day 2 pg 1

<u>Maxwell Eqns</u> $\vec{E}$ and $\vec{B}$ are fields with special properties

1. $\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$
2. $\nabla\times\vec{E} = -\frac{\partial B}{\partial t}$
3. $\vec{\nabla}\cdot\vec{B} = 0$
4. $\vec{\nabla}\times\vec{B} = \mu_0 \vec{J} + \epsilon_0\mu_0 \frac{\partial \vec{E}}{\partial t}$

optics: light is an Electromagnetic Wave!

- Helmholtz thm (?)

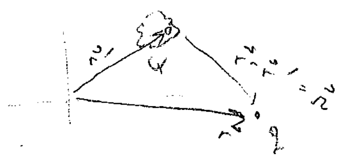
where do they come from? (Physics 441 in detail)

**I.** <u>Coulomb's law</u> force from <u>static</u> charges

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}\hat{r}$$

define $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$ then $\vec{F} = q\vec{E}$

more general



integrate

$$\vec{E}_{\text{from }Q} = \frac{1}{4\pi\epsilon_0}\int \frac{dQ\,\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \rho\frac{dv'\,\hat{\mathscr{r}}}{\mathscr{r}^2} \qquad \rho: \text{charge density}$$

Warning! $\rho$ = cyl. coord, $\rho$ = charge / vol unrelated

$\checkmark'$ because you integrate by varying $\vec{r}\,'$ not $\vec{r}$

from that, we get

1. <u>Gauss's law</u> $\oint_s \vec{E}\cdot d\vec{a} = q_{enc}/\epsilon_0$ → Typical problem on next pg
↓ div thm ↘ defn of $\rho$

$$= \int_v (\nabla\cdot\vec{E})\,dv = \frac{1}{\epsilon_0}\int_v \rho\, d\tau$$

true for all arbitrary volumes

$$\rightarrow \boxed{\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0}$$

could also be obtained by taking divergence of $\vec{E}$ directly

2. ~~If you take curl of Colomb's Law,~~ $\nabla\times\vec{E} = 0$ true for statics

~~3rd Faraday found!~~

~~<u>Faraday's law</u> $\mathcal{E} = -\frac{\partial\Phi_B}{\partial t}$~~

~~defn $\mathcal{E}$~~

~~$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a}$~~

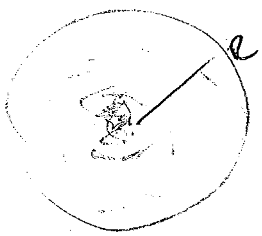
~~stokes thm~~

~~$\oint (\nabla\times\vec{E})\,da = \int\left(-\frac{\partial\vec{B}}{\partial t}\right)\cdot d\vec{a}$~~

~~true for all arbitrary surfaces~~

~~$\rightarrow \boxed{\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$~~

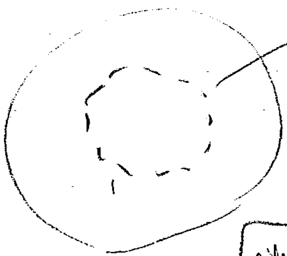
Typical Gauss's law problem from phys 260 (like P1.1 → that's a cylinder)



$$\rho(r) = \rho_0 \frac{R}{r} \quad \text{for } r<R$$
$$= 0 \quad \text{for } r>R$$

What is $E(r)$ for $r<R$? $r>R$?

(a) $r<R$



Gaussian surface

$$\oint \vec{E}\cdot d\vec{a} = q_{enc}/\epsilon_0$$
$$E \cdot 4\pi r^2 = q_{enc}/\epsilon_0$$

sidetrack: $q_{enc} = \int \rho \, dV$

$$\underbrace{\int_0^r \left(\rho_0 \frac{R}{\cancel{r}}\right)(4\pi r^{\cancel{2}} dr)}_{= \rho_0 \cdot 4\pi R \cdot \frac{1}{2} r^2}$$

$$E\, \cancel{4\pi r^2} = \frac{\rho_0 \cancel{4\pi} R \cdot \frac{1}{2} \cancel{r^2}}{\epsilon_0}$$

$$\boxed{\vec{E} = \frac{\rho_0}{\epsilon_0} \frac{1}{2} R \ \hat{r}}$$

(b) $r>R$ then $q_{enc} = \rho_0 4\pi \frac{R^3}{2}$ (same integral as before, but $\int_0^R$)

$$E\, \cancel{4\pi} r^2 = \rho_0 \cancel{4\pi} \frac{R^3}{2}$$

$$\boxed{\vec{E} = \rho_0 \frac{R^3}{2r^2} \ \hat{r}}$$

law 3. If you take curl of Coulomb's law, $\nabla \times \vec{E} = 0$

true for statics

However...

Faraday found

$$\mathcal{E} = \frac{-d\Phi_B}{dt} \qquad \text{"Faraday's law"}$$

induced EMF, aka voltage

Since $\mathcal{E} = \oint \vec{E} \cdot d\vec{l}$ (maybe non 441 haven't seen)

around a loop since $\Phi_B$ = flux through a loop

same $\vec{F} = -\vec{\nabla} U$

$\vec{E} = -\vec{\nabla} V \rightarrow V$ = integral of $\vec{E}$

And $\Phi_B = \int \vec{B} \cdot d\vec{a}$ by defn

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a}$$

$\Downarrow$ Stokes's Thm

$$\int (\vec{\nabla} \times \vec{E}) \cdot d\vec{a} = -\int \left(\frac{d\vec{B}}{dt}\right) \cdot d\vec{a}$$

true for all arbitrary surfaces

$$\rightarrow \boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \qquad \text{also "Faraday's Law"}$$

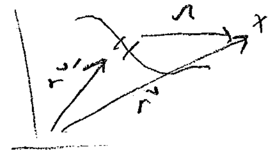
TOPS. 35500

~~VIIIa~~

II What causes mag fields? currents (field defined by $\vec{F} = q\vec{v} \times \vec{B}$)

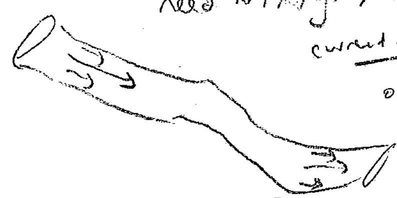
Biot Savart Law

$$\vec{B} = \frac{\mu_0}{4\pi} \int_{l} \frac{I\, d\vec{l} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$




if wire has thickness

need to integrate current density over a volume



current density $\vec{J} = \frac{I}{\text{volume}}$ $\frac{I}{\text{area}}$

$I\, d\vec{l} \rightarrow \vec{J}\, dV$

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi} \int_{V} \frac{\vec{J} \times \hat{\mathscr{r}}}{\mathscr{r}^2}\, dV'}$$

Law 2: can take $\vec{\nabla} \cdot \vec{B}$ and show it $= 0$

alternatively, recognize no magnetic monopoles

no isolated sources of $\vec{B}$ flux

surface will have as many field lines going in as going out

$\oint B \cdot da = 0$

$\boxed{\nabla \cdot B = 0}$

"Gauss's Law for $\vec{B}$"

Law 4: can take $\nabla \times \vec{B}$ and show it $= \mu_0 \vec{J}$ ← done in book (if $\vec{\nabla} \cdot \vec{B} = 0$, more on that next time)

matches Phys220 Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$