day 3 pg 1

3. No magnetic monopoles → "Gauss's law for $\vec{B}$"

$$\oint_S \vec{B}\cdot d\vec{a} = 0$$

$$\int_V (\vec{\nabla}\cdot\vec{B})\,dv = 0$$

true for all volumes

$$\rightarrow \boxed{\nabla\cdot B = 0}$$

II. Biot Savart Law, force between a current + a moving charge

$$\vec{F} = q\,\vec{v}\times\vec{B}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int_l \frac{d\vec{I}\times\hat{r}}{r^2} \quad (d\vec{I} = I\,d\vec{l})$$

true for "static currents"

$$\vec{B} = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}\times\hat{r}}{r^2}\,dv'$$

$\vec{J}$ = current density, current / area

can take $\nabla\cdot B$ to get $= 0$

can take $\nabla\times B$ to get $= \mu_0\vec{J}$ (if $\nabla\cdot J = 0$)

(done in book)

from that, we get

Law #4. Ampere's Law

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \quad \text{(if } I \text{ is not changing)}$$

defn J

Stokes thm

$$\int_S (\nabla\times B)\cdot d\vec{a} = \int_S \mu_0\vec{J}\cdot d\vec{a}$$

true for all surfaces

$$\rightarrow \vec{\nabla}\times\vec{B} = \mu_0\vec{J} \quad \text{true for "statics"}$$

Typical problem, next page

Colton: if a changing B field can produce an E field (Faraday's law) shouldn't a changing E field produce a B field?

Yes — something's missing

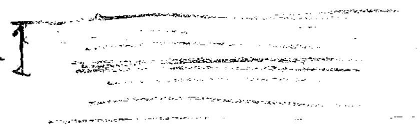
Review: $\nabla\cdot E = \rho/\epsilon_0$

$\nabla\times E = -\partial B/\partial t$

$\nabla\cdot B = 0$

$\nabla\times B = \mu_0 J + \ldots$

Typical Ampere's Law problem from Phy 220 (like P1.3)

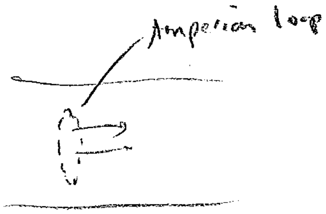
R 

$$J = J_0 \cdot \left(\frac{R}{\rho}\right) \quad \text{for } \rho < R$$
$$= 0 \quad \text{for } \rho > R$$

Warning! ρ = cylindrical coordinate not charge density here.

What is $B(\rho)$ for $\rho < R$? $\rho > R$?

(a) $\rho < R$



$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$B \cdot 2\pi\rho = \mu_0 I_{enc}$$

side track: find I_{enc}

$$I_{enc} = \int J\, da = \int_0^\rho \left(J_0 \frac{R}{\cancel{\rho}}\right)\left(2\pi \cancel{\rho}\, d\rho\right)$$

$$I_{enc} = J_0 \cdot 2\pi R\, \rho$$

$$B \cdot \cancel{2\pi\rho} = \mu_0 J_0 \cdot \cancel{2\pi} R \cdot \cancel{\rho}$$

$$\boxed{\vec{B} = \mu_0 J_0 R\, \hat{\phi}}$$

(b) $\rho > R$ $\quad I_{enc} = J_0 \cdot 2\pi R^2$ (same integral as before)

$$B \cdot \cancel{2\pi}\rho = \mu_0 J_0 \cdot \cancel{2\pi} R^2$$

$$\boxed{\vec{B} = \mu J_0 \cdot \frac{R^2}{\rho}\, \hat{\phi}}$$

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Maxwell's Fix to Ampere's Law

Note there's a fundamental mathematic problem

Vector thm: div of a curl = 0 always. HW P0.16 (not assigned)

$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{E})$ must $= 0$

test it out: $= \nabla\cdot\left(-\frac{\partial B}{\partial t}\right)$

$= -\frac{\partial}{\partial t}(\nabla\cdot B)$

$= 0$ ✓

what about $\nabla\cdot(\nabla\times B)$?

$\nabla\cdot(\nabla\times B) = \nabla\cdot(\mu_0 J + ?)$

$= \mu_0(\nabla\cdot\vec{J}) + \nabla\cdot(?)$

Not zero!

