Day 39 pg 1

Black body — "Hot objects glow"

(Skipping the UV catastrophe and Rayleigh Jeans incorrect formula. Just going to correct description.)

Boltzmann factor: Prob of an energy state $\propto e^{-E/k_BT}$

Plank: light at a particular frequency f, only available in integral multiples of hf

$3hf$
$n=2$ — $2hf$
$n=1$ — hf
0

Today: hf = photon energy

Plank: "act of desperation" to fit experimental data

$$\text{Prob of being in } n^{th} \text{ state} = \frac{e^{-E_n/k_BT}}{\sum \text{all B.F.s}}$$

(no chemical potential yet)

$$P_n = \frac{e^{-nhf/kT}}{e^0 + e^{-hf/kT} + e^{-2hf/kT} + \dots}$$

$$= e^{-nhf/kT}\underbrace{\left(1 - e^{-hf/kT}\right)}_{\text{result of infinite sum}}$$

actual energy included with n^{th} state $= (nhf) \times P_n$

Total energy of all f-related states

$$E = \sum_{n=0}^{\infty} (nhf)\left[e^{-nhf/kT}\left(1 - e^{-hf/kT}\right)\right]$$

$$= hf\left(1 - e^{-hf/kT}\right)\sum_{n=0}^{\infty} n\, e^{-nhf/kT}$$

Trick: $\sum_{x=0}^{\infty} x e^{-ax} = -\sum_{x=0}^{\infty} \frac{\partial}{\partial a}\left(e^{-ax}\right)$

$= -\frac{\partial}{\partial a}\sum_{x=0}^{\infty} e^{-ax}$

$\frac{1}{1-e^{-a}}$

* or sum with Mathematica

$$= hf\left(1 - e^{-hf/kT}\right)\left[\frac{e^{hf/kT}}{\left(1 - e^{hf/kT}\right)^2}\right]$$

Cancel, with (-1)

$\frac{e^a}{(1-e^a)(1-e^a)}$ $\frac{e^{-a}}{e^{-a}}$

$\frac{1}{(e^{-a}-1)(1-e^a)}$

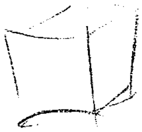
$$= \frac{hf}{e^a - 1}$$

$$\boxed{E = \frac{hf}{e^{hf/kT} - 1}}$$

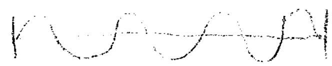
energy associated with all photons frequency f or plane waves with that freq.

Day 39 pg 2

"Box" (Could let $L \to \infty$)



radiation inside $\to$ which freqs are possible? We'll find not all freqs are equally likely

Bdy cond 

Integral # of half wavelengths $L_x = n\frac{\lambda_x}{2} \to \lambda_x = \frac{2L}{n}$

Allowed k's: $k_x = \frac{2\pi}{\lambda_x}$

$k_x = \frac{\pi}{L} \times n = k_0 n$ with $k_0 = \frac{\pi}{L}$ factor of 2 different from P+W since they're doing $e^{\pm ik_x}$ Fourier expansion, not standing waves

similarly $k_y = k_0 m$

$k_z = k_0 l$

$k = \sqrt{k_x^2 + k_y^2 + k_z^2} = k_0\sqrt{n^2+m^2+l^2}$

$= k_0 s \qquad s = \sqrt{n^2+m^2+l^2}$ (like $|\vec{r}|$ for normal space)

$\frac{\omega}{k} = c \to \quad k$ also $= \frac{\omega}{c} = \frac{2\pi f}{c}$

so $f = \frac{c}{2\pi} k_0 s$

$U_{tot} = \underset{\substack{\downarrow \\ \text{polar.}}}{2} \sum_{n,m,l} \frac{hf}{e^{hf/kT}-1}$ $\leftarrow$ Assumes each allowed k_x, k_y, k_z = equally probable

$= 2 \sum_{n,m,l=0}^{\infty} \frac{h\left(\frac{c}{2\pi}k_0 s\right)}{e^{\frac{h\left(\frac{c}{2\pi}k_0 s\right)}{kT}}-1}$

weighted towards higher s $\to$ higher (n,m,l) $\to$ higher (n,m,l) $\to$ More of the allowed modes can give you higher freqs than lower freqs

$= 2\frac{hc}{2\pi}k_0 \frac{1}{8}\int_0^\infty \frac{s}{e^{\frac{hc\,k_0 s}{2\pi kT}}-1} \times 4\pi s^2\, ds$

($\frac{1}{8}$: only positive n,m,l; $(\frac{1}{2})^3$ diff. than P+W)

$x = \frac{hc\,k_0 s}{2\pi kT} \qquad s = \frac{2\pi kT}{hck_0}x$

$ds = (\quad) dx$

$2\frac{hc}{2\pi}k_0 \frac{4\pi}{8}\left(\frac{2\pi kT}{hck_0}\right)^4 \int_0^\infty \frac{x^3\,dx}{e^x - 1}$

$$\frac{1}{2} \; K \cancel{d} \cancel{k_0} \; \frac{16 \pi^4 k^4 T^4}{h^3 c^3 k_0^3} \; \frac{\pi^4}{15}$$

$$\frac{8}{15} \frac{\pi^8}{h^3 c^3} \frac{k^4 T^4}{\pi^3 / L^3}$$

$$\frac{8}{15} \frac{\pi^5 L^3 k^4 T^4}{h^3 c^3}$$

$\hbar = \frac{h}{2\pi}$

$h = 4.\!\!\ldots$

$$u = \frac{U}{L^3} = \underline{\frac{8}{15} \frac{\pi^5 k^4}{h^3 c^3} T^4}$$

P+W Lec 13.2 Section 13.1

$$I = \frac{c}{4} \cdot u$$

I = total intensity of radiation flowing outward

c from speed of light moving away

$\frac{1}{2}$: half energy travelling away, half towards

$\frac{1}{2}$ energy from (eg x-dir) is dotted with the ...

"intensity observed along sphere will be actual intensity times cos θ"

So

$$I = \frac{2}{15} \frac{\pi^5 k_B^4}{h^3 c^2} T^4$$

$\sigma = 5.67 \cdot 10^{-8} \frac{W}{m^2 K^4}$ Stefan Boltzmann constant

correction factor emissitivity (from backwards surface reflections)

$I = \epsilon \sigma T^4$ Stefan-B Law

How much of u is at each frequency? ρ_f energy density in f ie $u = \int \rho_f \, df$.

Go back to integral

$$u = 2 \frac{hc}{2\pi} k_0 \; \frac{1}{8} \int_0^\infty \frac{s}{e^{\frac{hck_0 s}{2\pi kT}} - 1} 4\pi s^2 \, ds$$

Go back to fundstuff: substitute in $f = \frac{c}{2\pi} k_0 s$

Simplify ... $u = \int_0^\infty \rho_f \, df$

with $\boxed{\rho_f = \frac{8\pi h f^3}{c^3 (e^{hf/kT} - 1)}}$

"Plank radiation law" aka blackbody spectrum