What You Should Already Know About Optics

by Dr. Colton (last updated: Winter 2024)

From Phys 123 (some of this only in the majors section)

Complex numbers:

Euler's identity: $e^{ix} = \cos x + i \sin x$

Complex numbers as points in the complex plane; polar ↔ rectangular conversion

General wave properties:

what all of these parameters mean: x, t, A, λ, f, v, k, ω, ϕ

$f = A\cos(kx - \omega t + \phi) \leftrightarrow Ae^{i(kx - \omega t + \phi)}$

(and how to extend that to 3D for arbitrary wave direction and arbitrary oscillation direction)

$k = 2\pi/\lambda$; $\omega = 2\pi/T$

$v = \lambda f$

wave packets: $v_{phase} = \omega/k$; $v_{group} = \left(\frac{\partial\omega}{\partial k}\right)_{k_{ave}}$

Uncertainty relationships:

$\Delta x \Delta k \geq ½$; $\Delta x \Delta p \geq \hbar/2$

$\Delta t \Delta \omega \geq ½$; $\Delta t \Delta E \geq \hbar/2$

Reflection/transmission coefficients at normal incidence:

$r = \frac{v_2 - v_1}{v_2 + v_1} = \frac{n_1 - n_2}{n_1 + n_2}$; $t = \frac{2v_2}{v_2 + v_1} = \frac{2n_1}{n_1 + n_2}$

$R = |r|^2$; $T = 1 - R$

Fourier series:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{2\pi nx}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{2\pi nx}{L}$$

($2\pi/L = k_0$ = fundamental [spatial] frequency)

$$a_0 = \frac{1}{L}\int_0^L f(x)dx$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{2\pi nx}{L}dx$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{2\pi nx}{L}dx$$

Fourier series in time: $x \to t$; $L \to T$; $k_0 \to \omega_0$

Index of refraction, n

speed of light = c/n

$\lambda_{\text{material}} = \lambda_{\text{vacuum}}/n$

Laws of reflection/refraction:

$\theta_{incident} = \theta_{reflected}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ (θ measured from the perpendicular)

Total internal refraction: $\theta_{critical}$ of high index material is when $\theta_2 = 90°$

Polarization

Difference between linear and circular polarization

$\theta_{\text{Brewster}} = \tan^{-1}(\theta_2/\theta_1)$

Difference between *s*- and *p*-polarization

Lenses/mirrors

Thin lens equation: $1/f = 1/p + 1/q$

Mirror: $f = R/2$

Lensmaker's eqn: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

(R_1 = pos, R_2 = neg if convex-convex)

magnification: $M = h_i/h_o = -q/p$

f-number of a lens = f/D

Diffraction through slits/apertures:

Phase difference due to path-length difference: $\phi = 2\pi(\Delta PL/\lambda)$
Parallel ray approximation, if screen distance >> slit separation: $\Delta PL = d\sin\theta$ (d = distance to reference of phase)
Field at location on screen is sum of fields from each slit: $E = E_0\,(e^{i\phi 1} + e^{i\phi 2} + \ldots)$ (integrate if needed)
Intensity $I \sim |E|^2$
2 slit result: $I = I_0 \cos^2\left(\frac{2\pi}{\lambda}\frac{d}{2}\sin\theta\right)$; $d\sin\theta = m\lambda$ (maxima); $d\sin\theta = (m + ½)\lambda$ (minima)
1 wide slit result: $I = I_0\,\mathrm{sinc}^2\left(\frac{\pi a \sin\theta}{\lambda}\right)$; $a\sin\theta = m\lambda$ (minima)
2 wide slits result: I = (2 slit result) × (1 wide slit result)
Arbitrary number/arrangement of slits: how to apply this technique to get $I(\theta)$
Small angle approximation sometimes applies: $\theta \approx \sin\theta \approx \tan\theta = y/L$
Circular aperture result, Rayleigh criterion: $\theta_{\text{min.resolve}} = 1.22\lambda/D$
Grating result: $d\sin\theta_{\text{bright}} = m\lambda$
Spectrometer: $R = \lambda_{ave}/\Delta\lambda$ = #slits × m

