors, but the difference between individual colors is bigger than in sRGB (still $256 \times 256 \times 256$)

From Wikipedia,
"Adobe RGB color space"

ProPhoto RGB



“One of the downsides to this color space is that approximately 13% of the representable colors are imaginary colors that do not exist and are not visible colors.”

From Wikipedia,
“ProPhoto RGB color space”

Summary: Many Ways to Specify Color

- R,G,B (original color matching functions)
- X,Y,Z
- x,y,Y
- hue, saturation, Y
 - Complementary hue if needed
- hue, brightness, saturation
- sRGB R,G,B coordinates (if in sRGB color space)
- R,G,B coordinates of other color spaces

That's All, Folks