Conservation of charge: if charge flows out, ρ decreases

$Q_{inside} = \int \rho \, dv$

$I_{flowing\ out} = \int \vec{J}\cdot d\vec{a} = -\frac{dQ}{dt}$ (neglecting ... against $\vec{J}$ ... (Q))

$\int \vec{J}\cdot d\vec{a} = -\int \frac{\partial\rho}{\partial t}\, dv$

$\int(\nabla\cdot\vec{J})\, dv = \int\left(-\frac{\partial\rho}{\partial t}\right) dv$

true locally $\boxed{\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}}$ (continuity)

$0 = \mu_0\left(-\frac{\partial\rho}{\partial t}\right) + \nabla\cdot(?)$

$\rho = \epsilon_0\vec{\nabla}\cdot\vec{E}$ by Maxwell #1

$0 = -\mu_0\epsilon_0\nabla\cdot\left(\frac{\partial E}{\partial t}\right) + \nabla\cdot(?)$

$? = \mu_0\epsilon_0\frac{\partial E}{\partial t}$

$$\boxed{\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial E}{\partial t}}$$

→ related to "displacement current"

$\frac{\partial E}{\partial t}$ → seen in eg charging capacitor

"he saw that light could move through empty space

"a changing B field makes an E field changing

and vice versa all at the perfect pace

→ Ampere's Law Song

physicssongs.org

integral form $\oint_S(\nabla\times B)\cdot da = \mu_0 I + \mu_0\epsilon_0\frac{d}{dt}\int_S \vec{E}\cdot d\vec{a}$

circulate Φ_E

day 3 pg 4

Wave eqn Start w/ $\nabla\cdot E = \rho/\epsilon_0$
$\nabla\cdot B = 0$
$\nabla\times E = -\partial B/\partial t$
$\nabla\times B = \mu_0 J + \mu_0\epsilon_0 \frac{\partial E}{\partial t}$

Consider a vacuum, $\rho = 0$, $J = 0$

Then 1. $\nabla\cdot E = 0$
2. $\nabla\cdot B = 0$
3. $\nabla\times E = -\partial B/\partial t$
4. $\nabla\times B = \mu_0\epsilon_0 \frac{\partial E}{\partial t}$

Vector identity: Eqn 0.10 ~~HW Problem Part 7~~ $\nabla\times(\nabla\times\vec{A}) = \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A}$

Also on "What you should already know"

gradient of this scalar field

Laplacian $\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2}$

Take 3rd Maxwell Eqn

$$\nabla\times(\nabla\times E) = \nabla\times\left(-\frac{\partial B}{\partial t}\right)$$

$$\underbrace{\vec{\nabla}(\vec{\nabla}\cdot\vec{E})}_{0} - \nabla^2 E = -\frac{\partial}{\partial t}\underbrace{(\nabla\times B)}_{\mu_0\epsilon_0\frac{\partial E}{\partial t}}$$

$$\boxed{\nabla^2 E = \mu_0\epsilon_0 \frac{\partial^2 E}{\partial t^2}}$$ "the 3D wave equation" with $\mu_0\epsilon_0 = \frac{1}{v^2}$

Similarly $\nabla\times(\nabla\times B) \longrightarrow \boxed{\nabla^2 B = \mu_0\epsilon_0\frac{\partial^2 B}{\partial t^2}}$

$$\boxed{v = \frac{1}{\sqrt{\mu_0\epsilon_0}}}$$

$\mu_0 = 4\pi\, 10^{-7}$
$\epsilon_0 = 8.854\cdot 10^{-12}$
$\rightarrow v = 3\cdot 10^8$!!!

Why called wave eqn?

Consider eqn $\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$ "1D wave eqn"

solutions are travelling waves like $f = A\sin(kx - \omega t)$

$\frac{\partial^2 f}{\partial x^2} = -Ak^2\sin(kx-\omega t) = -k^2 f$

$\frac{\partial^2 f}{\partial t^2} = -A\omega^2\sin(kx-\omega t) = -\omega^2 f$

divide $\frac{\partial^2 f/\partial x^2}{\partial^2 f/\partial t^2} = \frac{k^2}{\omega^2}$

or $\frac{\partial^2 f}{\partial x^2} = \frac{1}{(\omega/k)^2}\frac{\partial^2 f}{\partial t^2}$ $v = \frac{\omega}{k}$ "phase velocity" from Phys 123

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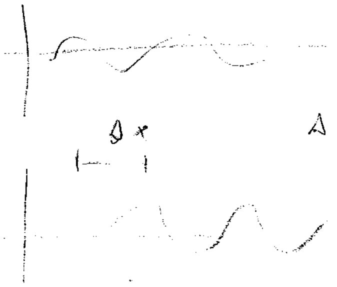
some time Δt later



$\Delta x = (\omega/k)\Delta t \Rightarrow v = \frac{\omega}{k}$ = wave speed

$\frac{d^2}{dx^2} \to \nabla^2$ then you have 3D wave eqn

Although I did this for sine wave, "one can show" that

any function of the form $\left.\begin{matrix} f(kx-\omega t) \\ \text{or } f(x-vt) \end{matrix}\right\}$ is a soln

eg. $\frac{1}{(kx-\omega t)^2} e^{-(kx-\omega t)^5}$