### Polarization: Jones and Stokes

<table>
<thead>
<tr>
<th>LIGHT</th>
<th>Jones</th>
<th>Stokes</th>
</tr>
</thead>
<tbody>
<tr>
<td>unpolar</td>
<td>[\begin{pmatrix} 1 \ 0 \ 0 \end{pmatrix}]</td>
<td>[\begin{pmatrix} 1 \ 0 \ 0 \end{pmatrix}]</td>
</tr>
<tr>
<td>Horiz</td>
<td>[\begin{pmatrix} 1 \ 0 \ 0 \end{pmatrix}]</td>
<td>[\begin{pmatrix} 1 \ 0 \ 0 \end{pmatrix}]</td>
</tr>
<tr>
<td>Vert</td>
<td>[\begin{pmatrix} 0 \ 1 \ 0 \end{pmatrix}]</td>
<td>[\begin{pmatrix} 0 \ 1 \ 0 \end{pmatrix}]</td>
</tr>
<tr>
<td>45°</td>
<td>[\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \ 1 \ 0 \end{pmatrix}]</td>
<td>[\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \ 1 \ 0 \end{pmatrix}]</td>
</tr>
<tr>
<td>-45°</td>
<td>[\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \ -1 \ 0 \end{pmatrix}]</td>
<td>[\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \ -1 \ 0 \end{pmatrix}]</td>
</tr>
</tbody>
</table>

#### Linear at θ

\[
\begin{pmatrix} \cos θ \\ -\sin θ \end{pmatrix}
\]

#### RCP

\[
\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}
\]

#### LCP

\[
\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}
\]

#### Elliptical

\[
\begin{pmatrix} A e^{i\delta} \end{pmatrix}
\]

\[
\begin{pmatrix}
\frac{A^2 + B^2}{2} & 0 \\
0 & \frac{A^2 - B^2}{2}
\end{pmatrix}
\]

\[\delta\text{ is the phase shift of vert relative to horiz}\]

#### OPTIC

<table>
<thead>
<tr>
<th>Jones</th>
<th>Mueller (Stokes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horiz linear</td>
<td>[\begin{pmatrix} 1 &amp; 0 \ 0 &amp; 0 \end{pmatrix}]</td>
</tr>
<tr>
<td>Vert linear</td>
<td>[\begin{pmatrix} 0 &amp; 0 \ 0 &amp; 1 \end{pmatrix}]</td>
</tr>
<tr>
<td>45° linear</td>
<td>[\begin{pmatrix} 1 &amp; 1 \ 2 &amp; 1 \end{pmatrix}]</td>
</tr>
<tr>
<td>-45° linear</td>
<td>[\begin{pmatrix} 1 &amp; -1 \ 2 &amp; -1 \end{pmatrix}]</td>
</tr>
</tbody>
</table>

\[\begin{pmatrix}
\cos θ & \sin θ \\
\sin θ & \cos θ
\end{pmatrix}
\]

\[\begin{pmatrix}
\cos 2θ & \sin 2θ \\
\sin 2θ & \cos 2θ
\end{pmatrix}
\]

\[\begin{pmatrix}
\cos 2θ & \sin 2θ \\
\sin 2θ & -\cos 2θ
\end{pmatrix}
\]

\[\begin{pmatrix}
\cos 4θ & 0 \\
0 & \cos 4θ
\end{pmatrix}
\]

\[\begin{pmatrix}
\cos 4θ & \sin 4θ \\
\sin 4θ & -\cos 4θ
\end{pmatrix}
\]

\[\begin{pmatrix}
\cos 4θ & 0 \\
0 & -\cos 4θ
\end{pmatrix}
\]

#### Jones:

**Angle of elliptical:** \[\alpha = \frac{1}{2}\tan^{-1}\left(\frac{2AB \cos δ}{A^2 - B^2}\right)\]

\[E_s = |E_{eff}| \sqrt{A^2 \cos^2 α + B^2 \sin^2 α + AB \cos δ \sin 2α}\]

\[E_{e\pm\theta} = |E_{eff}| \sqrt{A^2 \sin^2 α + B^2 \cos^2 α - AB \cos δ \sin 2α}\]

#### Stokes:

**Amount polarized:** \[\sqrt{t_1^2 + t_2^2 + t_3^2}\]

**Amount unpolarized:** \[\sqrt{t_1^2 + t_2^2 + t_3^2}\]

**Deg of polarization:** \[\frac{t_1^2 + t_2^2 + t_3^2}{t_1^2}\]

### Notes

The two quarter wave Jones matrices as given in the Wikipedia article [https://en.wikipedia.org/wiki/Jones_calculus](https://en.wikipedia.org/wiki/Jones_calculus) are slightly different than those here because there is an additional overall phase shift included there. I think Peatross & Ware’s equations are better and am using them here.

I’m convinced the Wikipedia article on Mueller matrices, [https://en.wikipedia.org/wiki/Mueller_calculus](https://en.wikipedia.org/wiki/Mueller_calculus), has a wrong equation for the “General Linear Retarder” matrix. It doesn’t reproduce the matrices given for quarter and half wave plates (which I do think are correct). On the other hand, I found a similar equation in a book by Gil and Ossikovski, *Polarized Light and the Mueller Matrix Approach*, which gives a similar equation on page 171, Eq (4.29) but differs from the Wikipedia equation by a few negative signs. It correctly reproduces the quarter and half wave plates matrices found both on Wikipedia and the additional ones given in this article on a Mueller matrix Python module, [https://mvropol.readthedocs.io/en/latest/06-Mueller-Matrices.html](https://mvropol.readthedocs.io/en/latest/06-Mueller-Matrices.html), so I trust it. I used that equation to generate the \[\lambda/4\text{ fast axis angle }\theta\] and \[\lambda/2\text{ fast axis angle }\theta\] equations.

I found three trustworthy references which all said that the Mueller R matrix given here should work like the Jones R matrix, namely \[M_{\text{angle}} = RM_{\text{horizontal}}R^{-1}\]. However, when I tested that equation against known results, it didn’t reproduce them. I was not able to determine what is going wrong with that. The references are the Gil and Ossikovski book; this paper Nee, “Decomposition of Jones and Mueller matrices in terms of four basic polarization responses”, J. Opt. Soc. Am. A 31, 2518 (2014) [http://dx.doi.org/10.1364/JOSAA.31.002518](http://dx.doi.org/10.1364/JOSAA.31.002518); and this website [https://www.fiber optic cables.com/encyclopedia/wave-optics/10470330-mueller-matrices-for-polarizing-elements](https://www.fiber optic cables.com/encyclopedia/wave-optics/10470330-mueller-matrices-for-polarizing-elements) (Note they all defined R using the opposite angle convention than I used so they give the equation as \[M_{\text{angle}} = R^{-1}MR\text{horizontal}\] but that doesn’t explain the issue.)