Fresnel’s Equation Examples, solving for n in crystals

by Dr. Colton, Physics 471 (last updated: Winter 2024)

Fresnel’s equation to find index of refraction for wave travel in crystals is this:

\[
\frac{1}{n^2} = \frac{u_x^2}{n^2 - n_x^2} + \frac{u_y^2}{n^2 - n_y^2} + \frac{u_z^2}{n^2 - n_z^2}
\]

where \((u_x,u_y,u_z)\) are components of the unit vector in the direction of travel and \((n_x,n_y,n_z)\) are the refractive index parameters with the coordinate system aligned to the principle axes for which the susceptibility tensor is diagonal.

Example 1.  \(n_x=1.2, n_y = 1.3, n_z = 1.4\). What are the two \(n\) values for wave travel in the \((1,1,1)\) direction?

\[
\text{In[1]} := \text{Solve}\left[\frac{1}{n^2} = \frac{1}{3} \left(\frac{1}{n^2 - 1.2^2} + \frac{1}{n^2 - 1.3^2} + \frac{1}{n^2 - 1.4^2}\right), n\right]
\]

\[
\text{Solve}: \text{Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.}
\]

\[
\text{Out[1]} := \left\{\left\{n \rightarrow 1.35397\right\}, \left\{n \rightarrow 1.23836\right\}, \left\{n \rightarrow 1.23836\right\}, \left\{n \rightarrow 1.35397\right\}\right\}
\]

\[
\text{In[2]} := \text{Plot}\left[\left\{\frac{1}{n^2} , \frac{1}{3} \left(\frac{1}{n^2 - 1.2^2} + \frac{1}{n^2 - 1.3^2} + \frac{1}{n^2 - 1.4^2}\right)\right\}, \\{n, 1.1, 1.5\}, \text{PlotRange} \rightarrow \{-10, 10\}\right]
\]

\[
\text{Out[2]} = \text{Plot}\left[\left\{\frac{1}{n^2} , \frac{1}{3} \left(\frac{1}{n^2 - 1.2^2} + \frac{1}{n^2 - 1.3^2} + \frac{1}{n^2 - 1.4^2}\right)\right\}, \\{n, 1.1, 1.5\}, \text{PlotRange} \rightarrow \{-10, 10\}\right]
\]
Example 2. nx=1.2, ny = 1.3, nz = 1.4. What are the two n values for wave travel in the (1,0,0) direction?

Example 3. nx=1.2, ny = 1.3, nz = 1.4. What are the two n values for wave travel in the (1,1,0) direction?
\( n[8] = \text{Plot}\left[\left\{\frac{1}{n^2}, \frac{1}{2} / (n^2 - 1.2^2) + \frac{1}{2} / (n^2 - 1.3^2)\right\},\right.\right.
\left.\{n, 1.1, 1.5\}, \text{PlotRange} \rightarrow \{-10, 10\}\right]\)

\( n[8] = \text{FindRoot}\left[\frac{1}{n^2} = \frac{1}{2} / (n^2 - 1.2^2) + \frac{1}{2} / (n^2 - 1.3^2), \{n, 1.25\}\right]\)

\( n[9] = \text{Solve}\left[\frac{1}{n^2} = \frac{10000}{20001} / (n^2 - 1.2^2) + \frac{10000}{20001} / (n^2 - 1.3^2) + \frac{1}{20001} / (n^2 - 1.4^2), n\right]\)

\( n[10] = \{\{n \rightarrow -1.39999\}, \{n \rightarrow -1.247\}, \{n \rightarrow 1.247\}, \{n \rightarrow 1.39999\}\}\)