Thin film interference:
OPL = PL × n (PL = "path length"; OPL = "optical path length")
ΔOPL + other phase shifts = $m\lambda$ (constructive); … = $(m + ½)\lambda$ (destructive)

Photons: (possibly not learned until Phys 222)
photon momentum $p = h/\lambda$
photon energy $E = pc = hc/\lambda$

From Phys 220

Coulomb's Law:
$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,\hat{\mathbf{r}} = \frac{1}{4\pi\epsilon_0}\frac{q\,\mathbf{r}}{r^3}$ (electric field from a point charge located at origin)
$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q\,(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$ (electric field from a point charge located at $\mathbf{r}'$)

Biot-Savart Law:
$\mathbf{B} = \frac{\mu_0}{4\pi}\int \frac{I\,d\boldsymbol{\ell}' \times (\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$ (magnetic field from a current-carrying wire; integrate over the primed variables)

Gauss's Law (Maxwell #1):
$\oint_S \mathbf{E}\cdot d\mathbf{a} = \frac{q_{enc}}{\varepsilon_0}$ (electric flux is proportional to q_{enclosed})

Gauss's Law for magnetism (Maxwell #2):
$\oint_S \mathbf{B}\cdot d\mathbf{a} = \frac{q_{enc}}{\varepsilon_0}$ (no magnetic monopoles)

Faraday's Law (Maxwell #3):
$\oint \mathbf{E}\cdot d\boldsymbol{\ell} = -\frac{d\Phi_{\mathrm{B}}}{dt}$ (induced EMF is –d(flux)/dt; minus sign is Lenz's Law)

Ampere's Law, with Maxwell correction (Maxwell #4):
$\oint \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0 I_{enc} + \epsilon_0\mu_0\frac{d\Phi_{\mathrm{E}}}{dt}$ (currents act as sources of magnetic fields; so do changing electric fields)

From Multivariable Calculus

Scalar and vector functions:
f = a scalar function of x, y, z. Example: $f(x,y,z) = x^2y + \sin z$.
$\mathbf{A}$ = a vector function of x, y, z. Example: $\mathbf{A}(x,y,z) = (x^2y + \sin z)\hat{\mathbf{x}} + xyz\hat{\mathbf{y}} + 4\hat{\mathbf{z}}$,
which means $A_x = x^2y + \sin z$, $A_y = xyz$, and $A_z = 4$

Gradient of a scalar function $\nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$ (which is a vector function)

Divergence of a vector function $\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$ (which is a scalar function)

Curl of a vector function $\nabla\times\mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$ (which is a vector function)

Laplacian of scalar function $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$ (which is a scalar function)
Laplacian of vector function $\nabla^2 \mathbf{A} = \nabla^2 \mathrm{A}_x \hat{\mathbf{x}} + \nabla^2 \mathrm{A}_y \hat{\mathbf{y}} + \nabla^2 \mathrm{A}_z \hat{\mathbf{z}}$ (which is a vector function)
Handy "curl of curl" formula: $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$
Gradient Theorem:

$$\int_{\substack{path\ from \\ \mathbf{a}\ to\ \mathbf{b}}} (\nabla f) \cdot d\boldsymbol{\ell} = f(\mathbf{b}) - f(\mathbf{a})$$

Divergence Theorem:

$$\int_{volume} \nabla \cdot \mathbf{A}\, dv = \oint_{\substack{surface\ bounding \\ the\ volume}} \mathbf{A} \cdot d\mathbf{a}$$

Stokes' Theorem, aka Curl Theorem:

$$\int_{surface} (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint_{\substack{path\ bounding \\ the\ surface}} \mathbf{A} \cdot d\boldsymbol{\ell}$$