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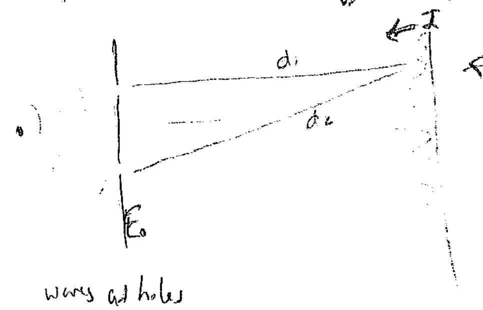
Section 8.6 Young's 2-slit experiment - first conclusive demo that light = wave

quasi-monochromatic light

like Michelson: 2 beams of light travel different distances



waves at holes spatially coherent (ie come from same wave fronts)

phase factors from both waves

$$E_{tot} = E_0 e^{ikd_1} + E_0 e^{ikd_2}$$

$$I \propto |E|^2$$

$$= |E_0|^2 \left(e^{ikd_1} + e^{ikd_2}\right)\left(e^{-ikd_1} + e^{-ikd_2}\right)$$

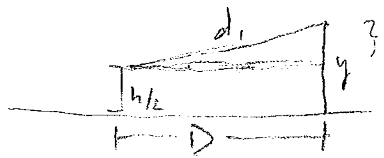
$$= |E_0|^2 \left(1 + 1 + \underbrace{e^{ik(d_2-d_1)} + e^{-ik(d_2-d_1)}}_{2\cos k(d_2-d_1)}\right)$$

$$\boxed{I = 2I_0\left[1 + \cos k(d_2-d_1)\right]}$$

if not completely symmetric, perhaps due to different phases at each slit,

$$\boxed{I = 2I_0\left[1 + \cos\left(k(d_2-d_1) + \Delta\phi\right)\right]}$$

Some geometry



$$\sin\theta_1 = \frac{y - \frac{h}{2}}{d_1}$$

$$d_1 = \sqrt{D^2 + \left(y - \frac{h}{2}\right)^2}$$

$$= D\left(1 + \left(\frac{y-\frac{h}{2}}{D}\right)^2\right)^{1/2}$$

$$= D\left(1 + \frac{1}{2}\left(\frac{y-\frac{h}{2}}{D}\right)^2\right) \quad \text{for large } D$$

$$d_2 - d_1 = D\left(\cancel{1} + \frac{1}{2}\left(\frac{y+\frac{h}{2}}{D}\right)^2\right) - D\left(\cancel{1} + \frac{1}{2}\left(\frac{y-\frac{h}{2}}{D}\right)^2\right)$$

$$= \frac{1}{2D}\left[\cancel{y^2} + yh + \cancel{\frac{h^2}{4}} - \cancel{y^2} + yh - \cancel{\frac{h^2}{4}}\right]$$

$$= \frac{yh}{D}$$

$$\boxed{I = 2I_0\left[1 + \cos\left(\frac{kyh}{D} + \Delta\phi\right)\right]}$$

Do this first

Alternate derivation from $|E_{tot}|^2$

$\Delta\phi = \frac{(\Delta PL)}{\lambda} 2\pi = \Delta PL \cdot k$

$h \sin\theta$

$\Delta PL \approx h\frac{y}{D}$

$E \sim e^{-i\frac{hyk}{2D}} + e^{ihyk/2D}$

$= 2\cos\frac{hyk}{2D}$

$I \sim E^2 \sim \cos^2\frac{hyk}{2D}$

$I = I_0\left(\frac{1}{2} + \frac{\cos\frac{kyh}{D}}{2}\right)$

because $\cos^2\frac{x}{2} = \frac{1}{2} + \frac{1}{2}\cos x$

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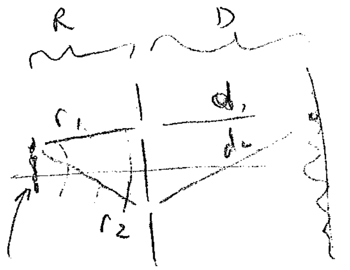
What if a finite source?



how coherent is the initial extended wavefront?

