Optics Winter 11 - Day 1

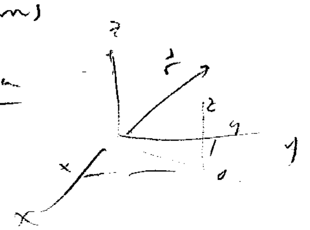
- ~~take att~~
- Go over syllabus
- ~~Report results of prerequisite survey~~
- ~~type up notes each day?~~

Multivariable Calc review

Vector: ordered triple

Coordinate Systems

Cartesian



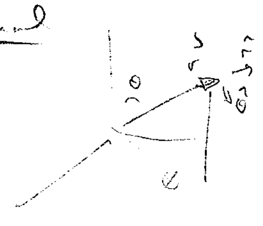
pt = (x, y, z)

$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

(unit vectors $\hat{x}, \hat{y}, \hat{z}$)

tiny volume element $dv = dx\,dy\,dz$ $\boxed{(\text{not } d\tau)}$

Spherical



pt = (r, θ, ϕ)

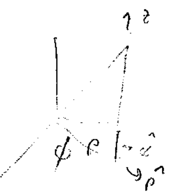
$\vec{r} = r\hat{r}$

$dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$

θ = tipping angle from z

ϕ = angle in xy plane

cylindrical

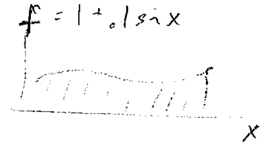


pt = (ρ, ϕ, z)

$\vec{r} = \rho\hat{\rho} + z\hat{z}$

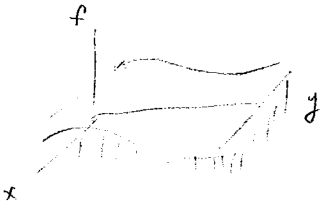
$dv = \rho\, d\rho\, d\phi\, dz$

1D integral :



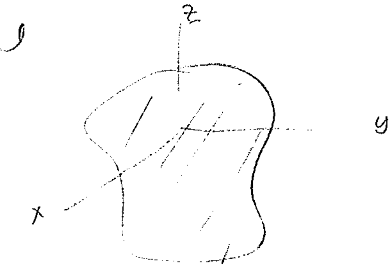
$\int f\,dx =$
area under curve

2D integral



$\int f\,dx\,dy$
volume under the surface
defined by limits of
integration

3D integral



$\int f\,dx\,dy\,dz$

if $f = 1$, $= \int dv =$ total volume inside

if $f =$ function $\wedge$ like a weighted average
$\int f\,dx\,dy\,dz$

simplest example $\rho =$ charge density

$\int \rho\,dv$ would give you total charge

Scalars vs Vectors

functions can be "scalar fields" like charge density, temperature, etc.

field — or "vector fields" where each pt in space has magnitude + direction

like eg wind. or electric field
Given a vector symbol $\vec{E} = E_x(x,y,z)\,\hat{x} + E_y(x,y,z)\,\hat{y} + E_z(x,y,z)\,\hat{z}$

Derivatives of fields

Scalar : gradient of a scalar field $\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$

E_x, E_y, E_z scalar fields (but not independent)

- partial derivatives
- gradient is a vector field
- pts in "down hill" direction.
  direction of eg. heat flow

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vector fields derivatives of $\vec{A}$ (could be wind)

divergence of a vector field $\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

- is a scalar field!
- represents how much vectors associated w/ $\vec{A}$ are "spreading out" at each pt in space

would have non zero divergence even though $\vec{A} = 0$ right there

also nonzero divergence

curl of a vector field $\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$

$$= \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - \hat{y}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

- is a vector field!
- represents whether vectors associated w/ $\vec{A}$ curl around your pt of interest

Integrals of fields specifically what do you get when you integrate a derivative?

