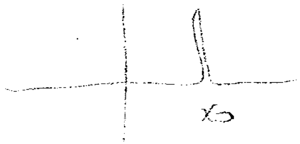
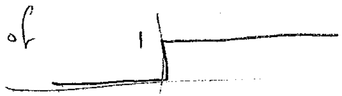
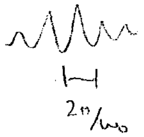
Delta functions! (quizzes)

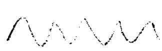
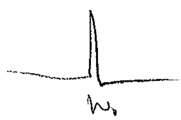
$\delta(x)$  area = 1

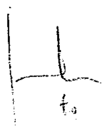
$\delta(x - x_0)$ 

Derivative of  step function

(may start here)

FT of  = 

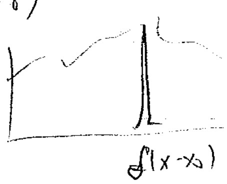
FT of  ? =  (with peak at $-\omega_0$ as well, up peak if cos, down peak if sin, no peak if $e^{i\omega_0 t}$)

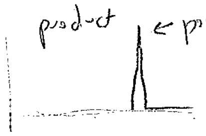
FT of  ? = $\frac{e^{i\omega t_0}}{\sqrt{2\pi}}$

proof $FT = \frac{1}{\sqrt{2\pi}} \int \delta(t - t_0)\, e^{i\omega t}\, dt$

$= \frac{1}{\sqrt{2\pi}} e^{i\omega t_0}$ by "sifting property"

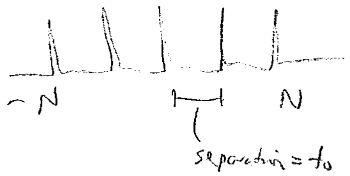
Sifting : $\int \delta(x - x_0) f(x)\, dx = f(x_0)$

graphical proof 

product  ← possible larger or smaller height than original delta function

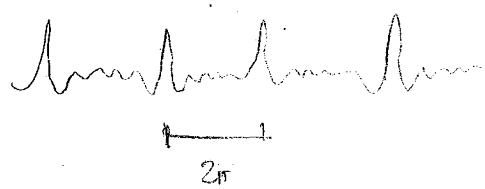
integral = area = 1 × value of f(x) at that location

$= f(x_0)$

FT of 

$N_{tot} = 2N+1$

in handout: $FT = \frac{1}{\sqrt{2\pi}} \frac{\sin(N_{tot}\, \omega t_0/2)}{\sin(\omega t_0/2)}$

$=$ 

→ w/ $t_0 = 1$!

Also note no $\sqrt{2\pi}$ factor

Otherwise $= \frac{2\pi}{t_0}$

Convolutions!

(reading quizzes)

- Context
- $g(t) = \int a(t')\, b(t-t')\, dt'$
  - $a(t')$ → function which gets modified
  - $b(t-t')$ → kernel
  - (or vice versa)
- if kernel = delta function, you get $a(t)$ back

(survey)

- website applet
- convolution theorem

The end.