Not just a simple convolution if phase of source is varying (randomly?)

Treat the extended source as a series of pt sources, $j = 1$ to N, each with slightly different phase ϕ_j

For each j $\quad E_j = |E_j|\left[e^{i(k(r_{1j}+d_1)-\omega t+\phi_j)} + e^{ik(r_{2j}+d_2)-\omega t+\phi_j}\right]$

$$E_{tot} = \sum_j E_j$$

$$I \propto |E_{tot}|^2 = \left[\sum_j |E_j| \left(e^{i(k(r_{1j}+d_1)-\omega t+\phi_j)} + e^{i(2)}\right)\right]\left[\sum_m \underbrace{}_{|E_m|} e^{-i[k(r_{1m}+d_1)-\omega t+\phi_m]} + e^{-i(2)}\right]$$

$$= \sum_{j,m}\Big[|E_j||E_m|\exp[i(k(r_{1j}-r_{1m})+\phi_j-\phi_m)]$$
$$+|E_j||E_m|\exp i[k(r_{2j}-r_{2m})+\phi_j-\phi_m]$$
$$+|E_j||E_m|\underbrace{\left[\exp i[k(r_{1j}-r_{2m})+k(d_1-d_2)+\phi_j-\phi_m]+c.c.\right]}_{2\,Re\{\exp i(\text{stuff})\}}\Big]$$

Assume: noncoherent source

like starlight or lightbulbs, not laser

$\rightarrow$ $\phi_j-\phi_m$ causes oscillations which average out unless $j=m$

(and when $j=m$, all $r_{1j}-r_{1m}$ terms go away)

$$|E_{tot}|^2 = \sum_j\left[|E_j|^2+|E_j|^2+|E_j|^2\,2\,Re\left[\exp i\left[k(\underbrace{r_{1j}-r_{2j}}_{-\frac{y_j' h}{R}})+k(\underbrace{d_1-d_2}_{-\frac{yh}{D}})\right]\right]\right]$$

$-\frac{yh}{D}$ already done

$-\frac{y_j' h}{R}$; y_j' = distance upward for the j^{th} source

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$$|E_{tot}|^2 = 2\sum_j |E_j|^2 + 2\,\mathrm{Re}\sum_j |E_j|^2 e^{-i\left(k\frac{y_j' h}{R} + k\frac{y h}{D}\right)}$$

take out of sum

put in multiplicative constants

$$I = 2\sum_j I_j + 2\,\mathrm{Re}\, e^{-ik\frac{yh}{D}} \sum_j I_j e^{-ik\frac{h y_j'}{R}}$$

$$\boxed{I(h) = \left(2\sum_j I_j\right)\left[1+\mathrm{Re}\left(\underbrace{e^{-ik\frac{yh}{D}}\frac{\sum_j I_j e^{-ik\frac{h y_j'}{R}}}{\sum_j I_j}}_{=\gamma(h)}\right)\right]}$$

$= \gamma(h)$ = "degree of coherence"

Looks a lot like our time coherence formula!

Recall $\gamma(\tau) = \dfrac{\int I(\omega)e^{-i\omega\tau}d\omega}{\int I(\omega)d\omega}$

$$\gamma = e^{ik\frac{hy}{D}}\frac{\int I(y')e^{-i\ldots}dy'}{\int I(y')dy'}$$

main difference • $\gamma(\tau)$ had continuous integral

→ can make $\sum_j$ into cont integral

• $\gamma(h)$ has $e^{-ik\frac{yh}{D}}$ factor

→ defines position of fringes on screen (rest defines depth of fringes)

$I(y')$: spatial intensity distribution (units I/length)

$\langle I(h)\rangle = 2\langle I_{one\ slit}\rangle(1+\mathrm{Re}\,\gamma(h))$

Notes: 1) when h large it's like when τ was large

→ amplitude of $\gamma \simeq 0$, no oscillations visible

2) Like before $V = |\gamma|$

3) like coherence time

$$h_c = \int_{-\infty}^{\infty} |\gamma(h)|^2 dh$$

sets boundary between large h (no oscillations) small h (oscillations)

"coherence separation"

That's all for exam!