Fundamental Thm of Calc.

$$\int_a^b \frac{df}{dx}\,dx = f(b) - f(a)$$

Geometric: add up small height changes, get total height change
pic on Griffiths pg 29

↓ In 3-D, we have 3 types of derivatives

Gradient Thm $\int_a^b (\vec{\nabla} f)\cdot d\vec{l} = f(b) - f(a)$ Geometric: same
pic on Griffiths pg 29

corollary 1: path independent
corollary 2: $\oint = 0$
$\vec{\nabla} f$ is like "conservative force" $W = \int \vec{F}\cdot d\vec{l}$

Divergence Thm $\int_V (\vec{\nabla}\cdot\vec{A})\,dv = \oint_S \vec{A}\cdot d\vec{a}$

S: closed body of volume, $d\vec{a}$ pts outward ⊥ to S

integral of derivative over a region = value of function at boundary
Geometric: source of flux, like a faucet, 2 ways to measure how much produced
(1) count up faucets in volume
(2) count up flows across body

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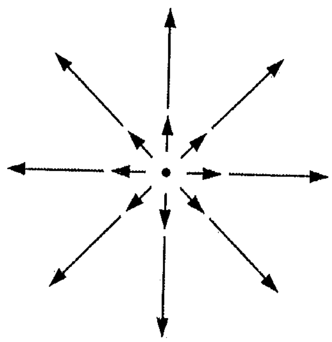
(in PowerPoint)

Figures from Griffiths, *Intro to Electrodynamics*



(a)

Divergence

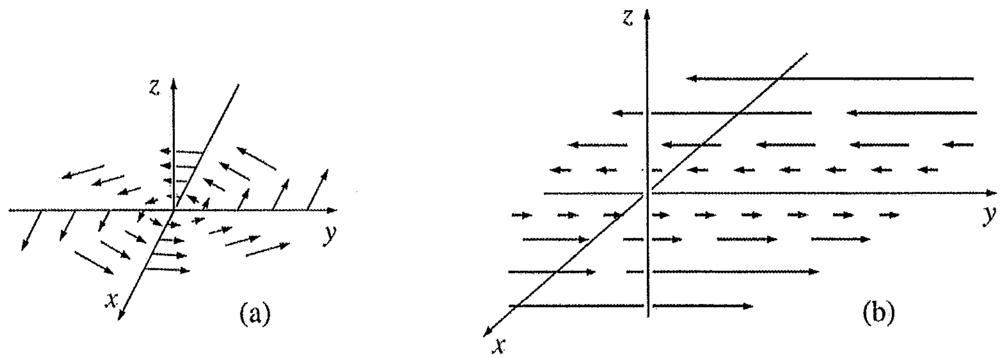


Figure 1.19

Curl

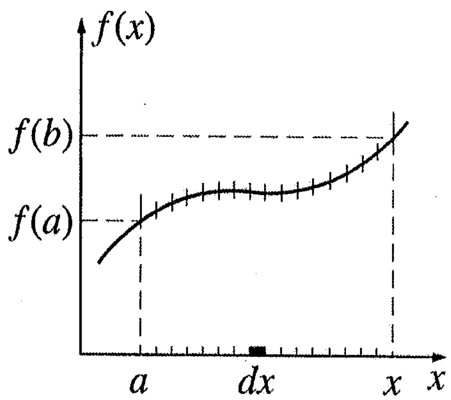


Figure 1.25

Fundamental Thm of Calc

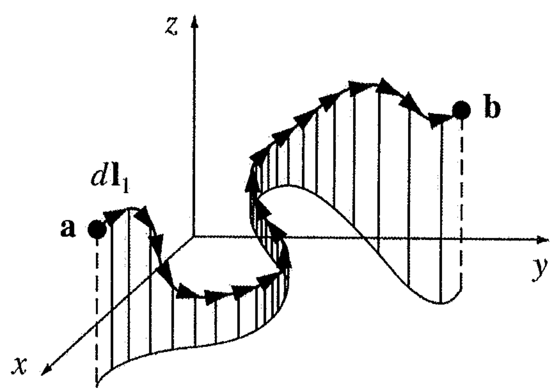


Figure 1.26

Gradient Theorem

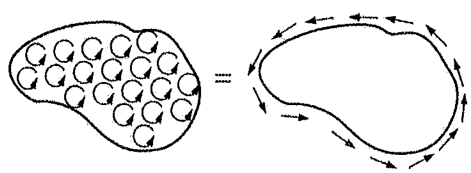


Figure 1.31

Curl Theorem (Stokes' Theorem)

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Curl thm aka Stokes thm

$$\int_S (\nabla \times \vec{A}) \cdot d\vec{a} = \oint_P \vec{A} \cdot d\vec{l}$$

P = perimeter

integral of derivative over a region = value of function on budy

Geometric: Griffiths pg 35 figure

total amount of swirl:

(1) add up each swirl source on the surface

or (2) go along edge to find out how much the flow is following the bady.