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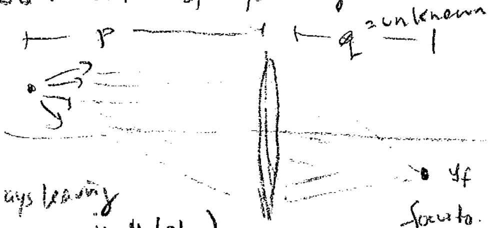
More general view of focusing



All rays leaving from a given $y_1 (= h_0)$

focus to the same y_f $(= h_i)$ at the same distance q

whole trip

$$\begin{pmatrix} y_f \\ \theta_f \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}$$

trans/focus/trans

$$= \begin{pmatrix} Ay_1 + B\theta_1 \\ Cy_1 + D\theta_1 \end{pmatrix} \leftarrow y_f$$

If y_f = constant for all θ_1, $\underline{B \text{ must} = 0}$ (otherwise $y_f = Ay_1 + B\theta_1$ depends on θ_1)

then $y_f = A y_1$

$\boxed{A = \text{magnification}}$

Let's compute the matrix

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & q \\ 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & p \\ 0 & 1 \end{pmatrix}}$$

$$\begin{pmatrix} 1 & p \\ -1/f & -p/f + 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 - \frac{q}{f} & p - \frac{qp}{f} + q \\ -1/f & 1 - \frac{p}{f} \end{pmatrix}$$

$B = 0 \rightarrow p - \frac{qp}{f} + q = 0 \rightarrow p + q = \frac{qp}{f}$

$$\frac{1}{f} = \frac{p+q}{qp}$$

$$\boxed{\frac{1}{f} = \frac{1}{p} + \frac{1}{q}}$$

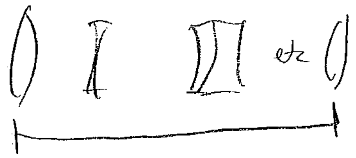
this q will force $B = 0$, will force image to occur

$$\frac{q}{f} = \frac{q}{p} + 1$$

$A = \text{Mag} = 1 - \frac{q}{f} = 1 - \left(\frac{q}{p} + 1\right)$

$\boxed{\text{Mag} = -q/p}$

"principal planes"

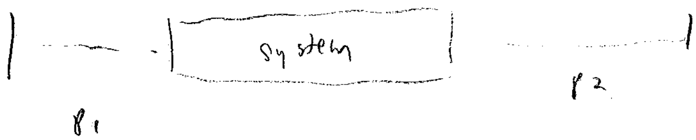


optical system.

determine matrix

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

book shows... if you choose p_1 and p_2 correctly

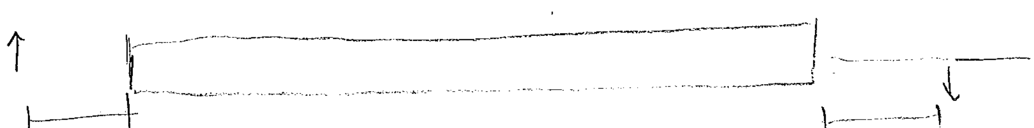


$$\underbrace{\begin{pmatrix} 1 & p_2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} 1 & p_1 \\ 0 & 1 \end{pmatrix}}$$

then. $\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$

$\boxed{\text{new system, now extending to principal planes}}$

acts like a lens!



can use thin lens eqn, etc

just measure p from first principal plane (can be negative!)
q from 2^{ND} " " (" ")

To make this work, need

$$\boxed{p_1 = \frac{1-D}{C}}$$

$$\boxed{p_2 = \frac{1-A}{C}}$$

then $\boxed{f = -\frac{1}{C}}$

using values from original combined $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ matrix

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9.8 stability of laser cavities

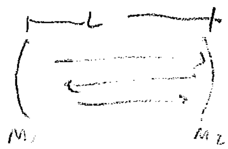
Light rays need to get trapped, so they can continually interact with lasing medium

obviously bad

possibly good!

Analysis

~~Note, consider [illegible] very similar to lens~~ $\cancel{\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}}$ $\cancel{\begin{pmatrix}1 & 0\\ -\frac{1}{f} & 1\end{pmatrix}}$

This  multiple bounces, is like this

(light path) $M_1 \to L \to M_2 \to L \to M_1 \to L \to M_2 \to$ etc

We have an infinite # of repetitions of this basic structure

$$\underbrace{M_1 \to L \to M_2 \to L}_{\text{matrices go in opposite order}}$$

$$M = \begin{pmatrix}1 & L\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ -\frac{1}{f_2} & 1\end{pmatrix}\begin{pmatrix}1 & L\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ -\frac{1}{f_1} & 1\end{pmatrix}$$

$f_1 = \frac{R_1}{2}$

$f_2 = \frac{R_2}{2}$

Multiply together

$$M = \begin{pmatrix}A & B\\ C & D\end{pmatrix}$$

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laser cavity, cont

Sylvester's Thm: if $\det M = 1$ (it is!)

then $M^N =$ complicated formula

$$\begin{pmatrix} A_N & B_N \\ C_N & D_N \end{pmatrix}$$

stable cavity: need matrix elements to remain finite as $N \to \infty$

or 88T ← Matrix elements like $\sin(N\theta)$ $\theta = \cos^{-1}\frac{A+D}{2}$ ← of original matrix

→ can blow up if θ = imaginary

stability condition: θ = real

or $\cos\theta$ between -1 and 1

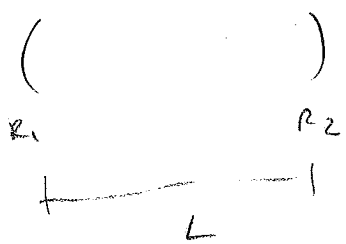
or $-1 < \frac{A+D}{2} < 1$

HW problem P9.15 (part of HW 20)

this: do matrix multiplication for 2 curved mirrors, then apply this condition

result:

stable if $0 < \left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right) < 1$



Finished 12 minutes early!